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Computation of rolling resistance caused by rubber hysteresis of truck radial tire

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Abstract: Applying the results of stress and strain calculated by 3D finite element model of truck radial tire 11.00R20, a MATLAB program used to compute rolling resistance of the tire caused by hysteresis rolling resistance (HRR) is worked out. The HRR distribution on different part of tire section, and the effects of speed, load, internal pressure and the width of the rim on HRR are analyzed. The analysis results showed that energy loss produced by tread rubber contributes the most part to HRR of the whole tire, and that to decrease the HRR, the hysteresis factor of the tread rubber should be reduced, and the distribution of the stress and strain on the section be optimized.

Key words: Truck radial tire, Hysteresis rolling resistance (HRR), Finite element analysis (FEA), Hysteresis factor, Tire structure design

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INTRODUCTION

Rolling resistance due to rubber hysteresis of tire causes a running truck to uselessly consume more fuel, and generates heat inside the tire materials (Schuring and Futamura, 1990). The unnecessary fuel consumption will result in waste of energy source, which is already short of in the world and worsen the environment pollution. In the meantime, the heat generation and accumulation inside the tire material will inevitably reduce the tire materials' strength and may cause the tire to be broken much earlier than its normal service life. Therefore, computation of rolling resistance caused by rubber hysteresis of truck radial tire is significant in the view of society and economy.

During recent decades, tire researchers have done a lot of work on hysteresis rolling resistance (HRR), focused mainly on the rubber properties (Amino and Uchiyama, 2000; Wang *et al.*, 2002; Naohiko, 1996; Hao *et al.*, 2001). Nevertheless, tire structure has significant influence on HRR of the whole tire, which can be proved by experiments and finite element analysis (FEA) results (Shida *et al.*, 1999; Ebbott *et al.*, 1999; Mc Allen *et al.*, 1996). In this paper, distributions of HRR on tire section and the effects of tire working conditions on the resistance are analyzed.

DEFINITION AND METHOD FOR COMPUTING A TIRE'S HRR

Rolling resistance is defined as heat energy transformed from mechanical energy when a tire has run a unit distance on a plane and straight road, that is, rolling resistance denotes the mechanical energy loss of a rolling tire. According to the definition of rolling resistance, its unit is N·m/m, simplified as N. Rolling resistance can be calculated by Eq.(1) (Ebbott *et al.*, 1999), which is the total mechanical energy loss divided by the corresponding distance after the tire had rolled one loop on the road surface.

$$RR=U''/(2\pi\rho_r)\,. \tag{1}$$

where RR is the rolling resistance; ρ_r is the valid rolling radius of the tire; and U'' is the total mechanical energy loss of rolling one loop.

Rolling resistance of tire consists of friction between road-surface and tread (including rolling friction and sliding friction), resistance produced by rubber hysteresis, resistance against air, and energy loss of tread pattern block knocking road-surface, among which, HRR accounts for above 80% when the truck runs at medium speed. Therefore, HRR has to be emphasized and studied firstly. In this paper, a method for computing HRR is developed, by which, HRR of the whole tire and any rubber parts on the section can be obtained, and effects of tire working conditions on HRR can also be analyzed.

Complicated structure and diversiform load-case of tire make it quite difficult to build the tire's FEA model. The perfect FEA of radial tire will consume extremely expensive computing time because not only the geometrical nonlinearities of large deformation and large strain and ground-tread contacting nonlinearity, but also the physical nonlinear property of rubber material have to be all considered in the FEA model for HRR analysis. In this paper, a simplified and approximate measure is taken to put the rubber material's nonlinearity from FEA to post-FEA HRR computing program of MATLAB so as to lessen largely the FEA computer running time and make the computation extraordinarily economical.

The method of computing HRR of radial tire developed in this paper consists of three steps as follows: (1) Build finite element model of radial tire without considering rubber hysteresis nonlinearity, then run it with the load case of stable rolling under vertical load; (2) Compute hysteresis energy loss of each node on the tire's section through MATLAB program, applying node stresses and strains obtained from the results of Step 1 and rubbers' hysteresis factors obtained from experiments; (3) Compute HRR of tire according to its definition.

NUMERICAL MODEL OF HRR OF TIRE

Concepts of hysteresis phase angle and hysteresis factor

The curves shown in Fig.1 are sinusoids of stress σ and strain ε, respectively, under alternate load with different phase angles θ. In Fig.1, angle δ is the phase angle difference between the phase angle of strain ε and phase angle of stress σ, with the ε phase angle lagging behind the σ phase angle by a phase angle of δ, and the phase angle difference δ is called hysteresis phase angle.

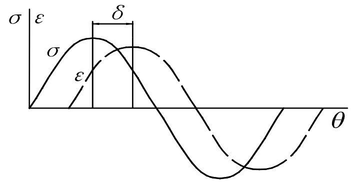


Fig.1 Sketch-map of sinusoids of stress and strain with strain lagging behind stress

Define E' as the ratio of stress to strain with the same phase angle, called energy storage modulus, which reflects energy stored in the material due to its property of elasticity, in the course of rubber being deformed. Define E'' as the ratio of stress to strain with 90° difference of phase angle, called energy loss modulus, which reflects energy lost in heat-generation due to its property of viscidity, in the course of rubber being deformed.

Generally, dynamic viscoelastic material is denoted by hysteresis factor $\tan\delta$ of the material, which is defined as the ratio of energy loss modulus to energy storage modulus, as shown in Eq.(2):

$$\tan\delta = E''/E'. \quad (2)$$

HRR of rubber materials is actually the energy loss produced by the performance of rubbers' strain lag behind stress during the periodical change of stress during the tire's rolling. From the description above, it is evident that HRR is directly relevant to hysteresis factor $\tan\delta$.

Radial tire comprises two main kinds of materials: rubbers structuring the matrix and connective, and steel-cord ply acting as enforcement, among which rubber exhibits strong viscoelastic property, and its hysteresis will affect HRR greatly; by contrast, steel-cord's contribution to HRR of tire is fractional and will be ignored in the following computing.

The testing condition should correspond to the radial tire's actual working condition as follows:

(1) Rotating frequency: 10 Hz;

(2) Environmental temperature: 25 °C;

(3) Maximum strain: 0.25%.

The data on rubbers' $\tan\delta$ of the radial tire, measured in laboratory, are listed in Table 1.

Table 1 Measured data of rubbers' tanδ and δ

Name of the part*	tanδ	δ (°)
Up tread	0.1354	7.71
Down tread	0.1130	6.45
Shoulder	0.1047	5.98
Sidewall	0.1565	8.89
Up apex	0.2172	12.25
Down apex	0.1125	6.42
Bead	0.1230	7.01

* referring to Fig.7, and tanδ of the other parts without testing will be determined according to some experience

Computing of HRR

When designing a new tire, what designers care for the most is the rubber HRR and the energy loss distribution over different rubber parts on a tire section, both of which are closely related to the radial section configuration of a tire; on the contrary, the tire-ground friction resistance and air drag resistance are related more closely to tire working conditions like truck running speed, road-surface condition, etc. However, testing in laboratory can only give the tire total rolling resistance, with which it is hard to improve tire structure design to obtain lower rolling resistance tire. On the other hand, computing rubber HRR by FEM (finite element method) can provide theoretical reference for more reasonable material properties configuration of the tire's radial section, and then, effectively guide structure optimizing, because it can not only analyze rubber HRR separately without being combined with other kind of rolling resistance, but also obtain energy loss density of different rubber parts on tire section.

HRR can be computed by developing a MATLAB program through the following steps:

(1) Fitting the strain-arc and stress-arc curves for nodes with Fourier series.

When the tire rolls one circle on the ground along a straight path, the strain and stress of each node on a radial section all experience a periodical change with the location of the section, which can be described using the tire's rolling angle as shown in Fig.2. The radial section of a radial tire and the nodes on the section are shown in Fig.7. From Fig.2 it can be seen that all the data on strain-rolling angle curve and the data on stress-rolling angle curve of each node on a section can be obtained from tire FEM results (Ma, 2005).

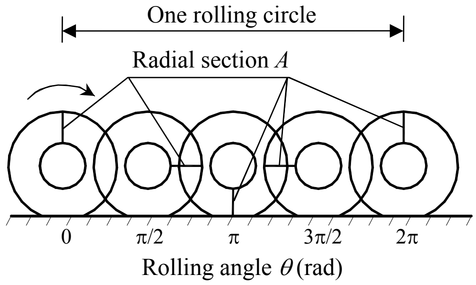


Fig.2 Rolling arc length in one rolling circle

The curves for certain typical nodes are shown in Fig.3, where the stress and strain are equivalent strain and stress of a node, respectively; and the node locations refer to the following Fig.7, i.e. node 1 situated on the centre of crown, node 167 on the widest point of the sidewall, nodes 245 and 246 both on contacting area between tire and rim (contact will occur on node 246 but not on node 245 when loading), node 336 on tire toe. The abscissa in Fig.3 is rolling angle, and the ordinate in Fig.3a and Fig.3b stress and strain, respectively. The peak-values of the curves indicate that the section is in the contacting area with ground.

To apply rubber hysteresis phase angle in computing energy loss, it is necessary to fit each curve of stress and strain vs rolling angle into a trigonometric function. In Fig.4a, the original stress vs rolling angle curve of node 1 indicated by legend "node" and its Fourier fitting curves with 1, 3, 4, 7 order indicated by "fit 1", "fit 3", "fit 4" and "fit 7", respectively, are shown. And in Fig.4b, the relevant strain vs rolling angle curves of node 1 are shown.

Fig.4a and Fig.4b show that the higher the order of the Fourier functions, the more precise the fitting curve is, but the extent of influence decreases with the increasing order. Considering the fitting precision and the computing cost jointly, order 4 Fourier function is chosen to fit the curves of stress and strain vs rolling angle for each node on a radial section.

Order 4 Fourier function can be expressed as:

$$f_{ij}(\theta)=a_0+a_1\cos\theta+b_1\sin\theta+a_2\cos(2\theta)+b_2\sin(2\theta)+a_3\cos(3\theta)+b_3\sin(3\theta)+a_4\cos(4\theta)+b_4\sin(4\theta), \quad (3)$$

where independent variable θ is the rolling angle (rad); the first subscript "i" is 1—stress, 2—strain; and the second subscript "j" is code of node.

(2) Computation of energy loss density for each node on section.

Introducing rubber hysteresis phase angle δ (see Table 1) into the Fourier fitting stress and strain curves with the same phase angle, the resulting stress vs rolling angle curve and strain vs rolling angle curve with a phase angle δ lagging behind for each node on a radial section can be obtained. A 4 order fitting stress curve and a 4 order strain curve with δ lag for node 1 are shown in Fig.5. With the two stress and strain curves in Fig.5, stress vs strain curve for node 1 can then be constructed, which forms 4 loops as shown in Fig.6 when the tire rolls over one circle. The total area of the loops represents the energy loss density of the node during a rolling circle, which can be easily computed by integrating over the loops' area. It is the rubber hysteresis that makes the stress vs strain curve to be a loop consisting of a loading stage and an unloading stage as shown in Fig.6.

The loops' area for node j can be obtained by

$$e_j = \int_0^{2\pi} f_{1j}(\theta)\mathrm{d}f_{2j}(\theta-\delta_j) = \int_0^{2\pi} f_{1j}(\theta) f'_{2j}(\theta-\delta_j)\mathrm{d}\theta, \tag{4}$$

where e_j is loops' area, i.e. hysteresis energy loss den-

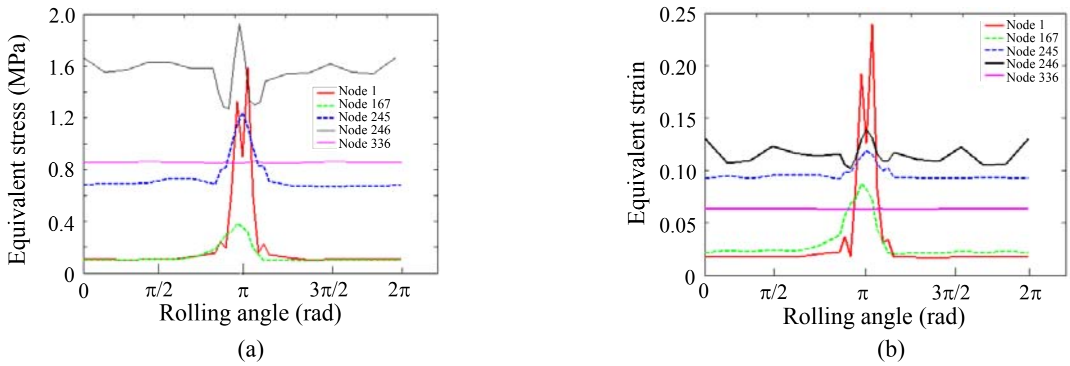


Fig.3 Curves of stress and strain vs rolling angle. (a) Equivalent stress; (b) Equivalent strain

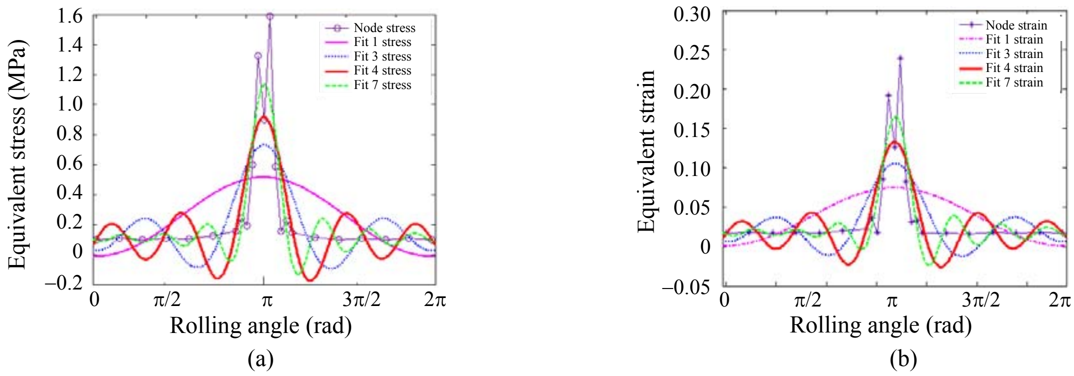


Fig.4 Contrasting graph of different order Fourier fitting curves of node 1. (a) Equivalent stress; (b) Equivalent strain

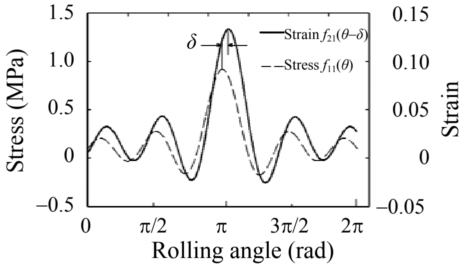


Fig.5 Fourier fitting curves of stress and strain with δ of node 1

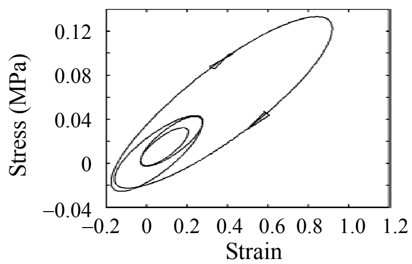


Fig.6 Loops of tire stress-strain of node 1 in one circle with the stress and strain curves as shown in Fig.5

sity of node j; $f_{1j}(\theta)$ is Fourier fitting curve of stress for node j; $f_{2j}(\theta-\delta_j)$ is Fourier fitting curve of strain for node j; and δ_j is hysteresis phase angle for node j (rad).

(3) Computation of energy loss for element.

The elements of rubber material of the tire are all simulated by 4-node axis-symmetry element, whose energy loss density is the average value of that of their 4 nodes, shown as Eq.(5):

$$E_i=(e_{i1}+e_{i2}+e_{i3}+e_{i4})/4, \quad (5)$$

where E_i is hysteresis energy loss density for element i on the section; e_{i1}~e_{i4} are hysteresis energy loss densities for 4 nodes of element i.

The volume of the solid revolving axis-symmetric element can be computed by

$$V_i=2\pi r_i A_i, \quad (6)$$

where V_i is volume of element i on the section; r_i is center radii of element i on the section; and A_i is area of element i on the section.

Element hyteresis energy loss, i.e. work consumed inside element, is the product of hyteresis energy loss density and element's volume, described as

$$W_i=E_i V_i, \quad (7)$$

where W_i is hysteresis energy loss of element i on the section.

According to definition, element HRR can be expressed by the ratio of element energy loss in rolling one circle to element center corresponding perimeter, see Eq.(8):

$$R_i=W_i/(2\pi\rho_i), \quad (8)$$

where R_i is rolling resistance of element i; ρ_i is valid rolling radius of each element on tire section.

Substituting Eqs.(6) and (7) into Eq.(8), yields Eq.(9) for computing element HRR:

$$R_i=E_i A_i r_i/\rho_i, \quad (9)$$

(4) Numerical model for tire HRR.

Summation of element HRR of all the elements on the section will yield that of the total tire; likewise, if summation of that of all elements of a certain part (such as tread, shoulder, sidewall, etc.) on the section, then that of different material part can be obtained, which is shown as

$$RR_k=\sum_{i=1}^{n}R_i=\sum_{i=1}^{n}(E_i A_i r_i/\rho_r), \quad (10)$$

where the subscript "k" in RR_k denotes different letter indicating different material part or the total tire; the adding superscript "n" is element number of different material part or the total tire.

ANALYSIS OF THE COMPUTING RESULTS OF HRR

The MATLAB program of computing rubber HRR was worked out based on a finite element model, which is described in detail in another paper of the same author (Ma, 2005). Applying the program, distribution of HRR of different parts on the section, for a truck radial tire (size 11.00R20) under standard pressure, rated load and special running speed, can be obtained; and the effects of tire working conditions, such as speed, load, pressure, rim width and rim installation on it can also be analyzed.

Distribution of HRR in different part on a radial section

The 11.00R20 truck radial tire section configuration of rubber material is shown in Fig.7 showing that the tire involves eleven parts of rubber materials, which play different roles during tire rolling on load.

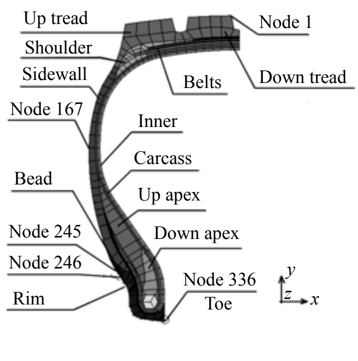
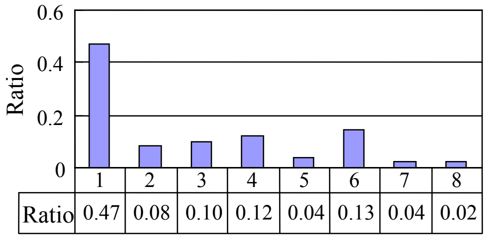


Fig.7 Truck radial tire section configuration of rubber material

HRR distribution in different parts on section, obtained from the MATLAB program, with a load-case of standard pressure 840 kPa, rated vertical load 3270 kg, and the proceeding speed 120 km/h, is shown in Fig.8, whose abscissa expresses different rubber parts on the tire section (refer to Fig.7), and ordinate the ratio of HRR of a part to the total HRR of the whole tire.



1: tread (up and down); 2: belts; 3: shoulder; 4: sidewall; 5: carcass; 6: apex (up and down); 7: bead; 8: inner

Fig.8 HRR distribution ratio of different parts to total

The computing results in Fig.8 show that the HRR of tread accounts for about 47%, which is the most important part of the tire; that of the apex and sidewall for about 14% and 12%, respectively; that of shoulder and belts for 10% and 8%; and that of the other parts of the section such as carcass ply, bead, inner, etc. accounts for only a small proportion.

The real values of HRR of different parts on the section are given in Table 2 showing that the total HRR is 100.98 N, i.e. the HRR factor is 0.006176, which is consistent with engineering practice and empirical value 0.005~0.015; that HRR of up apex accounts for a comparatively larger proportion in contrast to that of down apex; that among the 4 belts, HRR of the second and the third belt-plys are larger than that of the first and the fourth, which related to the stiffness of the steel-cord of the belt, for this tire, stiffness of the first and the fourth steel-cords are far bigger than that of the second and third. Referring to Fig.7 and Table 2, it could be seen that rubber parts nearer to rim like down apex and bead contribute only little to the total HRR, due to which, the rim-bind makes the near rubbers' deformation less.

Table 2 HRR real values of different parts on the section and the total tire

Name		HRR (N)
Total tire		100.984
Tread		47.533
Sidewall		12.365
Shoulder		9.968
Carcass		3.626
Inner		2.441
Apex	Up	13.161
	Down	0.010
Bead	Enforcement	1.142
	Steel ply*	0.665
	Fiber ply*	1.811
Belts	1st	0.577
	2nd	3.338
	3rd	4.010
	4th	0.337

*rubbers acting as matrixes of steel ply and fiber ply located in bead

Effect of tire working condition on HRR

The analysis above reveals that, among so many rubber parts on the tire section, such as bead and carcass contribute little to total HRR, both only accounting for 3.6%. To simplify the analyzing below, combine bead with apex into one part named "bead", and carcass with sidewall into "sidewall".

1. Effect of speed on HRR

Compared with static vertical load case, when the tire rolls at constant speed under specified pressure and load, the section profile will change, the width of grounding section will decrease, the overall diameter will increase, and the distribution of stress and strain on the section will also change, and then the distribution of rubber hysteresis energy loss density will differ.

Fig.9 shows the effect of speed on HRR of different part on the tire section and reveals that the change of speed makes little difference on HRR when proceeding speed is lower than 80 km/h; but when

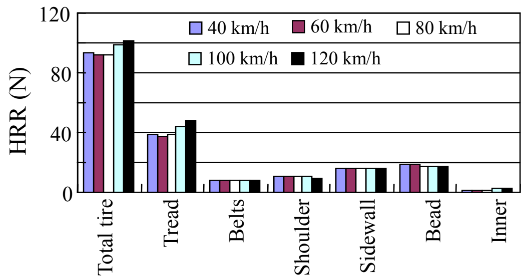


Fig.9 Effect of speed on HRR of different part on the tire section

speed goes up to above 100 km/h, the HRR of total tire and tread will increase with increasing speed; among different rubber parts, speed affects that of tread greatly, that of bead a little, and that of shoulder, sidewall, belts very little.

2. Effect of load on HRR

Load influence on HRR is great. The effect of load on HRR of different part on the tire section is shown in Fig.10 revealing that, under standard pressure and 120 km/h speed, HRR of the total tire and almost all parts on the section all linearly increase with increasing load.

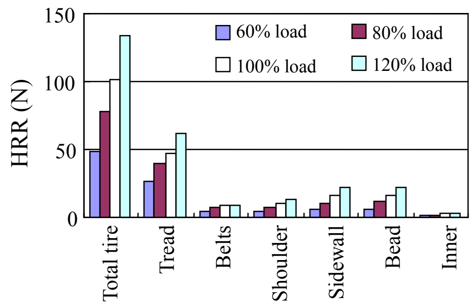


Fig.10 Effect of load on HRR of different part on the tire section

3. Effect of pressure on HRR

Fig.11 shows that the effect of pressure on HRR of different part on the tire section is nonlinear, that is, when the pressure is lower than the standard 840 kPa, the pressure affects the total tire's HRR a little; but when pressure goes up to 960 kPa, the HRR decreases greatly; the HRR of tread is the greatest when the pressure is 900 kPa; the HRR of belts, shoulder and inner are almost not influenced by pressure; and that of sidewall and bead slightly decreases with increasing pressure.

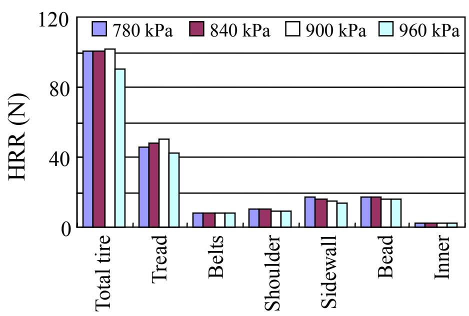


Fig.11 Effect of pressure on HRR of different part on the tire section

4. Effect of rim width on HRR

Rim width influences the deformation, stress and strain of the steadily rolling tire greatly, and the design rim width and the installation rim width will both affect HRR of tire and different part evidently, just as described in Fig.12.

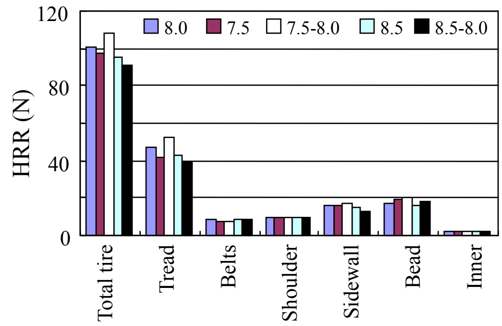


Fig.12 Effect of rim width on HRR of different part on the tire section

In Fig.12, legend "8.0" means that standard design rim width and standard installation rim width are used; "7.5" or "8.5" means that the design rim width and installation rim width both are the same, i.e. 7.5 inch or 8.5 inch, respectively; "7.5-8.0" or "8.5-8.0" means that the design rim width is 7.5 inch or 8.5 inch, and the corresponding installation rim width are both 8.0 inch.

Fig.12 shows that the total HRR of 7.5-8.0 is the biggest, and that of 8.5-8.0 the least; among different parts, the design rim width and installation rim width affect the HRR on tread the greatest, and on bead the second greatest, the HRR of 8.5-8.0 is larger on the bead, but is the lowest on the tread. Thus, it can be seen that the case of 8.5-8.0 is the best for low HRR tire design.

CONCLUSION

From the view of tire structure design, HRR distribution on tire section is influenced by tire structure parameters, mechanical properties of materials, especially the properties of rubbers used in different part, and all kinds of tire working conditions, which cannot be achieved and analyzed easily by experiment. The HRR numerical computation program developed in this paper can be used to compute HRR of total tire or that of any part on section, and accordingly to analyze effects of parameters, including material properties, structure, or tire working conditions, on tire total HRR and distribution of different part HRR on section.

From the computing and analyzing results, conclusions can be drawn as follows:

(1) The HRR of tread on the truck radial tire section accounts for 47% of that of the total tire, contributing the most hysteresis energy loss during tire running. Thus, in order to reduce the HRR of a radial tire more effectively, measures should be first taken to reduce the HRR of tread rubber. To accomplish this goal, not only should the hysteresis factor tanδ be reduced as low as possible, but also the tire radial section profile and configuration should be improved or optimized to make the stress and strain distribution more reasonable.

(2) Besides tread, up-apex and sidewall contribute the second greatest to the tire's total HRR among all the parts on the section.

(3) Among all kinds of tire working conditions mentioned in this paper, the load has the most remarkable effect on the total HRR and its distribution on the section.

(4) Only when speed is higher than 100 km/h, HRR will increase with increasing speed. Among all parts of tire section, tread is influenced the most by speed, and the other parts are all affected slightly by speed.

(5) As for pressure, once it is up to 960 kPa, HRR of the total tire decreases remarkably, and all parts on the section have similar changing trends; the HRR change extent of tread with pressure is also the most.

(6) It can be proved by the numerical computing results that how to design and install the tire with rim has much to do with the tire's HRR. From cases listed in this paper, it is obvious that the design case of 8.5-8.0 makes the lowest tire total HRR, but not the standard design of 8.0.

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