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Optimal restructuring strategies under various dynamic factors

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Abstract: Corporate restructuring was identified as a new industrial force that has great impact on economic values and that therefore has become central in daily financial decision making. This article investigates the optimal restructuring strategies under different dynamic factors and their numerous impacts on firm value. The concept of quasi-leverage is introduced and valuation models are built for corporate debt and equity under imperfect market conditions. The model's input variables include the quasi-leverage and other firm-specific parameters, the output variables include multiple corporate security values. The restructuring cost is formulated in the form of exponential function, which allows us to observe the sensitivity of the variation in security values. The unified model and its analytical solution developed in this research allow us to examine the continuous changes of security values by dynamically changing the coupon rates, riskless interest rate, bankruptcy cost, quasi-leverage, personal tax rate, corporate taxes rate, transaction cost, firm risk, etc., so that the solutions provide useful guidance for financing and restructuring decisions.

Key words: Restructuring, Capital structure, Financial decisions
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INTRODUCTION TO THE MODEL

This research deals with internal restructuring because this type of restructuring is an important financing decision facing financial managers. And internal restructuring theory is central in corporate finance. The value changes of external restructuring eventually happen via the internal restructuring. The models of the debt and equity value developed in the research consider restructuring costs, and the effects of personal and corporate taxes (Berger and Patti, 2006). The research focuses on optimal restructuring strategies and their impact on security values.

Contingent claims models can provide a consistent framework for multi-period valuation with proper consideration of risk (Douglas, 2006). In order to use this framework in restructuring, certain adaptations are necessary. A specific PDE must be found to value a firm's securities in restructuring. A number of researchers made efforts along those lines, e.g. (Leland and Toft, 1996). All of these studies represent a category of models that establish optimal capital structure by trading off tax advantage and potential costs associated with debt financing. While these papers and others explored one aspect or another of capital structure and demonstrated the existence of a theoretical optimum, they do not provide a detailed model that simultaneously measures all key factors for dynamic restructuring of a specific firm, so as to decide on the optimal restructuring policy for that firm.

To build a model measuring dynamic restructuring, a quasi-leverage ratio needs to be introduced (Meng, 1999). I define this ratio as the face value of the firm's debt divided by the sum of the face value of the firm's debt and the value of the firm's unlevered assets. The notations to be used are as follows: U is the firm's unlevered asset value; E is the market value of equity; D is the market value of debt; V is the total market value of the firm; F is the face value of the firm's debt; C is the instantaneous coupon payment; φ is the instantaneous corporate tax rate; δ is the instantaneous personal tax rate.

Introduce the firm's quasi-leverage ratio as:

$$L=F/(F+U).$$

Assuming that the unlevered value of the firm follows a logarithm Wiener process

$$\mathrm{d}U/U=\mu\mathrm{d}t+\sigma\mathrm{d}Z,$$

where μ is the expected instantaneous rate of return on the firm's unlevered assets U, σ is the instantaneous standard deviation, and Z is a standard Brownian motion. The stochastic process of the firm's unlevered assets U will not be influenced by the capital structure of the firm. The equity and debt value of the firm are claims on the firm's unlevered assets, i.e., they are functions of the unlevered value of the firm U, the face value of the debt F and time t. Let us denote these claims by $E(U, F, t)$ and $D(U, F, t)$. According to (Meng, 1999), the ODE for debt and equity can be derived as follows:

$$\frac{1}{2}U^2\sigma^2\frac{\partial^2 D}{\partial U^2}+U[\mu-r^*+r(1-\delta)]\frac{\partial D}{\partial U}-r(1-\delta)D+(1-\delta)C=0,$$

and

$$\frac{1}{2}U^2\sigma^2\frac{\partial^2 E}{\partial U^2}+U[\mu-r^*+r(1-\delta)]\frac{\partial E}{\partial U}-r(1-\delta)E+(1-\delta)C=0.$$

These forms of ODEs with boundary conditions are mathematically tractable, and with some transformation may have closed-form solutions. This property greatly facilitated the analysis in this research.

DYNAMIC RESTRUCTURING: OPTIMAL STRATEGY AND COMPARATIVE STATICS

Debt can be classified as risky or riskless according to the level of the quasi-leverage ratio L. Bankruptcy may occur in the case of risky debt. If the bankruptcy can be decided endogenously rather than being imposed by the limited liability or by a covenant that requires a certain positive asset value, the firm can either delay the bankruptcy until it can no longer afford to make the instantaneous coupon payment by issuing additional equity, or it can declare bankruptcy at a certain level of equity value chosen to maximize the total firm value (Dittmar, 2004). The following analysis about optimal restructuring policy and the effects upon the security values will focus on risky debt. Certain comparisons are conducted between the risky and riskless debt cases. The type of debt that yields the higher optimal firm value should be chosen in debt financing.

Now we solve for the numerical results for base optimum of value maximization. In both risky and riskless debt cases, the objective of restructuring is to maximize the firm value V less the restructuring costs $f(F)$, which is incurred in the restructuring as an increasing function of the debt face value:

$$\max[V-f(F)].$$

The firm value V, as well as the equity value E and the debt value D are functions of the lower quasi-leverage $L_{\min}$ and the upper quasi-leverage $L_{\max}$, the face value of the outstanding debt F, and the unlevered asset value U. This optimization problem is equivalent to finding the capital structure equilibrium based on the non-arbitrage theory (Howard, 2006). The constraints for this optimization problem are constructed below.

According to the non-arbitrage theory, the firm value before and after the restructuring activity should be equal, i.e., the firm value right after restructuring equals the sum of a firm's unlevered asset value and the associated restructuring cost. Based on this equality, a constraint is established:

$$V(L_0, F_0)=(1/L_0-1)F+f(F),$$

where L_0 denotes the optimal quasi-leverage ratio after restructuring, the first item on the right side of the equation is the unlevered assets value derived from the following:

$$L_0=F/(F+U).$$

The restructuring costs are given by an exponential function:

$$f(F)=F(1-\mathrm{e}^{-\beta F}),$$

where β is called the coefficient of the restructuring costs. The above function is a function that reflects the magnitude of restructuring costs.

The second constraint is constructed by using the requirement of limited liability (Huang and Song, 2006). With this requirement, the equity value will be limited to be nonnegative. It may be required to be a positive value by debt covenants. We have a constraint:

$$E(L, F)\geq\mu,$$

where $\mu\geq0$ is the bankruptcy equity value required by covenant. In the following formulation, let $\mu=0$.

The last constraint is on the newly issued debt. In solving the maximization problem, a group of parameters including the quasi-leverage ratios $L_{\min}$, $L_{\max,}$ the coupon rate γ, the rate of return surplus on unlevered asset ρ, and the initial optimal debt F will be found. With a proper coupon rate, the newly issued debt value can be adjusted to be equal to its par value, i.e., the following equality holds:

$$D(L, F)=F.$$

The problem formulation follows (Leland and Toft, 1996) with some modifications. Numerical solutions to these problems are resorted. To run the models, parameter inputs are needed. Historical data are referred in the parameter specification. The parameters are specified as follows: firm risk (standard deviation of the unlevered firm value) σ=0.22; instantaneous corporate tax rate φ=0.45; instantaneous personal tax rate δ=0.33; riskless interest rate r=0.05; bankruptcy cost coefficient m=0.03; restructuring cost coefficient β=0.0015. Where both corporate and personal taxes are instantaneous tax rates. The optimal restructuring strategy is computed when the size of unlevered asset value U_0=100. The coefficient

$$r^*=\beta_U(r_M-r)+r$$

is the equilibrium required rate of return on the firm's debt or equity, μ is the draft in the logarithm Wiener process and can be thought as the instantaneous rate of return on the firm's unlevered assets (Duffie, 2001). Then define

$$\rho=r^*-\mu$$

as the rate of return surplus on the unlevered assets (return surplus, thereafter) due to debt and equity financing. The optimal restructuring strategy can be established by solving the maximization problem formulated above (Jin, 2006). The numerical results for the optimal restructuring strategies of both risky debt and riskless debt are found, as shown in Table 1.

Table 1 The base optimal capital structure decisions with the specified firm parameters for both risky debt and riskless debt

Parameters	Values	
	Risk	Riskless
Coupon rate (%)	5.74	2.10
Lower leverage limit	0.18	0.15
Upper leverage limit	0.64	0.38
Span of leverage	0.46	0.23
Deficiency to return (%)	0.06	0.45
Initial optimal leverage	0.32	0.27

From Table 1, it can be seen that the coupon rate for risky debt is higher than that for riskless debt, 5.74% vs 2.1% . This result is reasonable because the risky debt involves a risk to go bankrupt while the riskless debt does not. The greater the risk, the higher the return. The quasi-leverage ratio of risky debt varies from 0.18 to 0.64. For riskless debt, this span is from 0.15 to 0.38. Clearly, risky debt has a wider variation in leverage and its leverage ratio can be more than 0.5. And the capital structure with risky debt enjoys a higher optimal initial leverage ratio. A less intuitively understandable result is that riskless debt reaches a larger return surplus P. This rate represents economic return on the unlevered assets. Its role is similar to that of coupon rate paid on the debt. The unlevered assets bear less risk for the riskless debt than for the risky debt. The return surplus for riskless debt should be larger than that for risky debt. The conclusions drawn here are consistent with observation from the real world.

OPTIMAL RESTRUCTURING STRATEGIES UNDER VARIOUS DYNAMIC FACTORS

The optimization model can be solved repeatedly with varying values of firm-specific parameters (Miao, 2005). The solutions of it then feature the optimal restructuring strategies in dynamic environments. The parameters are unlevered asset size (U), corporate tax rate (R), restructuring cost (β), firm risk (σ), and riskless interest rate (r). All these parameters have an impact on the firm's leverage, coupon rate, equity and debt values, the return surplus, and the yield spread of debt.

Effects of dynamic unlevered asset level

Changing the value of U (ranging from 70 to 160) and solving the optimization problem formulated repeatedly, we obtain Table 2. The firm's unlevered asset level represents the firm's initial size before it issues debt.

Table 2 Optimal restructuring strategy for different levels of unlevered assets

U	L_0	γ (%)	d (%)	$L_{\min}$	$L_{\max}$
70	0.40	6.36	0.06	0.18	0.59
80	0.37	6.11	0.06	0.19	0.60
90	0.34	5.92	0.06	0.18	0.62
100	0.32	5.74	0.06	0.18	0.64
110	0.29	5.61	0.06	0.17	0.66
120	0.29	5.53	0.06	0.16	0.68
130	0.28	5.44	0.06	0.15	0.70
140	0.26	5.33	0.06	0.14	0.73
150	0.23	5.26	0.05	0.13	0.75
160	0.24	5.24	0.05	0.13	0.76

U: unlevered asset; L_0: initial leverage; γ: coupon rate; d: deficiency to return; $L_{\min}$: lower leverage; $L_{\max}$: upper leverage

Table 2 shows the numerical results of the optimal restructuring strategy for risky debt with dynamic unlevered assets. This result shows that with an increase in the unlevered assets size, the initial optimal leverage drops steadily. The fact is that the optimal amount of debt is not proportional to the size of the company. When other firm-specific parameters are fixed, the growth of unlevered asset size makes it possible for a firm actually to use more debt than before in an absolute amount or to reach a higher optimal leverage ratio. The initial optimal leverage ratio, however, is relatively low as Table 2 demonstrates. Larger firms (in terms of unlevered assets) allow larger leverage spans because they possess greater ability of liquidation, which means converting assets to cash and terminating the outside claims. Debt is a corporate security with a general claim against the firm's total assets. If a firm defaults, it will be liquidated to repay the debt holders. Thus, firms with larger unlevered assets size have more advantages in financing options.

At various levels of unlevered assets, the coupon rate (γ) and leverage limits ($L_{\min}$, $L_{\max}$) do not significantly change in Table 2. In practice, we have seen corporate bonds from different firms have similar coupon rates no matter what size the firms are. Larger firms are thought to have a stronger ability to get their payoff on the unlevered assets. This situation is reflected in Table 2, i.e., the return surplus (s) drops slightly with an increase in the size of the unlevered assets.

Effects of dynamic corporate tax rate

Changing the value of corporate tax rate R (ranging from 0.35 to 0.49) and solving the optimization problem, we obtain Table 3. The solutions feature the optimal restructuring strategies for dynamic corporate tax rate.

Table 3 The optimal restructuring strategy for different levels of corporate tax rate

R	L_0	γ (%)	s (%)	$L_{\min}$	$L_{\max}$
0.35	0.08	5.01	0.00	0.03	0.93
0.37	0.16	5.03	0.01	0.07	0.90
0.39	0.21	5.06	0.03	0.08	0.88
0.41	0.28	5.14	0.05	0.10	0.85
0.43	0.29	5.33	0.05	0.16	0.78
0.45	0.32	5.74	0.06	0.18	0.64
0.47	0.33	5.89	0.06	0.19	0.60
0.49	0.34	5.96	0.06	0.20	0.57

R: corporate tax rate; L_0: initial leverage; γ: coupon rate; s: return surplus; $L_{\min}$: lower leverage; $L_{\max}$: upper leverage

It can be seen that with an increase in corporate tax rate, the leverage increases too. This is consistent with the trade-off theory of capital structure (Leary and Roberts, 2005). With other parameters fixed, as the corporate tax increases, the tax shield of debt increases because the coupon payments are tax deductible. Then the firms are better off if they incur more debt. Therefore, they respond by issuing more debt to take advantage of this tax shield. In order to attract enough debt, the issuing firms tend to offer higher coupon rates. In case of a high tax shield, it is in the firm's best interest to issue more debt to maintain a high leverage level. In this situation, the advantage of debt is apparent, and firms will be less hesitant to employ more debt and less likely to diversify their capital structure by seeking the other type of assets, e.g., equity. This advantage of debt financing explains why the leverage variation span is small when the corporate tax rate is high. At the lower range of corporate tax rates (35%~41%), the initial optimal leverage increases relatively fast; at the higher range (41%~49%), this increment slows down. This distinction can be explained by using a tax-shield-bankruptcy-equilibrium argument. As corporate tax increases, the risk and the cost of bankruptcy increase accordingly. Until a certain level of leverage is reached, an equilibrium between these two factors is established.

Table 3 also shows that the return surplus increases slightly as the leverage and corporate tax increase. This phenomenon makes intuitive sense. With an increase in corporate debt, the shareholders are influenced by offering return to debt holders, and therefore the return to unlevered assets drops. As a result, the return surplus increases.

Effects of dynamic restructuring cost

The optimization problem can be solved repeatedly for different levels of restructuring cost β. The numerical results are shown in Table 4. This table characterizes the dynamic process of varying restructuring cost.

The restructuring cost is incurred in the issuance and retirement of debt. It is measured by a convex increasing nonlinear function of the firm's debt:

$$f(F)=F(1-\mathrm{e}^{-\beta F}).$$

Table 4 The optimal restructuring strategy for different levels of restructuring cost

β	L_0	γ (%)	k (%)	$L_{\min}$	$L_{\max}$
0.0005	0.37	6.16	0.08	0.25	0.53
0.0010	0.34	6.04	0.07	0.21	0.59
0.0015	0.32	5.74	0.06	0.18	0.64
0.0020	0.29	5.51	0.05	0.15	0.69
0.0025	0.27	5.32	0.05	0.12	0.74
0.0030	0.24	5.19	0.04	0.10	0.78
0.0035	0.20	5.09	0.04	0.08	0.84
0.0040	0.18	5.06	0.04	0.07	0.87
0.0045	0.16	5.04	0.03	0.06	0.88

β: restructuring cost coefficient; L_0: initial leverage; γ: coupon rate; k:deficiency to return; $L_{\min}$: lower leverage; $L_{\max}$: upper leverage

Changing the restructuring coefficient β will simulate different levels of restructuring cost. In Table 4, β varies from 0.0005 to 0.0045. As the restructuring cost increases, the initial optimal leverage ratio decreases slightly. This effect is consistent with the fact that the higher the restructuring cost becomes, the more expensive the debt is. Therefore, the leverage shrinks. For the same reason, firms facing increasing restructuring cost are not in a position to offer the coupon rate as high as before. On the other hand, if firms still find debt favorable and are willing to bear this cost and continue to take advantage of the tax benefit of the debt, they have the option to use different levels of debt. This is why the leverage span is wide.

We have seen that with the increase in restructuring cost, the initial optimal leverage ratio drops. This means that debt accounts for a smaller portion of the capital structure than before and that unlevered assets will get a better return than before. That is why the return surplus drops.

Effects of dynamic firm risk

Changing the value of firm risk σ (ranging from 0.07 to 0.49), and solving the optimization problem repeatedly, we obtain the numerical results of the optimal restructuring strategy for dynamic bankruptcy cost. Data is shown in Table 5, and it illustrates the dynamic effects.

The firm risk means the standard deviation of the firm's total assets. With all increase in this standard deviation, the volatility of the firm's assets increases.

Table 5 The optimal restructuring strategy for different levels of firm risk

σ	L_0	γ (%)	k (%)	$L_{\min}$	$L_{\max}$
0.07	0.35	5.00	0.05	0.20	0.60
0.12	0.34	5.01	0.05	0.20	0.66
0.17	0.33	5.21	0.06	0.19	0.64
0.22	0.32	5.74	0.06	0.18	0.63
0.27	0.30	6.44	0.06	0.16	0.65
0.32	0.29	7.26	0.06	0.15	0.66
0.37	0.28	8.31	0.06	0.15	0.66
0.42	0.26	9.32	0.06	0.14	0.66
0.47	0.25	10.45	0.06	0.13	0.66
0.49	0.25	10.96	0.06	0.13	0.66

σ: firm risk; L_0: initial leverage; γ: coupon rate; k: deficiency to return; $L_{\min}$: lower leverage; $L_{\max}$: upper leverage

In response, firms sacrifice leverage: they lower the initial optimal leverage ratio to stabilize its security values. The fluctuation in the leverage span is not significant, but it can be recognized in the increasing direction.

At the same time the firm risk increases, the firm is willing to pay a much higher coupon rate to attract debt. This behavior explains "junk bonds," which are highly risky and also much higher in coupon payment. The higher the firm risk is, the more volatile the firm value is. So the return on debt should be adjusted upward.

Table 5 also shows that the dynamic firm risk does not have much impact on the return surplus.

Effects of dynamic riskless interest rate

Changing the value of riskless interest rate r (ranging from 0. 02 to 0.10), and solving the optimization problem repeatedly, we obtain the numerical results of the optimal restructuring strategy for dynamic riskless interest rate. Data is shown in Table 6, and it illustrates the dynamic effects.

Table 6 The optimal restructuring strategy for different levels of riskless interest rate

r	L_0	γ (%)	k (%)	L_{min}	L_{max}
0.02	0.28	2.80	0.06	0.16	0.83
0.03	0.29	3.77	0.06	0.17	0.76
0.04	0.30	4.62	0.06	0.19	0.75
0.05	0.32	5.74	0.06	0.18	0.64
0.06	0.34	6.72	0.06	0.16	0.58
0.07	0.38	7.80	0.06	0.14	0.58
0.08	0.39	8.69	0.06	0.12	0.55
0.09	0.41	9.76	0.06	0.10	0.55
0.10	0.43	10.84	0.06	0.07	0.56

r: riskless interest rate; L_0: initial leverage; γ: coupon rate; k: deficiency to return; L_{min}: lower leverage; L_{max}: upper leverage

Several insights can be drawn from Table 6. With an increase in riskless interest rate, the initial optimal leverage ratio also increases. The increase of riskless interest rate can be thought of as a signal that discourages the use of debt. On the other hand, it implies that the debt is greatly needed by the firms, and it is in the firm's best interest to borrow more. This explains why the leverage increases with an increase in the riskless interest rate.

There is a connection between the riskless interest rate and the coupon rate. The coupon rate is always higher than the riskless interest rate for a certain amount, because corporate bonds are definitely riskier than riskless deposits. But this difference in rates is small.

Effects of the 5 firm-specific parameters show that the tendency of optimal initial leverage ratios is always opposite to the tendency of the leverage span. This tells us that decisions to increase the leverage are concentrated, while the decisions to decrease the leverage are diversified.

CONCLUSION

In the past two decades corporate restructuring has become an important element in today's economy (Meng, 2002). A prominent phenomenon accompanying these processes is an increase in the issuance of corporate debt in terms of both investment grade and speculative grade bonds. As a result, corporate leverage is increased. Existing capital structure theories based on information asymmetry, agency cost, incomplete markets, moral hazard, and corporate control considerations do not answer why companies, both small and large, have substituted debt for equity in financing. This research deals with internal restructuring because this type of restructuring is an important financing decision facing financial executives and internal restructuring is central in corporate finance. This research demonstrates the impacts of restructuring on corporate values, and the model employed here provides analytical solutions for financing decisions. Theses implications give practical guidance to financial executives.

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