Journal of Zhejiang University SCIENCE A
ISSN 1009-3095 (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Inspection-replacement policy of system under predictive maintenance*

LIU Bao-you[†1], FANG You-tong[†‡2], WEI Jin-xiang[3], WANG Yong-liang[1]

([1]Department of Mathematics and Physics, Shijiazhuang Railway Institute, Shijiazhuang 050043, China)

([2]College of Electrical Engineering, Zhejiang University, Hangzhou 310027, China)

([3]Extrahigh Tension Transmittal and Transformation Company Branch, Hebei Electric Power Company, Shijiazhuang 050051, China)

[†]E-mail: liubyde@tom.com; youtong@zju.edu.cn

Received Dec. 19, 2006; revision accepted Jan. 5, 2007

Abstract: In general, every system is in one of the three states: normal, abnormal, or failure state. When the system is diagnosed as abnormal state, it needs predictive maintenance. If the system fails, an identical new one will replace it. The predictive maintenance cannot make the system "as good as new". Under these assumptions, the reliability index and the inspection-replacement policy of a system were studied. The explicit expression of the reliability index and the average income rate (i.e., the long-run average income per unit time) are derived by using probability analysis and vector Markov process method. The criterion of feasibility for the optimal inspection-replacement policy under the maximum average income rate is obtained. The numerical example shows the optimal inspection-replacement policy can raise the average income rate when the optimal inspection-replacement policy is feasible.

Key words: Inspection-replacement policy, Inspection and diagnosis, Monotone stochastic process, Reliability index, Vector Markov process method

doi:10.1631/jzus.2007.A0495 **Document code:** A **CLC number:** TB114.3; O224; O211.6

INTRODUCTION

It was assumed that systems could be repaired as good as new in earlier studies on the maintenance of a system. Because this assumption is far from actuality, some new models have been developed continuously (Barlow and Hunter, 1960; Brown and Proschan, 1983; Yeh, 1988a; 1988b; Stadje and Zuckerman, 1990). These new models make study of repair close to actual situation. Inspecting and diagnosing the system to find the omen of failure and to avoid happening of failure is the newest effective measure to raise reliability, safety and economy. The failures of any system can be classified to three categories. The first one is a slow progressing failure, which can be found by inspection equipments and can be remained for long time without repairing but just for watching, studying and later treatment. The second one is the failure that occurs suddenly without any warning signs and cannot be found by inspection. The third kind of failure is between the above two, which may be found by inspection and avoided with proper measures. When the system is in developing failure, it is in an abnormal state. The repair for an abnormal system after it has been inspected and diagnosed is called predictive maintenance. In this state, though the system is not damaged seriously, it cannot be repaired as good as new. Because the system can be damaged seriously after it fails, an identical new one replaces it. This paper studied the reliability and the optimal inspection-replacement policy of the system. The optimal inspection-replacement policy is to determine how often a system is inspected and how many predictive maintenances a system is replaced

‡ Corresponding author

* Project supported by the National Hi-Tech Research and Development Program (863) of China (No. 2005AA505101-506) and the National Natural Science Foundation of China (No. 50477030)

such that the average income rate of a system is maximized. There are many results about inspection and diagnosis technology (Chen, 2003; Liu *et al*., 2003), and some results about reliability and inspection-replacement policy (Su, 1997; Chelbi and Ait-kadi, 1999; Biswas *et al*., 2003; Su, 2003), but many important problems are remained to be resolved. The aim of this paper is to build a model by probability analysis and vector Markov process method (Shi, 1999) and to study reliability and inspection-replacement policy.

DEFINITION AND CHARACTER DESCRIPTION OF SYSTEM

Definition 1 X, Y are random variables, their cumulative distribution functions (abbreviated c.d.f.) are $F(t)$ and $H(t)$, $\bar{F}(t)=1-F(t)$, $\bar{H}(t)=1-H(t)$, if for all real $t\geq0$, $\bar{F}(t)\leq\bar{H}(t)$, then X is called stochastically smaller than Y, denoted by $X\leq_{st}Y$; otherwise X is called stochastically larger than Y, and denoted by $X\geq_{st}Y$.

Definition 2 Let $\{X_n, n=1,2,\ldots\}$ be a sequence of non-negative and independent random variables. If $X_n\leq_{st}X_{n+1}$, $n=1,2,\ldots$, then $\{X_n, n=1,2,\ldots\}$ is called a monotonously increasing stochastic process. If $X_n\geq_{st}X_{n+1}$, $n=1,2,\ldots$, then $\{X_n, n=1,2,\ldots\}$ is called a monotonously decreasing stochastic process.

The lifetime sequence of actual repairable system is a monotonously decreasing stochastic process, but the repair time sequence is a monotonously increasing stochastic process.

The system has the following characters:

(1) The system has 3 modes—normal, abnormal and failure. The system can transfer from normal to failure directly, or from normal to failure via abnormal. Normal and abnormal are the working states of the system. Whether the system is in normal or abnormal state can be known through exact inspection and diagnosis. When the system fails, it can be known without inspection and diagnosis.

(2) After a new system (system at the beginning of operation or after replacement) begins its ith normal, it is inspected and diagnosed every random time period T_i to know whether it is in normal or abnormal. The c.d.f. of T_i is $H_i(x)$. Density function of T_i is $h_i(x)$. Failure rate of T_i is $\alpha_i(x)$. Inspection and diagnosis can be finished instantaneously. When the system is inspected and diagnosed in abnormal, it accepts the ith times predictive maintenance. The c.d.f. of the Y_i of the ith times predictive repair time is $G_i(y)$. Density function is $g_i(y)$. Repair rate is $\mu_i(y)$. The mean is $E(Y_i)=\mu_i$. After the new system begins its ith times normal, it transfers from normal to abnormal at failure rate λ_{i01} and to failure at λ_{i02}. When the new system is in abnormal after its ith times normal, it transfers to failure at failure rate λ_{i12}. $\lambda_{i01}\leq\lambda_{i+101}$, $\lambda_{i02}\leq\lambda_{i+102}$, $\lambda_{i12}\leq\lambda_{i+112}$, $i=1,2,3,\ldots$ $\{Y_i, i=1,2,\ldots\}$ is a monotonously increasing stochastic process. If the system fails during working, it is replaced by an identical new one. The c.d.f. of Y of replacement time is $G(y)$. Density function is $g(y)$. Replacement rate is $\mu(y)$. The mean is $E(Y)=\mu$.

(3) All random variables are independent.

(4) At $t=0$, a new system is installed and begins normal work. When the system is inspected and diagnosed in abnormal after it begins the Nth times normal, it will not need predictive maintenance and is replaced by an identical new one. $G_N(y)$ is c.d.f of replacement time. $G_N(y)=G(y)$.

(5) The normal working reward, abnormal working reward per unit time are K_0 and K_1 respectively. Average cost for inspection and diagnosis each time, replacement each time, predictive repair each time are E_1, E_2, E_3, respectively.

The state of system is defined as follows. State $(i,0,n)$ means that a new system is in the ith normal and inspected and diagnosed n times. State $(i,1)$ means that the system is in abnormal after the ith times normal, $i=1,2,\ldots,N$. State $(i,3)$ means that the system is in its ith times predictive repair, $i=1,2,\ldots,N-1$. State $(N,3)$ means that the system is in replacement state after inspected and diagnosed in abnormal after the Nth times normal. State $(i,2)$ means the system is in replacement state after failure during its ith times working state, $i=1,2,\ldots,N$.

Let $S(t)$ be the state of the system at time t. Define supplementary variables as follows. $X_{i0n}(t)$ denotes the inspection and diagnosis interval time when $S(t)=(i,0,n)$. $X_{i1}(t)$ denotes the inspection and diagnosis interval time when $S(t)=(i,1)$, $i=1,2,\ldots,N$. $Y_{i3}(t)$ denotes the predictive repair time of system at time t when $S(t)=(i,3)$, $i=1,2,\ldots,N-1$. $Y_{i2}(t)$ denotes the replacement time of system at time t when $S(t)=(i,2)$, $i=1,2,\ldots,N$. $Y_{N3}(t)$ denotes the replacement time of system at time t when $S(t)=(N,3)$. After supplementary variables are introduced, the process is a vector Markov process (Shi, 1999).

The state probability density of system is defined as follows:

$P_{i0n}(t,x)\mathrm{d}x=P\{S(t)=(i,0,n),\ x\le X_{i0n}(t)<x+\mathrm{d}x\}$,
$P_{i1}(t,x)\mathrm{d}x=P\{S(t)=(i,1),\ x\le X_{i1}(t)<x+\mathrm{d}x\}$,
$i=1,2,\ldots,N$;
$P_{i3}(t,y)\mathrm{d}y=P\{S(t)=(i,3),\ y\le Y_{i3}(t)<y+\mathrm{d}y\}$,
$i=1,2,\ldots,N-1$;
$P_{N3}(t,y)\mathrm{d}y=P\{S(t)=(N,3),\ y\le Y_{N3}(t)<y+\mathrm{d}y\}$,
$P_{i2}(t,y)\mathrm{d}y=P\{S(t)=(i,2),\ y\le Y_{i2}(t)<y+\mathrm{d}y\}$,
$i=1,2,\ldots,N$.

STATE PROBABILITY DENSITY EQUATIONS OF THE SYSTEM

By probability analysis, we can get the state probability density differential equations as follows:

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_{i01}+\lambda_{i02}+\alpha_i(x)\right]P_{i0n}(t,x)=0,\quad n=0,1,2,\cdots;\ i=1,2,\cdots,N, \tag{1}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_{i12}+\alpha_i(x)\right]P_{i1}(t,x)=\lambda_{i01}\sum_{n=0}^{\infty}P_{i0n}(t,x),\quad i=1,2,\cdots,N, \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu(y)\right]P_{i2}(t,y)=0,\ i=1,2,\cdots,N, \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_i(y)\right]P_{i3}(t,y)=0,\ i=1,2,\cdots,N. \tag{4}$$

The boundary conditions are

$$P_{100}(t,0)=\delta(t)+\sum_{i=1}^{N}\int_0^{\infty}\mu(y)P_{i2}(t,y)\mathrm{d}y+\int_0^{\infty}\mu_N(y)P_{N3}(t,y)\mathrm{d}y, \tag{5}$$

$$P_{i00}(t,0)=\int_0^{\infty}\mu_{i-1}(y)P_{i-13}(t,y)\mathrm{d}y,\ i=1,2,\cdots,N, \tag{6}$$

$$P_{i0n}(t,0)=\int_0^{\infty}\alpha_i(x)P_{i0n-1}(t,x)\mathrm{d}x\quad n=1,2,\cdots,\ i=1,2,\cdots,N, \tag{7}$$

$$P_{i1}(t,0)=0,\ i=1,2,\cdots,N, \tag{8}$$

$$P_{i2}(t,0)=\lambda_{i02}\sum_{n=0}^{\infty}\int_0^{\infty}P_{i0n}(t,x)\mathrm{d}x+\lambda_{i12}\int_0^{\infty}P_{i1}(t,x)\mathrm{d}x, \tag{9}$$

$$P_{i3}(t,0)=\int_0^{\infty}\alpha_i(x)P_{i1}(t,x)\mathrm{d}x,\ i=1,2,\cdots,N. \tag{10}$$

The initial conditions are as follows:

$$P_{100}(0,x)=\delta(x), \tag{11}$$

the others are 0.

Let $B_0(s)=D_0(s)=1$,

$$B_i(s)=\prod_{k=1}^{i}\lambda_{k01}[h_k^*(s+\lambda_{k01}+\lambda_{k02})-h_k^*(s+\lambda_{k12})]\prod_{k=1}^{i}g_k^*(s), \tag{12}$$

$$D_i(s)=\prod_{k=1}^{i}(\lambda_{k12}-\lambda_{k01}-\lambda_{k02})[1-h_k^*(s+\lambda_{k01}+\lambda_{k02})], \tag{13}$$

$$f_i(s)=(\lambda_{i01}+\lambda_{i02})(\lambda_{i12}-\lambda_{i02})\bar{H}_i^*(s+\lambda_{i01}+\lambda_{i02})-\lambda_{i01}\lambda_{i12}\bar{H}_i^*(s+\lambda_{i12}),\quad i=1,2,\ldots,N, \tag{14}$$

$$C_i(s)=\frac{B_{i-1}(s)}{D_i(s)}\left[1-\sum_{i=1}^{N}\frac{f_i(s)B_{i-1}(s)g^*(s)}{D_i(s)}-\frac{B_N(s)}{D_N(s)}\right]^{-1}, \tag{15}$$

$$f^*(s)=\int_0^{\infty}\mathrm{e}^{-st}f(t)\mathrm{d}t,\ \bar{f}(t)=1-f(t). \tag{16}$$

Using Shi (1999)'s method, we can get the state probability density of the system in terms of the Laplace transform as follows:

$$\sum_{n=0}^{\infty}P_{i0n}^*(s,x)=C_i(s)(\lambda_{i12}-\lambda_{i01}-\lambda_{i02})\bar{H}_i(x)\mathrm{e}^{-(s+\lambda_{i01}+\lambda_{i02})x}, \tag{17}$$

$$P_{i1}^*(s,y)=\lambda_{i01}C_i(s)\bar{H}_i(x)[\mathrm{e}^{-(s+\lambda_{i01}+\lambda_{i02})x}-\mathrm{e}^{-(s+\lambda_{i12})x}], \tag{18}$$

$$P_{i2}^*(s,y)=C_i(s)f_i(s)\bar{G}(y)\mathrm{e}^{-sy}, \tag{19}$$

$$P_{i3}^*(s,y)=\lambda_{i01}C_i(s)[h_i^*(s+\lambda_{i01}+\lambda_{i02})-h_i^*(s+\lambda_{i12})]\bar{G}_i(y)\mathrm{e}^{-sy},\quad i=1,2,\cdots,N. \tag{20}$$

RELIABILITY INDEX AND OPTIMAL INSPECTION-REPLACEMENT POLICY OF THE SYSTEM

Let

$$C_i=\frac{B_{i-1}(0)}{D_i(0)}\left\{\sum_{i=1}^{N}\frac{B_{i-1}(0)[(\lambda_{i12}-\lambda_{i02})\bar{H}_i^*(\lambda_{i01}+\lambda_{i02})]}{D_i(0)}-\sum_{i=1}^{N}\frac{B_{i-1}(0)\lambda_{i01}\bar{H}_i^*(\lambda_{i12})}{D_i(0)}+\mu\sum_{i=1}^{N}\frac{f_i(0)B_{i-1}(0)}{D_i(0)}+\frac{B_N(0)}{D_N(0)}\mu+\sum_{i=1}^{N-1}\frac{B_i(0)\mu_i}{D_i(0)}\right\}^{-1}. \quad (21)$$

Availability

The probability that the system is working in normal is called N-availability, whereas the probability that the system is working in abnormal is called A-availability. Denoting $A_0(t)$ as the instantaneous N-availability at time t, we have

$$A_0(t)=\sum_{i=1}^{N}\sum_{n=0}^{\infty}\int_0^{\infty}P_{i0n}(t,x)\mathrm{d}x. \quad (22)$$

Using Eq.(17), we can get the Laplace transform of $A_0(t)$

$$A_0^*(s)=\sum_{i=1}^{N}C_i(s)(\lambda_{i12}-\lambda_{i01}-\lambda_{i02})\bar{H}_i^*(s+\lambda_{i01}+\lambda_{i02}). \quad (23)$$

Denote A_0 as the steady N-availability. By applying the limiting theorem of the Laplace transform, we have

$$A_0=\lim_{t\to\infty}\frac{\int_0^t A_0(x)\mathrm{d}x}{t}=\lim_{s\to 0}sA_0^*(s)=\sum_{i=1}^{N}C_i(\lambda_{i12}-\lambda_{i01}-\lambda_{i02})\bar{H}_i^*(\lambda_{i01}+\lambda_{i02}). \quad (24)$$

By applying the same method, we can obtain the steady A-availability

$$A_1=\sum_{i=1}^{N}C_i\lambda_{i01}[\bar{H}_i^*(\lambda_{i01}+\lambda_{i02})-\bar{H}_i^*(\lambda_{i12})]. \quad (25)$$

Inspection and diagnosis frequency

Denote $W_1(t)$ as the instantaneous inspection frequency at time t, we have

$$W_1(t)=\sum_{i=1}^{N}\sum_{n=0}^{\infty}\int_0^{\infty}\alpha_i(x)P_{i0n}(t,x)\mathrm{d}x+\sum_{i=1}^{N}\int_0^{\infty}\alpha_i(x)P_{i1}(t,x)\mathrm{d}x. \quad (26)$$

Using Eqs.(17) and (18), we can get the Laplace transform of $W_1(t)$

$$W_1^*(s)=\sum_{i=1}^{N}C_i(s)[(\lambda_{i12}-\lambda_{i02})h_i^*(s+\lambda_{i01}+\lambda_{i02})-\lambda_{i01}h_i^*(s+\lambda_{i12})]. \quad (27)$$

Inspection and diagnosis frequency in (0,t] is

$$M_1(t)=\int_0^t W_1(x)\mathrm{d}x. \quad (28)$$

Let M_1 be the steady inspection frequency. Applying the limiting theorem of the Laplace transform, we have

$$M_1=\lim_{t\to\infty}\frac{\int_0^t W_1(x)\mathrm{d}x}{t}=\lim_{s\to 0}sW_1^*(s)=\sum_{i=1}^{N}C_i[(\lambda_{i12}-\lambda_{i02})h_i^*(\lambda_{i01}+\lambda_{i02})-\lambda_{i01}h_i^*(\lambda_{i12})]. \quad (29)$$

Replacement frequency of the system

Denoting $W_2(t)$ as the instantaneous replacement frequency at time t, we have

$$W_2(t)=\sum_{i=1}^{N}\sum_{n=0}^{\infty}\lambda_{i02}\int_0^{\infty}P_{i0n}(t,x)\mathrm{d}x+\sum_{i=1}^{N}\lambda_{i12}\int_0^{\infty}P_{i1}(t,x)\mathrm{d}x+\int_0^{\infty}\alpha_N(x)P_{N1}(t,x)\mathrm{d}x. \quad (30)$$

Using Eqs.(17) and (18), we can obtain the Laplace transform of $W_2(t)$

$$W_2^*(s)=\lambda_{N01}C_N(s)[h_N^*(s+\lambda_{N01}+\lambda_{N02})-h_N^*(s+\lambda_{N12})]+\sum_{i=1}^{N}f_i(s)C_i(s). \quad (31)$$

Denoting $M_2(t)$ as replacement frequency in (0,t], then we have

$$M_2(t)=\int_0^t W_2(x)\mathrm{d}x. \quad (32)$$

Denote M as the steady replacement frequency. By applying the limiting theorem of the Laplace transform, we get

$$M_2 = \lim_{t\to\infty}\frac{M_2(t)}{t} = \lim_{s\to 0} sW_2^*(s) = \sum_{i=1}^{N} C_i f_i(0) + C_N\lambda_{N01}[h_N^*(\lambda_{N01}+\lambda_{N02}) - h_N^*(\lambda_{N12})]. \quad (33)$$

Frequency of predictive repair

Denoting $W_3(t)$ as the instantaneous predictive repair frequency at time t, then we have

$$W_3(t) = \sum_{i=1}^{N-1}\int_0^\infty \alpha_i(x)P_{i1}(t,x)\mathrm{d}x. \quad (34)$$

Using Eq.(18) we can obtain the Laplace transform of $W_3(t)$

$$W_3^*(s) = \sum_{i=1}^{N-1} C_i(s)\lambda_{i01}[h_i^*(s+\lambda_{i01}+\lambda_{i02}) - h_i^*(s+\lambda_{i12})]. \quad (35)$$

Predictive repair frequency in $(0,t]$ is

$$M_3(t) = \int_0^t W_3(x)\mathrm{d}x. \quad (36)$$

Denote M_3 as the steady predictive repair frequency. By applying the limiting theorem of the Laplace transform, we get

$$M_3 = \lim_{t\to\infty}\frac{\int_0^t W_3(x)\mathrm{d}x}{t} = \lim_{s\to 0} sW_3^*(s) = \sum_{i=1}^{N-1} C_i\lambda_{i01}[h_i^*(\lambda_{i01}+\lambda_{i02}) - h_i^*(\lambda_{i12})]. \quad (37)$$

Optimal inspection-replacing policy for the system

We can obtain the average income rate of the system by using the results obtained above. The expected total income generated by the system during $(0,t]$ is

$$R(N,H_1,\cdots,H_N,t) = K_0\int_0^t A_0(x)\mathrm{d}x + K_1\int_0^t A_1(x)\mathrm{d}x - E_1M_1(t) - E_2M_2(t) - E_3M_3(t). \quad (38)$$

We can obtain the average income rate:

$$\begin{aligned} D(N,H_1,H_2,\cdots,H_N) &= \lim_{t\to\infty}\frac{R(N,H_1,\cdots,H_N,t)}{t} \\ &= K_0A_0 + K_1A_1 - E_1M_1 - E_2M_2 - E_3M_3 \\ &= \sum_{i=1}^{N} C_i\{K_0(\lambda_{i12}-\lambda_{i01}-\lambda_{i02})\bar{H}_i^*(\lambda_{i01}+\lambda_{i02}) \\ &\quad + K_1\lambda_{i01}[\bar{H}_i^*(\lambda_{i01}+\lambda_{i02}) - \bar{H}_i^*(\lambda_{i12})] \\ &\quad - E_1[(\lambda_{i12}-\lambda_{i02})h_i^*(\lambda_{i01}+\lambda_{i02}) - \lambda_{i01}h_i^*(\lambda_{i12})] \\ &\quad - E_2f_i(0) - E_3\lambda_{i01}[h_i^*(\lambda_{i01}+\lambda_{i02}) - h_i^*(\lambda_{i12})]\} \\ &\quad + (E_3-E_2)C_N\lambda_{N01}[h_N^*(\lambda_{N01}+\lambda_{N02}) - h_N^*(\lambda_{N12})]. \end{aligned} \quad (39)$$

Obviously, $D(N,H_1,H_2,\ldots,H_N)$ is an explicit expression of N and $H_1,H_2,\ldots,H_N$. We can find the optimal inspection-replacing policy $(N^*,H_1^*,H_2^*,\cdots,H_N^*)$ to make $D(N,H_1,H_2,\ldots,H_N)$ maximized.

By using the method in this paper, we can also obtain the average income rate without inspection and diagnosis:

$$RW = \frac{\lambda_{112}k_0 + \lambda_{101}k_1 - E_2(\lambda_{101}+\lambda_{102})\lambda_{112}}{\lambda_{112}+\lambda_{101}+\mu(\lambda_{101}+\lambda_{102})\lambda_{112}}. \quad (40)$$

Comparing $D(N^*,H_1^*,\cdots,H_N^*)$ and RW, we can obtain the criterion for feasibility of optimal inspection-replacement policy.

If $D(N^*,H_1^*,\cdots,H_N^*)>RW$, then optimal inspection-replacing policy is feasible.

It is difficult to search out the optimal inspection-replacement policy from all $(H_1,H_2,\ldots,H_N)$. In practice, the inspection time interval is often taken as a constant for sake of conveniences. When the inspection time interval is a constant of u, $\bar{H}_i^*(\lambda) = (1-\mathrm{e}^{-\lambda u})/\lambda$, $h_i^*(\lambda) = \mathrm{e}^{-\lambda u}$. Substituting them into $D(N,H_1,H_2,\ldots,H_N)$, $D(N,H_1,H_2,\ldots,H_N)$ would be expressed as a function of variables u and N, which is denoted as $L(N,u)$. Using the analytical or numerical method, we can get the optimal inspection-replacing policy (N^*,u^*). The largest average income rate is $L(N^*,u^*)$.

If $L(N^*,u^*)>RW$, the optimal inspection-replacement policy is feasible.

NUMERICAL EXAMPLE

Assume the data of some electrical products as follows: λ_{i01}=0.00069×1.05^{i-1}; λ_{i02}=0.00002×1.05^{i-1};

λ_{i12}=0.004×1.01^{i-1} h^{-1}; μ_i=26×1.06^{i-1}, i=1,2,…,N−1; μ=6 h; k_0=2900, k_1=2050 ¥/h (¥: RMB); E_1=4000, E_2=1220000, E_3= 73000 ¥/once.

With a microcomputer, we can obtain the optimal inspection-replacing policy N^*=12, u^*=20.7446 h, and the maximum average income rate of the system $L(N^*,u^*)$=2620.36 ¥/h, RW=2028.81 ¥/h. Obviously, the optimal inspection-replacing policy in this example is feasible.

CONCLUSION

Two hot topics are concerned by scholars. One is the reliability and inspection policy of a system with perfect maintenance. Another is the reliability and replacement policy of a system with no inspection and imperfect maintenance. Few research results are reported on the challenging problem of the reliability and inspection-replacement policy of a system with imperfect maintenance. This paper deals with the optimal inspection-replacement policy of a system with imperfect predictive maintenance and replacement after failure. The average income rate of the system is obtained. Maximizing the average income rate can therefore develop the optimal inspection-replacement policy. We also obtain the criterion for feasibility of the optimal inspection-replacement policy. The numerical example shows that the inspection-replacement policy can raise the average income rate when it is feasible.

References

Barlow, R.E., Hunter, L.C., 1960. Optimum preventive maintenance policy. *Operations Res.*, **8**:90-100.

Biswas, A., Sarkar, J., Sarkar, S., 2003. Availability of a periodically inspected system, maintained under an imperfect-repair policy. *IEEE Transaction Reliability*, **52**(3): 311-318. [doi:10.1109/TR.2003.818716]

Brown, M., Proschan, F., 1983. Imperfect repair. *J. Appl. Probabil.*, **20**(4):851-859. [doi:10.2307/3213596]

Chelbi, A., Ait-kadi, D., 1999. An optimal inspection strategy for a randomly failing equipment. *Reliability Engineering and System Safety*, **63**(2):127-131. [doi:10.1016/S0951-8320(98)00031-3]

Chen, S., 2003. Intelligence fault, prediction and maintains system of mechanical equipment. *Journal of Southwest Jiao Tong University*, **38**(5):540-543 (in Chinese).

Liu, R.D., Tao, D.H., Hu, S.H., 2003. Application of oil analysis in the failure diagnosis of vibrating sieve. *Lubrication Engineering*, (2):66-68 (in Chinese).

Shi, D.H., 1999. Density Devolution Method in Stochastic Models. Science Press, Beijing (in Chinese).

Stadje, W., Zuckerman, D., 1990. Optimal strategies for some repair replacement models. *Advances in Appl. Probabil.*, **22**(3):641-656. [doi:10.2307/1427462]

Su, B.H., 1997. On a two-dissimilar-unit system with three modes and random check. *Microelectronics and Reliability*, **37**(8):1233-1238. [doi:10.1016/S0026-2714(96) 00121-7]

Su, B.H., 2003. Research on the inspection polices of 2-failure-mode systems. *Acta Automatica Sinica*, **29**(4):544-549.

Yeh, L., 1988a. A note on the optimal replacement problem. *Advances in Appl. Probabil.*, **20**(2):479-482. [doi:10.2307/ 1427402]

Yeh, L., 1988b. Geometric processes and replacement problem. *Acta Mathematicae Applicatae Sinica (English Series)*, **4**(4):366-377. [doi:10.1007/ BF02007241]