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Random vibration analysis of switching apparatus based on Monte Carlo method*

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Abstract: The performance in vibration environment of switching apparatus containing mechanical contact is an important element when judging the apparatus's reliability. A piecewise linear two-degrees-of-freedom mathematical model considering contact loss was built in this work, and the vibration performance of the model under random external Gaussian white noise excitation was investigated by using Monte Carlo simulation in Matlab/Simulink. Simulation showed that the spectral content and statistical characters of the contact force coincided strongly with reality. The random vibration character of the contact system was solved using time (numerical) domain simulation in this paper. Conclusions reached here are of great importance for reliability design of switching apparatus.

Key words: Contact system, Random vibration, Monte Carlo simulation, Matlab/Simulink
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INTRODUCTION

Switching apparatus, as important apparatus in today's industry, is widely used in the field of aerospace and national defense where high reliability is required. Anti-vibration performance of the switching apparatus, especially that of the contact system is the decisive element in reliability studies. The cases with nonlinear condition or random external vibration in working environment, which are difficult to analyze by using some traditional methods, can be readily and directly analyzed by random vibration method statically and by spectral method. Systemic analysis of vibration characteristics of contact system has not come into being. The nonlinear mathematical model of contact system was established and the criterion of loss contact was presented in (Zhai *et al.*, 2003; Ren *et al.*, 2006), but only the sinusoidal excitation was discussed and the results are little different from the actual situation. Different types of nonlinear model were investigated under external or parametric random excitation by using analytical methods, such as the perturbation method, the Fokker-Planck approach or stochastic averaging technique and solutions are given under special condition (Bellizzi and Bouc, 1999; Zhu *et al.*, 2004; Rigaud and Perret-Liaudet, 2003), but were either not convenient or are difficult to use for analyzing non-linear multi-degree of freedom (MDOF) mechanical systems. Some complex stochastic processes which are difficult to solve analytically are investigated by numerical method like Monte Carlo method, and statistical conditions of convergence are given (Zhu *et al.*, 2004; Perret-Liaudet and Rigaud, 2003; Wu *et al.*, 2006). Monte Carlo simulation is a technique using sampling from a random number sequence to simulate characteristics or events or outcomes with multiple possible values. Complex stochastic processes are suitably analyzed by Monte Carlo simulation which has the advantages of being simple and flexible.

A contact system mathematical model containing nonlinear (piecewise linear) contact was built. Dynamic characteristics of the model subjected to

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Gaussian white noise was analyzed with stochastic processes theory by using Monte Carlo simulation, with the statistical results and spectral results being investigated in detail. Furthermore, the relations between the input excitation level and contact loss probability, input level and spectral content were discussed. The results showed the validity of the mathematical model built.

CONTACT SYSTEM MATHEMATICAL MODEL

A model of a two-degrees-of-freedom oscillator which describes the contact system dynamic characteristics is shown in Fig.1. M_1 and M_2 denote equivalent mass of contact, δ_1 and δ_2 denote the initial deflection of the contact position, δ_0 is the initial elastic deformation between contact. The co-ordinate y_1 represents the displacement of the mass M_1 with respect to its fixed end, while y_2 stands for the displacement of the mass M_2 with respect to its fixed end. As the contact region can be assumed as an invariable rectangle, a segment linear spring with elastic coefficient k_j was used to represent the contact status of the contact system.

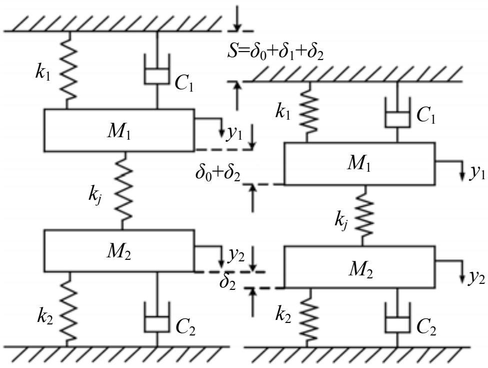

Fig.1 Two-degrees-of-freedom oscillator of contact system

If the exciting displacement is described by

$$w_{1,2}=A\sin(\Omega t+\varphi), \tag{1}$$

then we have $F_{s1}=-M_1A\Omega^2$, $F_{s2}=-M_2A\Omega^2$. Each side of the contact system is excited by a harmonic force F_{s1} and F_{s2}. By the Principle of D'Alembert, the governing equations of motion for the system are

$$\begin{cases} M_1\ddot{y}_1+c_1\dot{y}_1+k_jH(\delta_0+y_1-y_2)(\delta_0+y_1-y_2) \\ \qquad\qquad =k_1(\delta_1-y_1)+F_{s1}; \\ M_2\ddot{y}_2+c_2\dot{y}_2-k_jH(\delta_0+y_1-y_2)(\delta_0+y_1-y_2) \\ \qquad\qquad =-k_2(\delta_2+y_2)+F_{s2}, \end{cases} \tag{2}$$

where $H(x)$ is the Heaviside unit step-function defined by $H(x\geq0)=1$, $H(x<0)=0$. Suppose that $y_0=M_1A\Omega^2/k_1$, introduce the following dimensionless parameters: $\tau=\Omega t$, $\eta_i=y_i/y_0$, $\omega_i^2=k_i/M_i$, $\varpi_i=\omega_i/\Omega$, $\xi_i=c_i/(2\sqrt{k_iM_i})$, $\zeta_i=\xi_i\varpi_i$, $\omega_{ji}^2=k_j/M_i$, $\varpi_{ji}=\omega_{ji}/\Omega$, $\lambda=\varpi_1^2$, where ϖ_i is the non-dimensional frequency and ω_i (i=1,2) are the natural frequencies of the contact system. Now the dimensionless governing equations of the system motion can be rewritten as

$$\begin{cases} \ddot{\eta}_1+\zeta_1\dot{\eta}_1+H(\delta_0/y_0+\eta_1-\eta_2)\varpi_{j1}^2 \\ \cdot(\delta_0/y_0+\eta_1-\eta_2)+\varpi_1^2(\eta_1-\delta_1/y_0)=\lambda F_{s1}; \\ \ddot{\eta}_2+\zeta_2\dot{\eta}_2-H(\delta_0/y_0+\eta_1-\eta_2)\varpi_{j2}^2 \\ \cdot(\delta_0/y_0+\eta_1-\eta_2)+\varpi_2^2(\eta_2+\delta_2/y_0)=\lambda F_{s2}. \end{cases} \tag{3}$$

Eq.(3) can be solved by Explicit Runge-Kutta algorithm, with the elastic restoring force in contact system being obtained by

$$\begin{aligned} N&=H(\delta_0+y_1-y_2)\varpi_{j1}^2(\delta_0/y_0+\eta_1-\eta_2) \\ &=H(\delta_0+y_1-y_2)\varpi_{j2}^2(\delta_0/y_0+\eta_1-\eta_2). \end{aligned} \tag{4}$$

When $\delta_0+y_1-y_2<0$, N=0, contact loss occurs.

MONTE CARLO SIMULATION OF CONTACT SYSTEM

The contact system's vibration characteristic is judged by the probability distribution and RMS spectra of contact force around resonance. Monte Carlo simulations conducted to estimate the relevant results. In order to simulate the Gaussian white-noise excitation, 500 samples of wide band pseudo-random signal were generated by using Eq.(5):

$$\begin{cases} F_{s1}=\omega_1(\tau)=W_M\sum_{k=1}^{M}\cos(\varpi_{k1}\tau+\phi_{k1}); \\ F_{s2}=\omega_2(\tau)=W_M\sum_{k=1}^{M}\cos(\varpi_{k2}\tau+\phi_{k2}), \end{cases} \tag{5}$$

where variables ϖ_{k1}, ϖ_{k2} and ϕ_{k1}, ϕ_{k2} are independent and uniformly distributed in [0, $\varpi_{\max}$] and [0, π]. W_M is related with the frequency resolution

$$W_M = \sqrt{\Delta f h}\ . \tag{6}$$

The output random signal $\omega_{1,2}(\tau)$ generated in this way should obey normal distribution. After substituting it into Eq.(3), the dynamic system can be written as

$$\begin{cases} \ddot{\eta}_1 + \zeta_1\dot{\eta}_1 + H(\delta_0/y_0+\eta_1-\eta_2)\varpi_{j1}^2 \\ \cdot(\delta_0/y_0+\eta_1-\eta_2) + \varpi_1^2(\eta_1-\delta_1/y_0) = \lambda\omega_1(\tau); \\ \ddot{\eta}_2 + \zeta_2\dot{\eta}_2 - H(\delta_0/y_0+\eta_1-\eta_2)\varpi_{j2}^2 \\ \cdot(\delta_0/y_0+\eta_1-\eta_2) + \varpi_2^2(\eta_2+\delta_2/y_0) = \lambda\omega_2(\tau). \end{cases} \tag{7}$$

As a case of reed relay (Zhai *et al.*, 2003), k_j=5.98×10^9 N/m, M_1=M_2=2.0721×10^{-5} kg, δ_1=δ_2=2×10^{-4} m, k_1=k_2=220.264 N/m, δ_0=7.36622×10^{-12} m.

Equivalence model of contact system is shown in Fig.2, module Random1, Random2 are external excitations, module inh is the gain of excitation.

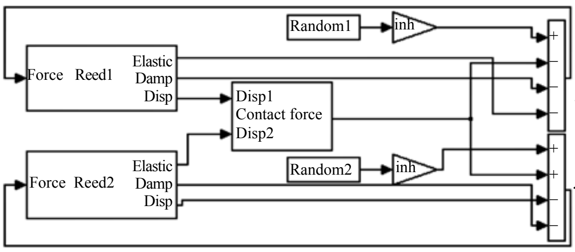

Fig.2 Equivalence model of contact system

Statistical results of the probability densities of the contact force from Monte Carlo simulation are shown in Fig.3 showing that when h_1=4.85×10^{-6}, h_2=1.94×10^{-5}, h_3=4.85×10^{-4}, h_4=1.94×10^{-3}, no contact loss occurs.

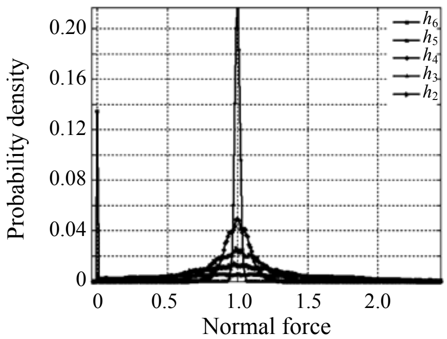

Fig.3 Distribution of normal contact force under different input level

However, h_5=7.76×10^{-3}, h_6=4.85×10^{-2}, h_7=1.94×10^{-1} intermittent contact loss occurs, corresponding to the probability distribution at N=0. Some statistical results are listed in Table 1.

Table 1 Statistical results of Monte Carlo simulation

Input level h	Expectation μ	Variance σ	Probability $p(\delta_0+y_1-y_2<0)$ (%)
h_1	0.99752	0.0088	0
h_2	0.99754	0.0176	0
h_3	0.99769	0.0884	0
h_4	0.99788	0.1769	0
h_5	0.99805	0.3486	0.21
h_6	0.99215	0.7196	8.556
h_7	0.98105	1.4550	21.694

The spectral content of contact force is obtained by Welch's method (Hayes, 1996; Welch, 1967) with 2048 samples. The sample vector is segmented into four sections of equal length, each with 50% overlap. Each segment is windowed with a Hamming window that is the same length as the segment. Average RMS spectrum obtained with a number of spectra up to 500 is shown in Fig.4. RMS spectrum can be computed by using Matlab command below:

```
[H,f]=psd(x,Nfft,1/Fs,Nfft,round(Nfft/2)),
RMS_spectrum=sqrt(H/Nfft)×2×sqrt(sum(hanning(Nfft)^2)/Nfft×2).
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The contact loss ratio affected by seven different levels external excitation is denoted by one-side RMS spectrum (shown in Fig.4). Spectral results are obtained around the main dimensionless circular frequency ϖ_1=1.

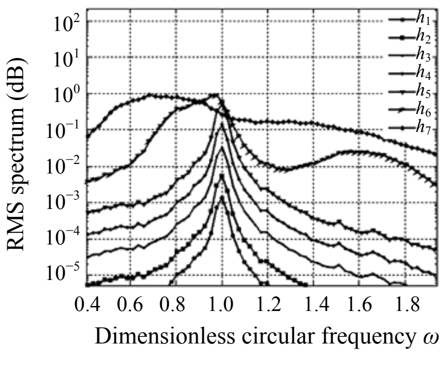

Fig.4 RMS spectrum of contact force under different excitation levels

As shown in Fig.4, with increasing input level, the spectral content at $\varpi_1=1$ increases, and at certain input level, the broadening of the resonant peaks is clearly observed and the spectral content at $\varpi_1<1$ and $\varpi_1=1.6$ increases. The system now is acting as a nonlinear one, corresponding to occurrence of contact loss.

CONCLUSION

A two-degrees-of-freedom mathematical model was built for analyzing the dynamic characteristic of contact system. The relation between probability of contact loss and input level under random excitation was investigated by Monte Carlo simulation.

In Monte Carlo simulation, the relation between the excitation level and the contact loss probability is described by statistical method and spectral method. When input force has low amplitude, the system behaves as a linear one, and loss contact does not occur. As the excitation level increasing, with increasing of the third harmonic spectral peak, broadening of the spectral peaks was observed; this behavior is known to be an essential property of systems with large non-linearity and low damping. From the results of Monte Carlo simulation the validity of the mathematical model was verified. A problem for future analytical solution is presented.

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