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Novel approach for determining the optimal axial preload of a simulating rotary table spindle system

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Abstract: This paper presents a new theoretical model to determine the optimal axial preload of a spindle system, for challenging the traditional method which relies heavily on experience of engineers. The axial preloading stiffness was treated as the sum of the spindle modal stiffness and the framework elastic stiffness, based on a novel concept that magnitude of preloads can be controlled by measuring the resonant frequency of a spindle system. By employing an example of a certain type of aircraft simulating rotary table, the modal stiffness was measured on the Agilent 35670A Dynamic Signal Analyzer by experimental modal analysis. The equivalent elastic stiffness was simulated by both finite element analysis in ANSYS® and a curve fitting in MATLAB®. Results showed that the static preloading stiffness of the spindle was 7.2125×10^7 N/m, and that the optimal preloading force was 120.0848 N. Practical application proved the feasibility of our method.

Key words: Three-axis simulating rotary table, Axial position preload, Stiffness, Experimental modal analysis, Finite element analysis
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INTRODUCTION

As the main part of a three-axis simulating rotary table, a spindle has significant influence on the accuracy, mobility and reliability of the system (Alasty and Abedi, 2003). A three-axis rotary table has three spindles: the inner one, the middle one and the outer one. Each spindle consists of a shaft, bearings, a framework and relevant screw fasteners.

In recent years, emphases have been focused on the axial stiffness which is critical to the precision of the system. The axial stiffness, which is 2.83 times of that one without preload based on the Hertz contact theory (Johnson, 1985), can be greatly increased under an axial preload (Jorgensen and Shin, 1998; Hernot *et al*., 2000). However, a too small preloading force does not make sense, while an excessive preloading force cuts down the service life of the system by increasing the contact stress and the friction. The conventional method for determining the axial preloading force is the handwork which relies heavily on the personal experience of engineers. Accordingly, no exact magnitude of the preload can be determined by the conventional method. In order to overcome this limitation, it is necessary to develop a method for determining the optimal preloading force.

Li (1995) mentioned a novel concept that the magnitude of the preload can be controlled by using the measured resonant frequency of a spindle system. It was just an idea, with the details on how to achieve it not being described. Mannan and Stone (1998) described a method for checking the spindle assembly by measuring the dynamic response, which can possibly be used to adjust the preload and check the frequency for the quality control of a machine tool spindle system. Alfares and Elsharkawy (2000; 2003) highlighted a mathematical model based on a five-degree-of-freedom dynamic system to study the effects of axial preloads on the vibration behavior of a grinding machine spindle. However, the configuration of the spindles in a three-axis rotary table significantly differs from that of the said spindles. Consequently, based on the concept, the current paper presents a model where the static preloading stiffness is the summation of the spindle's modal stiffness and the elastic stiffness of the framework. The modal stiffness will be obtained by the experimental modal analysis and the developed mathematical model of a spindle system in our previous work. Moreover, finite element analysis in ANSYS will be used to calculate the equivalent elastic stiffness. By treating the framework as a spring element which conforms well to Hooke's Law, the relationship between deformation and preloading forces will be obtained by a curve fitting in MATLAB.

THEORETICAL STIFFNESS MODEL OF A SPINDLE SYSTEM

The stiffness of a spindle system within elastic deformation boundaries was assumed as the summation of the modal stiffness of the spindle and the framework elastic stiffness

$$K_{as}=K_{am}+K_{e}, \quad (1)$$

where K_{as} is the stiffness of a spindle system, K_{am} the modal stiffness of a spindle system and K_e is the equivalent stiffness of the framework by the finite element analysis.

Dynamic modelling for the modal stiffness

Dynamic analysis and optimization are usually used to design higher accuracy, flexibility and reliability rotary table frameworks. Moreover, an equivalent handling, which keeps the mass and the moment of inertia equal, is usually applied in the structural dynamic analysis. Fig.1 is a 2D sketch of the outer framework of a three-axis rotary table on which the experimental modal analysis would be performed.

However, in the conventional dynamic analysis, bearings in a spindle are often replaced by radial and axial spring elements with stiffness but no mass (Zverv *et al.*, 2005; Padmanabhan *et al.*, 2006), by neglecting the damping, such as the external friction from the structure and the supporting, and the internal friction from materials and the medium (Dietl *et al.*, 2000). A new model taking the damping into account, therefore, was presented in (Xie *et al.*, 2006) (Fig.2).

For an angular contact bearing, the axial deformation per unit load is considerably higher than that of the radial deformation. And in most situations, the principal concentration is focused on the axial load and the axial deformation. Consequently, a simplified dynamic model of a spindle was presented by neglecting the radial spring element and the cross-couple stiffness. Fig.3 shows a simplified axial dynamic model of a spindle.

According to Fig.3, the dynamic characteristics differential equation of the spindle-bearing system can be defined as Eq.(2):

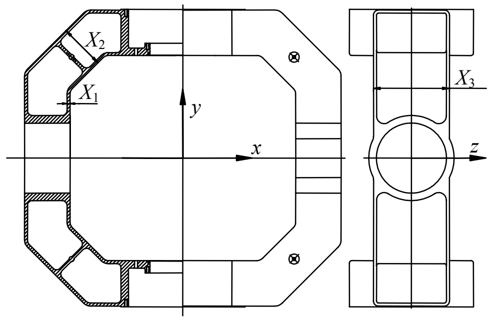

Fig.1 2D sketch of a three-axis rotary table's outer framework

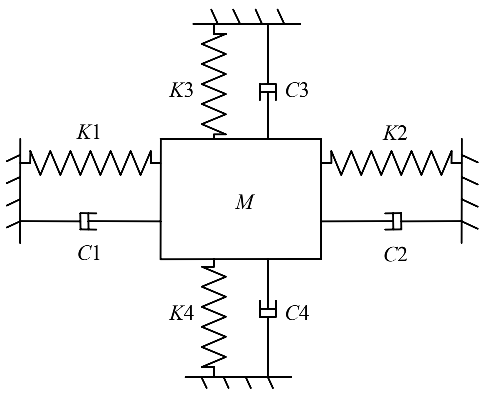

Fig.2 Dynamic model of a spindle

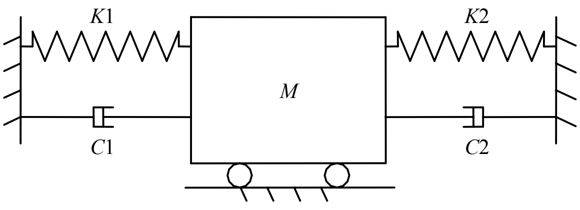

Fig.3 Axial dynamic model of a spindle

$$M\ddot{x} + C\dot{x} + K_{am}x = F, \tag{2}$$

where M is the modal mass of a spindle system, C the modal damping of a spindle system and F is the exciting force.

The natural frequency, the modal damping and the modal shape can be obtained by experimental modal analysis (Dutta *et al*., 2001; Forrest, 2006). The axial stiffness and the natural frequency of the spindle system are

$$K_{am} = M\omega_n^2, \tag{3}$$

where ω_n is the modal frequency of a spindle system.

Finite element modelling

The framework, which is constructed from cast duralumin with a low Young's modulus (78 GPa), has O-shaped structure (as shown in Fig.1). Axial loads mainly yield axial elastic deformation of the framework (Wang and Chen, 2005). Therefore, we assumed that the framework is a spring with high stiffness, which conforms to Hooke's Law.

According to the above assumption, the relationship between the preloading force F_{a0} and the bearing axial deformation $\delta(F_{a0})$ can be described by a general equation

$$\delta(F_{a0}) = aF_{a0}^3 + bF_{a0}^2 + cF_{a0} + d, \tag{4}$$

where a, b, c and d are constant coefficients, F_{a0} is the preloading force of a spindle system and $\delta(F_{a0})$ is the bearing axial deformation.

The stiffness of a framework K_e is obtained by the derivative of Eq.(4) with respect to the variable F_{a0} based on Hooke's Law. A function between the preloading force and the stiffness is therefore

$$\frac{d\delta(F_{a0})}{dF_{a0}} = 3aF_{a0}^2 + 2bF_{a0} + c, \tag{5}$$

$$K_e = \frac{d\delta(F_{a0})}{dF_{a0}}, \tag{6}$$

$$K_e = 3aF_{a0}^2 + 2bF_{a0} + c. \tag{7}$$

In order to determine the constant coefficients a, b, c and d, a finite element method is used. Accordingly, a 3D finite element model of the said rotary table framework was developed, as shown in Fig.4. Fig.5 illustrates the application of a preloading force on nodes in x direction of the framework. Four nodes at the terminal face of the model in the positive x direction are considered, which are N_1, N_2, N_3 and N_4, respectively. Simultaneously, other four nodes at the terminal face in the negative x direction are selected, which are N_5, N_6, N_7 and N_8, respectively. The deformation of the above eight nodes can be simulated by the finite element analysis in ANSYS.

RESULTS AND DISCUSSION

Results of the experimental modal analysis

An example of a certain type of three-axis aircraft simulating rotary table was provided. The experimental modal analysis was performed on the outer spindle system. The resulting power spectrum was measured on the Agilent 35670A Dynamic Signal Analyzer. Fig.6 shows that the natural frequency of the spindle-bearing system is equal to 152 Hz. Fig.7 shows an example of the finite element deformation graph of the framework. Table 1 shows the result of the finite element analysis.

According to Table 1, the total axial deformation of the framework was calculated by the weighted-average method at four points. Moreover, the polynomial coefficients in Eq.(7) were calculated by the polynomial curve fitting in MATLAB. The result is $a=b=d=0$ and $c=0.0162$.

Fig.8 shows the graph with respect to the deformation of the eight nodes and the polynomial fitting curve. According to the polynomial fitting curve, a linear relationship between the total deformation of the outer framework and the preloading force was observed.

Optimal preloading force

The axial load is always varying nonlinearly in practical application. Consequently, an approximation algorithm has been developed by researchers in response to the impossibility of computing the bearings stiffness exactly. When an axial preload F_{a0} acts on a pair of angular contact ball bearings, the empirical equation between the axial bearing stiffness and the preloading force is

$$K_{ab} = 3.44\times10^6(F_{a0}Z^2D_b\sin^5\alpha_0)^{1/3}, \tag{8}$$

where K_{ab} is the axial bearing stiffness obtained by the empirical equation, Z the number of balls in an angular contact ball bearing, D_b the diameter of the ball and α_0 is the contact angle.

Eqs.(1) and (8) were used to calculate the optimal preloading force of the spindle system. Table 2 shows the parameters and results. Fig.9 shows that the curve of the function $K_{ab}(F_{a0})$ cuts the line of the function $K_{as}(F_{a0})$ at a unique point which is the theoretical optimal result.

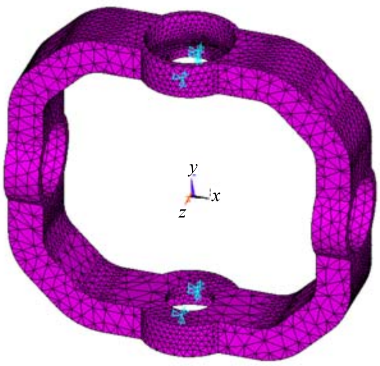

Fig.4 3D finite element model of the rotary table framework

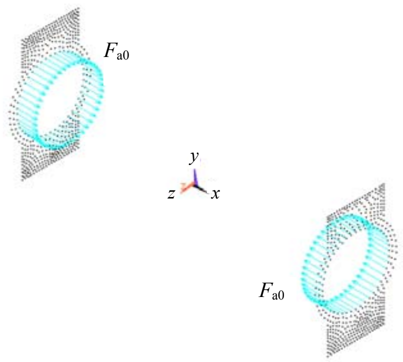

Fig.5 Apply a preloading force on nodes in x direction

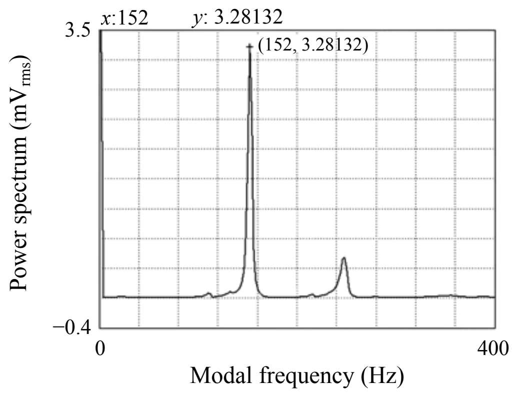

Fig.6 Resonance spectrum of the spindle system

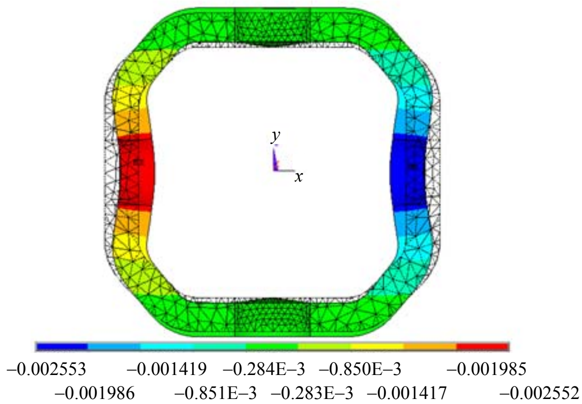

Fig.7 An example of the finite element deformation of the simulating rotary table framework

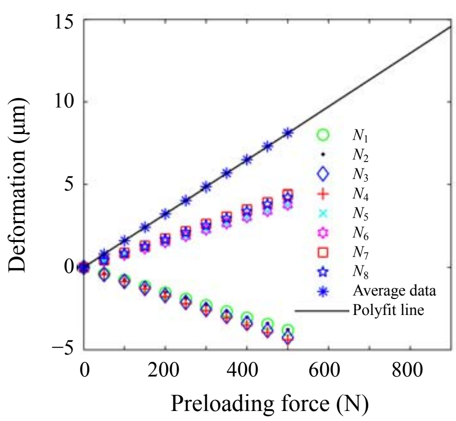

Fig.8 Nodes deformation and the fitting curve

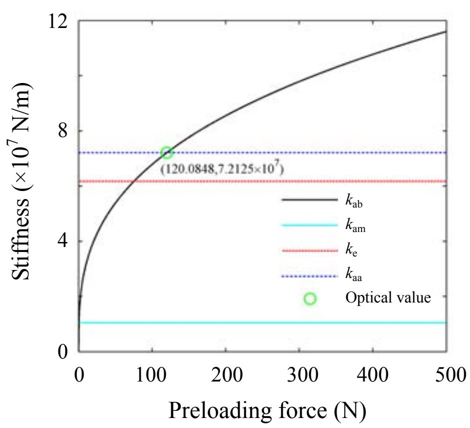

Fig.9 Optimal preloading force

Table 1 Nodes and their deformation

Preloads (N)	Deformation of nodes (μm)							
	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
50	−0.3814	−0.3779	−0.4254	−0.4393	0.3780	0.3804	0.4396	0.4248
100	−0.7627	−0.7558	−0.8509	−0.8786	0.7559	0.7607	0.8791	0.8496
150	−1.1441	−1.1337	−1.2763	−1.3179	1.1339	1.1410	1.3187	1.2744
200	−1.5255	−1.5116	−1.7018	−1.7572	1.5118	1.5214	1.7583	1.6991
250	−1.9068	−1.8894	−2.1272	−2.1965	1.8898	1.9017	2.1978	2.1239
300	−2.2882	−2.2673	−2.5526	−2.6358	2.2677	2.2821	2.6374	2.5487
350	−2.6696	−2.6452	−2.9781	−3.0751	2.6457	2.6624	3.0770	2.9735
400	−3.0509	−3.0231	−3.4035	−3.5144	3.0236	3.0428	3.5165	3.3983
450	−3.4323	−3.4010	−3.8289	−3.9537	3.4016	3.4231	3.9561	3.8231
500	−3.8137	−3.7789	−4.2544	−4.3930	3.7795	3.8035	4.3957	4.2479

Table 2 Parameters and analytical results

Parameters	Values
Young's modulus of the cast duralumin E (GPa)	78
Poisson's ratio of the cast duralumin ε	0.33
Number of balls in an angular contact ball bearing Z	20
Diameter of the ball D_b (mm)	14.23
Contact angle α_0 (rad)	0.44
Modal mass of a spindle system M (kg)	450
Modal frequency of a spindle system ω_n (Hz)	152
Modal stiffness of a spindle system K_{am} ($\times 10^7$ N/m)	1.0397

CONCLUSION

This paper presents a novel theoretical model in which the static preloading stiffness is the summation of the modal stiffness and the elastic stiffness for a spindle system. By employing an example of a certain type of three-axis aircraft simulating rotary table, the modal stiffness, the equivalent elastic stiffness and the relationship between deformation and preloading forces were obtained. Several conclusions were drawn:

(1) The static preloading stiffness of the spindle is K_{as}=7.2125×10^7 N/m with the optimal preloading force being 120.0848 N.

(2) Our method offers an effective theoretical and experimental basis to control the magnitude of axial preload for the research and development of simulating rotary tables. It is also appropriate for a general spindle system with angular contact ball bearings.

(3) In the present study, the coupling stiffness between the axial and radial forces is neglected. Further research will be focused on the coupling stiffness.

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