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An analytical model for predicting sheet springback after V-bending

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Abstract: Springback is caused by the redistribution of stress in sheet material after the tooling is removed. Precise prediction of sheet springback is very important in die design. Based on Hill's yielding criterion and plane strain condition, an analytical model is proposed in this paper which takes into account the effects of contact pressure, the length of bending arm between the punch and die, transverse stress, neutral surface shifting and sheet thickness thinning on the sheet springback of V-bending. The predicted results by this analytical model indicated that the contact pressure and transverse stress have much effect on the springback when the bending ratio (the ratio of punch radius to sheet thickness) is less than five. The contact pressure declined when the length of bending arm goes up, which means that shorter length of bending arm will result in larger springback. The effect of neutral surface shifting on the springback is less than that of contact pressure and decreases with the bending ratio. However, this research showed that the influence of thickness thinning on the springback can be ignored. Comparison with finite element method (FEM) simulating results shows that the predicted results by the analytical model accord well with simulation results by FEM. In addition to that, the bending ability—the limit bending ratio for a given sheet thickness and material properties was also determined.

Key words: Springback, V-bending, Contact pressure, Neutral surface shifting, Transverse stress, Bending ratio
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INTRODUCTION

As an important manufacturing method, bending has been widely used in modern industries to produce stamping parts such as frames, channels, braces, brackets and other structural parts. The understanding and development of bending mechanics are aimed at achieving two kinds of information which are very important for industrial production. One is to predict springback for dies design and compensation in order to obtain high dimension accuracy of bending parts. The other is to determine the limit bending ratio R_i/t_0 for a given sheet thickness and material properties.

Different methods such as analytical method, semi-analytical method and finite element method (FEM) have been applied to analyze the bending process (Gerdeen and Duncan, 1986; Huang and Gerdeen, 1994). FEM is a time-consuming method and also is very sensitive to numerical parameters such as element type and size, algorithms, contact definition and convergence criterion for solution, etc. (Li and Wagoner, 1998; Hamouda *et al*., 2004). Analytical method is a time-saving method and has been widely used for predicting springback of bending parts. But most of these researches ignored the effects of contact pressure, transverse stress, neutral surface shifting and thickness thinning on sheet springback of bending parts. Gardiner (1957) presented a mathematical model to calculate the springback of pure bending for perfect plastic material. Johnson and Yu (1981a; 1981b; 1982) further developed Gardiner's work on tension-bending problem, using linearly and exponentially hardening material model. Based on plane strain condition, Wang *et al*.(1993) established a mathematical model for predicting the springback, bend ability, strain and stress distributions, and maximum loads on the punch for air-bending problem. By using three rules for material hardening (kinematic hardening, isotropic hardening and directional hardening), Zhang and Hu (1998) developed a mathematical model for predicting sheet springback of bending and calculated residual stress distribution through thickness after springback. According to exponential hardening law and Von Mises yield criterion, Buranathiti and Cao (2004) proposed an analytical model for predicting springback after sheet straight flanging. However, since these researches ignored the effects of neutral surface shift and contact pressure between the sheet and die, they were limited to the springback problem of bending ratio $R_i/t_0 \geq 5$. For the bending ratio $R_i/t_0 \geq 5$, the effects of contact pressure, transverse stress, neutral surface shifting on sheet springback of bending are significant, with the analysis models which ignore them being inaccurate as the bending ratio decreases. Hill (1950) proposed a bending theory in which the effect of transverse stress was considered, but applied the ideal elastic-plastic material model, which cannot reflect the real material property. Chen (1962) extended Hill's work to exponential hardening model, but he ignored the effects of bending arm and contact pressure. Robinson (2000) estimated the errors introduced by the ignorance of transverse stress and concluded that the transverse stress was an important factor in the bending analysis. And the contact pressure is another factor ignored by previous analysis (Tekaslan *et al*., 2006; Tekiner, 2004). By using a simplified method in which the transverse stress of shell element is considered, Cho *et al*.(2002) concluded that transverse stress induced by contact pressure has much influence on the sheet metal forming and springback analysis.

An analytical model is proposed in this paper to predict the sheet springback of V-bending. This model is based on Hill's yielding criterion and plane strain condition, and takes into account contact pressure, transverse stress, neutral surface shifting and sheet thickness thinning. The effects of contact pressure, the length of bending arm between the punch and die, neutral surface shifting and sheet thickness thinning on sheet springback were studied. The predicted results by the analytical model were compared with FEM simulating results. And the calculation of bending ability—the limit bending ratio (R_i/t_0) for a given sheet metal thickness and material properties is also presented in this paper.

ANALYSIS OF SHEET BENDING

The round corner of sheet V-bending can be considered as bending under the actions of contact pressure $\boldsymbol{q}$ and bending moment $\boldsymbol{M}$ as shown in Fig.1. The following assumptions are applied: (1) The sheet is wide enough relative to its thickness. Therefore the strain in the width direction is zero; (2) Straight lines perpendicular to the neutral surface remain straight during the V-bending process; (3) Volume conservation is kept during V-bending process; (4) Bauschinger effect is neglected and only elastic deformation occurs during the unloading process.

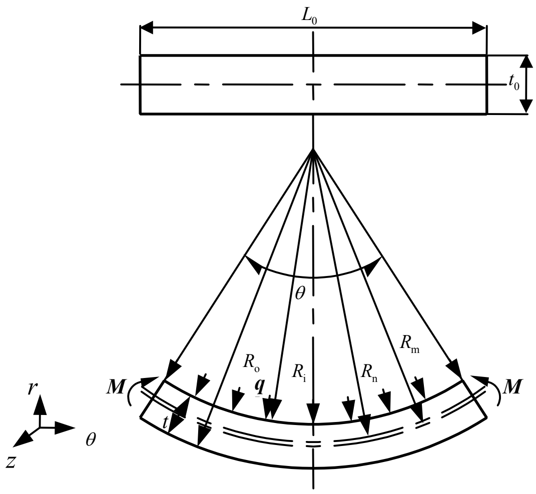


Fig.1 The scheme of sheet bending

Sheet thinning and neutral surface shifting after V-bending

According to Assumptions (1) and (3), the area of sheet cross-section remains constant during V-bending process, that is:

$$L_0 t_0 = (R_o^2 - R_i^2)\theta / 2, \tag{1}$$

where L_0, t_0 are initial sheet length and thickness, respectively; θ is the bending angle; R_i, R_o are the radii of concave and convex surface.

Since the neutral layer length remains constant during V-bending, the radius of neutral surface R_n can be defined as:

$$R_n = L_0 / \theta. \tag{2}$$

Substitution of Eq.(2) into Eq.(1) yields:

$$R_m / R_n = t_0 / t, \tag{3}$$

where t is the sheet thickness after V-bending; R_m is the radius of middle surface and can be written as:

$$R_m = (R_i + R_o)/2.$$

Assuming the distribution of tangential strain through thickness is:

$$\varepsilon_\theta = \begin{cases} (r - R_n)/R_n, & |r - R_n| < c; \\ \ln(r/R_n) = \ln(r/R_m) + \ln(R_m/R_n), & |r - R_n| \ge c, \end{cases} \tag{4}$$

where ε_θ is the tangential strain; r is the radius of the studied bending layer; c is half thickness of elastic region.

Substitution of Eq.(3) into Eq.(4) yields:

$$\varepsilon_\theta = \begin{cases} (r - R_n)/R_n, & |r - R_n| < c; \\ \ln(r/R_n) = \ln(r/R_m) + \ln(t_0/t), & |r - R_n| \ge c. \end{cases} \tag{5}$$

It can be seen from Eq.(5) that the tangential strain in the plastic region has two parts. One is bending strain under the condition that the sheet thickness is not changed. The other is unique strain through thickness caused by sheet thickness thinning.

Based on Hill's bending analysis with ideal elastic-plastic material, the neutral layer radius can be expressed as:

$$R_n = \sqrt{R_i R_o}. \tag{6}$$

It means that the neutral surface does not coincide with the middle surface and shifts to the concave surface during V-bending.

From Eqs.(1), (2) and (6), a relationship of sheet thickness before and after V-bending can be established:

$$4R_i(R_i + t)t_0^2 = (2R_i + t)^2 t^2. \tag{7}$$

The sheet thickness t can be solved from Eq.(7). Substituting t into Eqs.(1), (2) and (3) yields the values of θ, R_i, R_o and R_n.

Calculation of stress, strain and bending moment

According to Hill's quadratic yielding function, for anisotropic sheets, the material yields when

$$\begin{aligned} &\varphi^2(\sigma_{ij}, \varepsilon_{ij}) \\ &= \sigma_\theta^2 + \sigma_z^2 - \frac{2\bar{r}}{1+\bar{r}}\sigma_\theta\sigma_z + \frac{2}{1+\bar{r}}(\sigma_r^2 - \sigma_\theta\sigma_r - \sigma_z\sigma_r) - \bar{\sigma}^2 \\ &= 0, \end{aligned} \tag{8}$$

where σ_θ, σ_z, σ_r are the tangential, width and transverse stresses, respectively; $\bar{\sigma}$, $\bar{\varepsilon}$ are equivalent stress and strain; $\bar{r}$ is the transverse anisotropy coefficient; φ is yielding function.

Because the plastic strain in transverse direction $\varepsilon_z^p=0$, so that

$$d\varepsilon_z^p = d\lambda \frac{\partial\varphi}{\partial\sigma_z} = 0.$$

Combined with Eq.(8), σ_z can be expressed as:

$$\sigma_z = (\bar{r}\sigma_\theta + \sigma_r)/(1+\bar{r}), \tag{9}$$

where $d\lambda$ is a coefficient relative to a material hardening rule (Wang and Shao, 1997).

Substitution of Eq.(9) into Eq.(8) yields:

$$\bar{\sigma} = \frac{\sqrt{1+2\bar{r}}}{1+\bar{r}}|\sigma_\theta - \sigma_r| = \begin{cases} (\sigma_\theta - \sigma_r)/f, & R_n + c \le r \le R_o; \\ (\sigma_r - \sigma_\theta)/f, & R_i \le r \le R_n - c. \end{cases} \tag{10}$$

From plastic work formulation, the equivalent strain is:

$$\bar{\varepsilon} = \begin{cases} f\varepsilon_\theta, & R_n + c \le r \le R_o; \\ -f\varepsilon_\theta, & R_i \le r \le R_n - c, \end{cases} \tag{11}$$

where $f = (1+\bar{r})/\sqrt{1+2\bar{r}}$, which is related to the transverse anisotropy in plane strain condition.

Assuming that the plastic region material follows exponential strain hardening law, namely:

$$\bar{\sigma} = k(\varepsilon_0 + \bar{\varepsilon})^n, \tag{12}$$

where k, n are the hardening coefficient and exponent; ε_0 is the initial strain.

Substitution of Eqs.(11) and (12) into Eq.(10)

yields:

$$\boldsymbol{\sigma}_\theta-\boldsymbol{\sigma}_r=\begin{cases} fk\left(\varepsilon_0+f\ln(r/R_\mathrm{n})\right)^n, & R_\mathrm{n}+c\le r\le R_\mathrm{o}; \\ -fk\left(\varepsilon_0-f\ln(r/R_\mathrm{n})\right)^n, & R_\mathrm{i}\le r\le R_\mathrm{n}-c. \end{cases} \tag{13}$$

For the layer of bending radius equaling to $R_\mathrm{n}+c$, the sheet metal was just yielded:

$$k\left(\varepsilon_0+f\ln\frac{R_\mathrm{n}+c}{R_\mathrm{n}}\right)^n=\bar{\boldsymbol{\sigma}}_\mathrm{s}.$$

where $\bar{\boldsymbol{\sigma}}_\mathrm{s}$ is the initial yielding stress, and c can be solved from the above equation.

Then the transverse stress in this layer can be expressed as:

$$\boldsymbol{\sigma}_r=\boldsymbol{\sigma}_\theta-fk\left(\varepsilon_0+f\ln\frac{R_\mathrm{n}+c}{R_\mathrm{n}}\right)^n, \tag{14}$$

If the elastic region material follows Hooke's law, the tangential stress in just yielding layer can be written as follows:

$$\boldsymbol{\sigma}_\theta=\frac{E}{1-\nu^2}\frac{c}{R_\mathrm{n}}+\frac{\nu}{1-\nu}\boldsymbol{\sigma}_r. \tag{15}$$

Substitution of Eq.(15) into Eq.(14) yields:

$$\boldsymbol{\sigma}_r=\frac{E}{(1+\nu)(1-2\nu)}\frac{c}{R_\mathrm{n}}-\frac{1-\nu}{1-2\nu}fk\left(\varepsilon_0+f\ln\frac{R_\mathrm{n}+c}{R_\mathrm{n}}\right)^n. \tag{16}$$

Assuming that σ_r is constant in the elastic region, the distribution of tangential stress $\boldsymbol{\sigma}_\theta$ through thickness in $R_\mathrm{n}-c\le r\le R_\mathrm{n}+c$ region can be expressed as:

$$\boldsymbol{\sigma}_\theta=E_1\frac{r-R_\mathrm{n}}{R_\mathrm{n}}+E_1\frac{\nu}{1-2\nu}\frac{c}{R_\mathrm{n}}-\frac{\nu}{1-2\nu}fk\left(\varepsilon_0+f\ln\frac{R_\mathrm{n}+c}{R_\mathrm{n}}\right)^n, \tag{17}$$

where ν is Poisson's ratio; E, E_1 are the Young's modulus and the corresponding Young's modulus in plane strain condition ($E/(1-\nu^2)$) respectively.

The stress equilibrium equation during V-bending progress is:

$$\frac{\mathrm{d}\boldsymbol{\sigma}_r}{\mathrm{d}r}=\frac{\boldsymbol{\sigma}_\theta-\boldsymbol{\sigma}_r}{r}. \tag{18}$$

Substituting Eq.(13) into Eq.(18) and integrating, according to the boundary conditions of $\boldsymbol{\sigma}_r=-\boldsymbol{q}$ at $r=R_\mathrm{i}$ and $\boldsymbol{\sigma}_r=0$ at $r=R_\mathrm{o}$, the distributions of σ_r and σ_θ in plastic region are defined as follows:

(1) For $R_\mathrm{n}+c\le r\le R_\mathrm{o}$,

$$\begin{cases} \boldsymbol{\sigma}_r=\dfrac{k}{n+1}\left(\varepsilon_0+f\ln\dfrac{r}{R_\mathrm{n}}\right)^{n+1}-\dfrac{k}{n+1}\left(\varepsilon_0+f\ln\dfrac{R_0}{R_\mathrm{n}}\right)^{n+1}, \\ \boldsymbol{\sigma}_\theta=\boldsymbol{\sigma}_r+fk\left(\varepsilon_0+f\ln\dfrac{r}{R_\mathrm{n}}\right)^n. \end{cases}$$

(2) For $R_\mathrm{i}\le r\le R_\mathrm{n}-c$,

$$\begin{cases} \boldsymbol{\sigma}_r=\dfrac{k}{n+1}\left(\varepsilon_0-f\ln\dfrac{r}{R_\mathrm{n}}\right)^{n+1}-\dfrac{k}{n+1}\left(\varepsilon_0-f\ln\dfrac{R_\mathrm{i}}{R_\mathrm{n}}\right)^{n+1}-\boldsymbol{q}, \\ \boldsymbol{\sigma}_\theta=\boldsymbol{\sigma}_r-fk\left(\varepsilon_0-f\ln\dfrac{r}{R_\mathrm{n}}\right)^n. \end{cases} \tag{19}$$

The bending moment is:

$$\boldsymbol{M}=\boldsymbol{M}_\mathrm{e}+\boldsymbol{M}_\mathrm{p}, \tag{20}$$

where $\boldsymbol{M}_\mathrm{e}$ is elastic moment and $\boldsymbol{M}_\mathrm{p}$ plastic moment, which can be calculated as:

$$\boldsymbol{M}_\mathrm{e}=\int_{R_\mathrm{n}-c}^{R_\mathrm{n}+c}(\boldsymbol{\sigma}_\theta-\boldsymbol{\sigma}_\mathrm{m})(r-R_\mathrm{m})\mathrm{d}r, \tag{21}$$

$$\boldsymbol{M}_\mathrm{p}=\int_{R_\mathrm{n}+c}^{R_0}(\boldsymbol{\sigma}_\theta-\boldsymbol{\sigma}_\mathrm{m})(r-R_\mathrm{m})\mathrm{d}r+\int_{R_\mathrm{i}}^{R_\mathrm{n}-c}(\boldsymbol{\sigma}_\theta-\boldsymbol{\sigma}_\mathrm{m})(r-R_\mathrm{m})\mathrm{d}r, \tag{22}$$

where $\boldsymbol{\sigma}_\mathrm{m}$ is the tangential stress in sheet middle surface.

Calculation of contact pressure

Assuming that the V-bending arm remains straight before and after springback, the pressure acting on it can be ignored. Because of the symmetry of the V-bending part, one-half model is analyzed.

And the force applied on the V-bending part during forming is shown in Fig.2a, where point c is support point. Fig.2b shows the moment equilibrium at support point b. The reaction force $\boldsymbol{N}$ at point c can be solved as:

$$\boldsymbol{N} = \boldsymbol{M}_b / L, \tag{23}$$

where $\boldsymbol{M}_b$ is the bending moment at cross-section b; L is the length of bending arm between the punch and die.

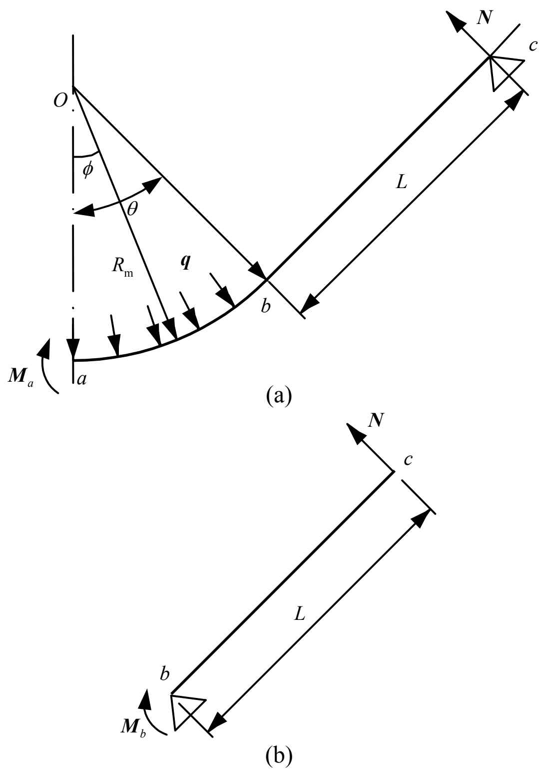


Fig.2 The moment equilibrium scheme of sheet V-bending. (a) Moment equilibrium at support point *a*; (b) Moment equilibrium at support point *b*

The moment equilibrium at support point a as shown in Fig.2a can be expressed as:

$$\boldsymbol{M}_a + \boldsymbol{M}_q = \boldsymbol{M}_N, \tag{24}$$

where $\boldsymbol{M}_a$ is the bending moment at cross-section a; $\boldsymbol{M}_N$, $\boldsymbol{M}_q$ are the moments caused by reaction force $\boldsymbol{N}$ and contact pressure $\boldsymbol{q}$, which are calculated as follows:

$$\boldsymbol{M}_N = \boldsymbol{N}(L + R_n \sin\theta), \tag{25}$$

$$\boldsymbol{M}_q = \int_0^\theta R_n \sin\phi \cdot \boldsymbol{q} \cdot R_n \mathrm{d}\phi. \tag{26}$$

Assuming that the contact pressure along the bending surface $\overset{\frown}{ab}$ follows linear distribution:

$$\boldsymbol{q} = \boldsymbol{q}_0 \frac{\overset{\frown}{ab} - x}{\overset{\frown}{ab}} = \boldsymbol{q}_0 \frac{\theta - \phi}{\theta}. \tag{27}$$

Substitution of Eq.(27) into Eq.(26) yields:

$$\boldsymbol{M}_q = \int_0^\theta R_n \sin\phi \cdot \boldsymbol{q}_0 \frac{\theta - \phi}{\theta} \cdot R_n \mathrm{d}\phi = \frac{\boldsymbol{q}_0 R_n^2}{\theta}(-\sin\theta + \theta),$$

The distribution of contact pressure $\boldsymbol{q}$ along $\overset{\frown}{ab}$ can be resolved by combining Eqs.(19), (20) and (24).

Calculation of springback

The non-uniform distribution of stress through sheet thickness during forming process will change the part profile and cause springback when the loading is removed. Assuming that the arm of V-bending is straight before and after springback, its springback can be ignored. Therefore the springback of sheet V-bending occurs only in the region of the bottom round corner. Since the distribution of contact pressure and bending moment $\boldsymbol{M}(\alpha)$ along the bending surface has been solved, the springback angle of sheet V-bending is:

$$\Delta\theta = 2\int_0^\theta \frac{\boldsymbol{M}(\alpha)}{E_1 \boldsymbol{I}} R_n \mathrm{d}\alpha, \tag{28}$$

where $\boldsymbol{I}$ is inertia moment of cross-section per unit width.

Limit bending ratio

The sheet bending ability is restricted by the material strain limit ($\bar{\varepsilon}_{\lim}$). Cracks will occur when the tensile strain on the outer layer of sheet reaches the strain limit. According to Eqs.(4) and (11), the tensile strain on the outer layer ε_{θ_o} is:

$$\varepsilon_{\theta_o} = \ln\frac{R_o}{R_n} = \ln\frac{R_o}{\sqrt{R_o R_i}} = \frac{1}{2}\ln\left(1 + \frac{t}{R_i}\right) \le \frac{1}{2}\ln\left(1 + \frac{t_0}{R_i}\right) \le \frac{\bar{\varepsilon}_{\lim}}{f}.$$

Therefore the limit bending ability $(R_i/t_0)_{lim}$ can be calculated as:

$$(R_i/t_0)_{lim} = (e^{2\bar{\varepsilon}_{lim}/f} - 1)^{-1}. \tag{29}$$

RESULTS AND DISCUSSIONS

The profile of sheet V-bending studied in this work is shown in Fig.3. The angles of the two side arms before and after springback are θ and θ' respectively, as shown in Fig.4. The bending angle θ is 90°. The material is an aluminum alloy (Al6111-T4), of which E is 70.5 GPa, ν is 0.346, YS is 194.1 MPa, K is 550.4 MPa, n is 0.223, t_0 is 1 mm, $\bar{r}$ is 0.894 and $\bar{\varepsilon}_{lim}$ is 0.4 respectively. The limit bending ability $(R_i/t_0)_{lim}$ is 0.9762. Therefore the radii of punch (R_i) applied in this paper are 1, 2, 3, 4, 5, 8, 10 mm, respectively.

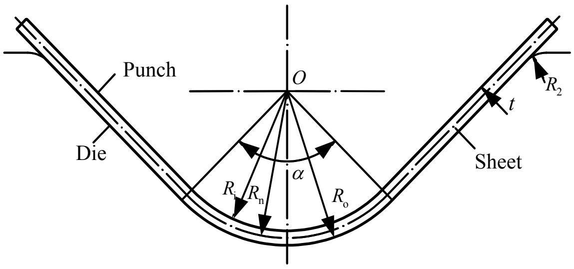


Fig.3 The benchmark scheme of sheet V-bending

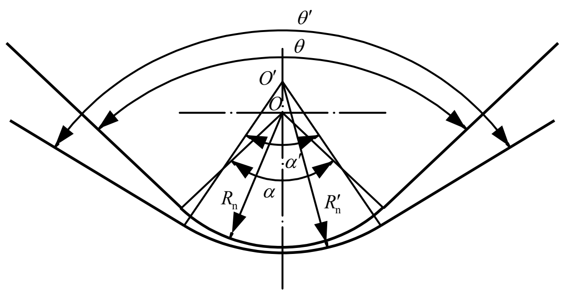


Fig.4 Angle measurement of sheet V-bending before and after springback

The effect of contact pressure on sheet springback of V-bending is summarized in Table 1 assuming that the length of the bending arm between the punch and die equals half of the punch radius. It can be seen from Table 1 that the relative error obtained by considering and without considering contact pressure decreases rapidly with increasing bending ratio (R_i/t_0). Only when the bending ratio is smaller than 5, the contact pressure has much effect on the sheet springback of V-bending. However for bending ratio $R_i/t_0 \geq 5$, its effect can be ignored.

Table 1 The effect of contact pressure on sheet springback of V-bending

R_i/t_0	$\Delta\theta_{cp}^{*}$ (°)	$\Delta\theta_{cp}^{**}$ (°)	Relative error (%)
1	2.2603	3.0555	26.025
2	3.3902	3.8719	12.441
3	4.4108	4.7411	6.967
4	5.3649	5.6103	4.374
5	6.2717	6.4642	2.978
8	8.7974	8.9107	1.272
10	10.3669	10.4547	0.840

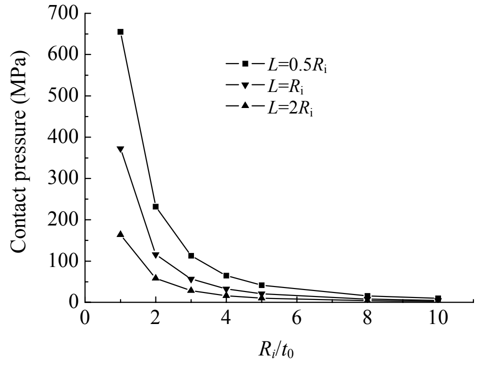
$\Delta\theta_{cp}^{*}$: Springback angle without considering contact pressure $\boldsymbol{q}$; $\Delta\theta_{cp}^{**}$: Springback angle of the analysis model proposed in this paper (L=0.5R_i)

Fig.5a and Fig.5b show the effect of bending arm length between the punch and die on contact pressure and on sheet springback of V-bending, respectively. It can be seen from Fig.5a that the shorter the bending arm length is, the larger the contact pressure is, and the more is the sheet springback since larger bending arm means less plastic deformation and should result in larger springback. And when the bending ratio is smaller than 5, the effect of bending arm length on sheet springback is prominent.

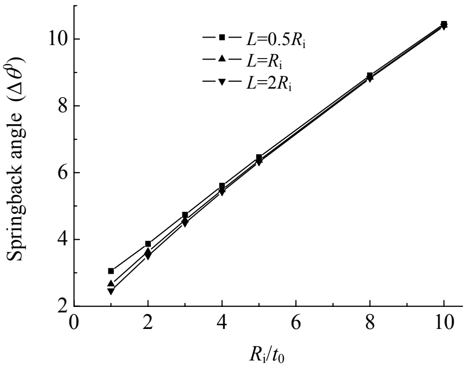
Fig.6 shows the relationship between the ratio of relative shifting amount of neutral surface to sheet thickness $(R_m-R_n)/t$ and bending ratio R_i/t_0. Table 2 gives the effect of neutral surface shifting on sheet springback of V-bending when the length of bending arm equals half of punch radius. It can be concluded that the effect of neutral surface shifting on sheet springback decreases rapidly when the bending ratio increases. And its effect is less than that of contact pressure.

Assuming that the length of the bending arm equals half of punch radius, Table 3 shows the effect of thickness thinning on sheet springback of V-bending. Table 3 indicates that ignoring of thickness thinning has little influence on sheet springback of V-bending.

When the length of the bending arm between the punch and die equals 1.6R_i, the predicting results of



(a)



(b)

Fig.5 The effect of bending arm length on contact pressure (a) and on sheet springback of V-bending (b)

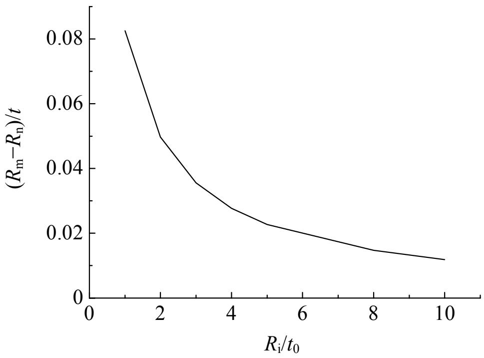


Fig.6 The relationship between $(R_m-R_n)/t$ and R_i/t_0

Table 2 The effect of neutral surface shifting on sheet springback of V-bending

R_i/t_0	$\Delta\theta_{nss}^{*}$ (°)	$\Delta\theta_{nss}^{**}$ (°)	Relative error (%)
1	3.3169	3.0555	8.560
2	3.9867	3.8719	2.960
3	4.8140	4.7411	1.540
4	5.6622	5.6103	0.925
5	6.5041	6.4642	0.617
8	8.9334	8.9107	0.254
10	10.4719	10.4547	0.164

$\Delta\theta_{nss}^{*}$: Springback angle without considering neutral surface shifting; $\Delta\theta_{nss}^{**}$: Springback angle of the analysis model proposed in this paper (L=0.5R_i)

Table 3 The effect of thickness thinning on sheet springback of V-bending

R_i/t_0	$\Delta\theta_{tt}^{*}$ (°)	$\Delta\theta_{tt}^{**}$ (°)	Relative error (%)
1	3.0107	3.0555	1.4460
2	3.8371	3.8719	0.8990
3	4.7141	4.7411	0.5690
4	5.5889	5.6103	0.3810
5	6.4467	6.4642	0.2710
8	8.8998	8.9107	0.1223
10	10.4461	10.4547	0.0820

$\Delta\theta_{tt}^{*}$: Springback angle without considering thickness thinning; $\Delta\theta_{tt}^{**}$: Springback angle of the analysis model proposed in this paper (L=0.5R_i)

Table 4 Comparison of sheet V-bending springback between the analytical model and FEM

R_i/t_0	$\Delta\theta_{am}$ (°)	$\Delta\theta_{FEM}$ (°)
1	2.5125	2.6259
2	3.5412	3.6578
3	4.5142	4.5700
4	5.4418	5.5219
5	6.3322	6.5024
8	8.8337	9.0125
10	10.3955	10.5424

$\Delta\theta_{am}$: Springback angle of the analysis model proposed in this paper (L=1.6R_i); $\Delta\theta_{FEM}$: Springback angle of FEM

sheet V-bending springback by the analytical model proposed in this paper were also compared with FEM simulating results shown in Table 4. The FEM simulation was done by ABAQUS/Standard solver. In view of the simulation result's consistency with the analytical model, the plane strain element CPE4R and exponent hardening material model were applied to the simulation. The enhancing control method of hourglass energy in ABAQUS is used to reduce the effect of hourglass energy on sheet springback. To increase the predicted accuracy, eight plane strain elements CPE4R were applied in the thickness direction to simulate the forming and springback of sheet U-bending. Table 4 indicates that the predicted results by the analytical model are in good agreement with the FEM simulating data.

CONCLUSION

An analytical model based on Hill's yielding criterion and plane strain condition was proposed for predicting sheet springback of V-bending. The effects of contact pressure, neutral surface shifting and thickness thinning on sheet springback were investigated. The predicted results by the analytical model showed that the contact pressure is related to the length of the bending arm between the punch and die. For the same bending ratio, the shorter the arm length is, the larger the contact pressure is, which results in larger sheet springback of V-bending. And the contact pressure has much more influence on sheet springback for bending ratio $R_i/t_0 \leq 5$. However for bending ratio $R_i/t_0 \geq 5$, its effect can be ignored.

The effect of neutral surface shifting on sheet springback of V-bending is less than that of contact pressure. The relative shifting amount of neutral surface to sheet thickness $(R_m - R_n)/t$ and its effect on sheet springback decreases with the bending ratio R_i/t_0 increases. And for bending ratio $R_i/t_0 \geq 5$, ignoring its effect will not result in much error.

The effect of sheet thickness thinning on sheet springback of V-bending was also discussed. Its effect is little and need not be considered.

References

Buranathiti, T., Cao, J., 2004. An effective analytical model for springback prediction in straight flanging processes. *International Journal of Materials and Product Technology*, **21**(1/2/3):137-153. [doi:10.1504/IJMPT.2004.004748]

Chen, Y.Z., 1962. Pure bending of exponential hardening wide sheet. *Acta Mechanica Sinica*, **5**:107-115 (in Chinese).

Cho, J.W., Yang, D.Y., Chung, W.J., 2002. A simplified approach for incorporating thickness stress in the analysis of sheet metal forming using shell elements. *International Journal for Numerical Methods in Engineering*, **53**(10):2311-2327. [doi:10.1002/nme.379]

Gardiner, F.J., 1957. The springback of metals. *Journal of Applied Mechanics*, **79**:1-9.

Gerdeen, J.C., Duncan, J.L., 1986. Springback in Sheet Metal Forming. AISI Report, p.1201-465.

Hamouda, A.M.S., Khadra, F.A., Hamadan, M.M., Imhemed, R.M., Mahdi, E., 2004. Springback in V-bending: A finite element approach. *International Journal of Materials and Product Technology*, **21**(1/2/3):124-136. [doi:10.1504/IJMPT.2004.004747]

Hill, R., 1950. The Mathematical Theory of Plasticity. Oxford, London.

Huang, M., Gerdeen, J.C., 1994. Springback of Doubly Curved Developable Sheet Metal Surface—An Overview. SAE Technical Paper No. 940938, p.718-728.

Johnson, W., Yu, T.X., 1981a. Springback after the biaxial elastic-plastic pure bending of a rectangular plate—I. *International Journal of Mechanical Sciences*, **23**(10):619-630. [doi:10.1016/0020-7403(81)90042-4]

Johnson, W., Yu, T.X., 1981b. On springback after the pure bending of beams and plates of elastic work-hardening materials—III. *International Journal of Mechanical Sciences*, **23**(11):687-695. [doi:10.1016/0020-7403(81)90022-9]

Li, K., Wagoner, R.H., 1998. Simulation of Springback. Proceedings of NUMIFORM'98, the Netherlands, p.21-31.

Robinson, M., 2000. An evaluation of the errors in the yield surface for a rotationally symmetric thin shell due to neglecting transverse normal stresses and shell curvature. *International Journal of Mechanical Sciences*, **42**(6):1087-1095. [doi:10.1016/S0020-7403(99)00037-5]

Tekaslan, Ö., Şeker, U., Özdemir, A., 2006. Determining springback amount of steel sheet metal has 0.5 mm thickness in bending dies. *Materials and Design*, **27**(3):251-258. [doi:10.1016/j.matdes.2004.10.006]

Tekiner, Z., 2004. An experimental study on the examination of springback of sheet metals with several thicknesses and properties in bending dies. *Journal of Materials Processing Technology*, **145**(1):109-117. [doi:10.1016/j.jmatprotec.2003.07.005]

Wang, C.T., Kinzel, G., Altan, T., 1993. Mathematical modeling of plane-strain bending of sheet and plate. *Journal of Materials Processing Technology*, **39**(3-4):279-304. [doi:10.1016/0924-0136(93)90164-2]

Wang, X.C., Shao, M., 1997. The Basic Principle and Numerical Simulation of Finite Element Method. Tsinghua University Press, Beijing (in Chinese).

Yu, T.X., Johnson, W., 1982. Influence of axial force on the elastic-plastic bending and springback of a beam. *Journal of Mechanical Working Technology*, **6**(1):5-21. [doi:10.1016/0378-3804(82)90016-X]

Zhang, Z.T., Hu, S.J., 1998, Stress and residual stress distributions in plane strain bending. *International Journal of Mechanical Sciences*, **40**(6):533-543. [doi:10.1016/S0020-7403(97)00075-1]