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Effect of boundary conditions and convection on thermally induced motion of beams subjected to internal heating

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Abstract: Numerical exercises are presented on the thermally induced motion of internally heated beams under various heat transfer and structural boundary conditions. The dynamic displacement and dynamic thermal moment of the beam are analyzed taking into consideration that the temperature gradient is independent as well as dependent on the beam displacement. The effect of length to thickness ratio of the beam on the thermally induced vibration is also investigated. The type of boundary conditions has its influence on the magnitude of dynamic displacement and dynamic thermal moment. A sustained thermally induced motion is observed with progress of time when the temperature gradient being evaluated is dependent on the forced convection generated due to beam motion. A finite element method (FEM) is used to solve the structural equation of motion as well as the heat transfer equation.

Key words: Thermal induced oscillations, Natural convection, Forced convection, Finite element analysis
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INTRODUCTION

It is well known that the topic of thermally induced vibrations is of great concern for structural components of spacecrafts and has been extensively investigated by Thornton and co-authors (Thornton and Foster, 1992; Thornton and Kim, 1993; Gulick and Thornton, 1995; Johnston and Thornton, 2000). Pioneering works of Boley (1956), Boley and Barber (1957) have shown the influence of time dependence of the temperature on the structural transients. Manolis and Beskos (1980) used the Laplace transform and method of Papoulis to obtain thermally induced vibrations of beam subjected to rapid heating. They also discussed the effects of axial load, internal viscoelastic damping and external viscous damping on thermal vibrations of simply supported beam subjected to rapid heating. Boley (1972) evolved an approximate method by deriving a simple formula for the ratio of the maximum dynamic to static deflection in order to study the thermally induced vibrations of beams and plates. In the same article the effect of damping and axial (or in-plane) load on the thermal vibration of beams and plates was also discussed. Dynamic stresses and deformations were evaluated by Stroud and Mayers (1971) for a rapidly heated rectangular plate using the dynamic thermo-elastic variational principle. Lyons (1966) suggests that the best practical way of providing sudden heat input to beams, plates and shells is by instantaneous supply of electrical energy and by gamma radiation. Associated governing equation of motion for infinitely long cylindrical shell and the displacement response solution has been presented. Seibert and Rice (1973) carried out studies on thermally induced vibration of a simply supported beam using the uncoupled and coupled thermoelastic governing equations for thin and thick beams. Kidawa-Kukla (1997; 2003) analyzed the thermally induced vibration of uniform simply supported beam heated by a harmonically moving laser beam (mobile heat source). The solution to the problem in analytical form was obtained by using the properties of the Green functions and also a time partitioning method was used to improve the convergence of the series solution to the heat conduction problem.

Thus it is noted that, sufficient analytical studies on the thermal induced motion of beams and plates are available. This article attempts a detailed investigation on the effect of boundary condition and free and forced convection effects on the thermal induced motion of beam. Numerical results are presented based on the finite element formulation for an Euler-Bernoulli beam subjected to thermal load. The study considers the beam with insulated surface and the opposite surface subjected to convective heat transfer. A thermal moment arising from the temperature variation across the thickness of the beam is the source of forcing function for the structure. Dynamic response of the beam due to temperature transients is presented for various boundary conditions. The dynamic thermal moment for each case is examined providing an insight on the mechanism and its relation on the dynamic response of the internally heated beam.

EQUATION OF MOTION OF BEAM SUBJECTED TO INTERNAL HEAT SOURCE

Fig.1 shows the simply supported beam subjected to internal heating and exposed to ambient conditions on one side and insulated on the other side. Practically, internal heating may be achieved by several means. One method is the instantaneous supply of large amount of electrical energy to a structure by applying very high current across the thickness of the structure, and second method is by supply of current of desired amperage and voltage. This would allow each molecule of the structure to act as the interior heat source. Third method of inducing vibrations caused due to internal heat sources is by the instantaneous exposure of the structure to radiation or gamma rays (Lyons, 1966), as in nuclear power plants. Fig.2 shows the free body diagram of a differential element, dx, of the thin beam under the action of mechanical, inertial and thermal loads. T_T and T_B correspond to temperatures on top and bottom surfaces respectively, F is the shear force and P is the load intensity.

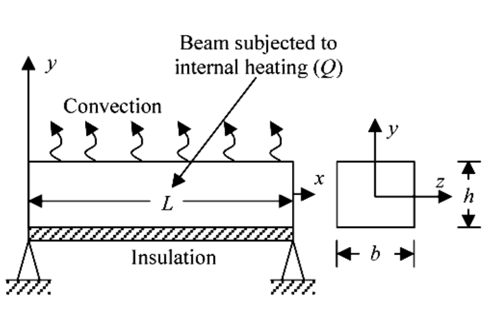

Fig.1 Simply supported beam subjected to internal heating

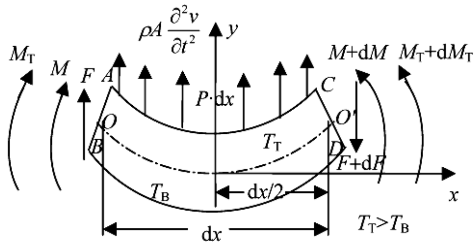

Fig.2 Free body diagram of a beam subjected to mechanical and thermal loads

The governing equation of motion for a beam in the transverse direction in the presence of thermal moment is given by (Boley, 1956)

$$\frac{\partial^2 M_T}{\partial x^2} = \frac{\partial^2}{\partial x^2}\left(EI\left(\frac{\partial^2 v}{\partial x^2}\right)\right) + \rho A\left(\frac{\partial^2 v}{\partial t^2}\right), \tag{1}$$

where, ρ is the mass density, M is the bending moment produced by the applied forces, M_T is the thermal moment, v is the transverse deflection in the y direction, E the Young's modulus and I the moment of inertia of beam cross section. The boundary and initial conditions for the problem are as follows

$$\begin{aligned} &v(0,t) = v(L,t) = v(x,0) = \dot{v}(x,0) = 0, \\ &v''(0,t) = v''(L,t) = m_T, \end{aligned} \tag{2}$$

where m_T is the non-dimensional thermal moment.

For the beam subjected to internal heat source and insulated on one side and undergoing convection heat loss on the other side, the thermal moment acts as a forcing function which is given as

$$\begin{aligned} M_T &= \int_A E\alpha\Delta T y \mathrm{d}A = b\int_y E\alpha\Delta T y \mathrm{d}y \\ &= E\alpha b\Delta T(y_{i+1}^2 - y_i^2)/2, \end{aligned} \tag{3}$$

where, ΔT is the change in temperature, α is the coefficient of thermal expansion and A is the cross sectional area, b is the width of the beam and y_i indicates thickness at ith layer measured along the y axis (Fig.3). The thermal moment is calculated at uniform intervals across the thickness from the top to bottom surfaces of the beam and it is summed up in order to get the total thermal moment across the section. The thermal moment along the length is assumed to be constant as there is no temperature variation along the length of the beam hence, $M_T=M_T(t)$. The following non-dimensional parameters are defined (Boley, 1956): The non-dimensional time τ is:

$$\tau = \kappa t / h^2, \tag{4}$$

where, $\kappa=k/(\rho c_p)$ is thermal diffusivity, k is thermal conductivity, c_p the specific heat and h the total thickness of beam. The non-dimensional displacement V is given as

$$V = \pi^4 kv/(192Q\alpha L^2), \tag{5}$$

where, Q is the heat flux in W/m^2 and L is the length of the beam. m_T is given as

$$m_T(\tau) = \pi^4 kM_T/(192EIQ\alpha). \tag{6}$$

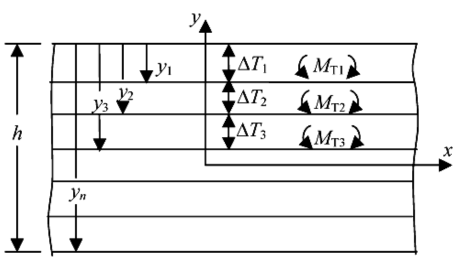

Fig.3 Geometry and temperature details for the calculation of thermal moment

The parameter B is the square root of the ratio of the characteristic time h^2/κ of heat transfer problem to characteristic time $(\rho AL^4/EI)^{1/2}$ of the vibration problem (or proportional to the natural period of vibration). Thus B is large for beams with low diffusivity, low density and high bending rigidity; it is low if the beam is slender or dense.

Determination of temperature distribution across the beam thickness

The evaluation of the temperature distribution across the thickness of the beam is found by using the finite element idealization as illustrated in Fig.4.

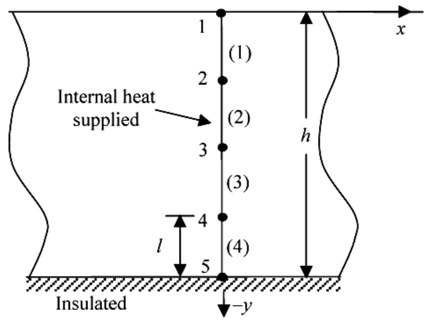

Fig.4 Finite element idealization across the beam thickness for thermal analysis

The finite element equation for temperature evaluation across beam thickness when the beam is subjected to sudden internal heating, exposed to ambient condition on one side and insulated on other side is as follows:

$$\left\{\frac{kA}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & h_c A\end{bmatrix}\right\}\begin{Bmatrix}T_1\\ T_2\end{Bmatrix}+\frac{\rho c_p Al}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{Bmatrix}\dot{T}_1\\ \dot{T}_2\end{Bmatrix} = \frac{\dot{q}Al}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix}+h_c AT_\infty\begin{Bmatrix}0\\ 1\end{Bmatrix}. \tag{7}$$

In Eq.(7) the second matrix on LHS and second vector on RHS are contribution from convection and will be taken into consideration only for last element, h_c is convective heat transfer coefficient, T_1 and T_2 are the nodal temperatures and T_∞ is the ambient temperature. The global finite element equation for time dependent temperature distribution has the following form:

$$\boldsymbol{K}_{\text{comb}}\boldsymbol{T} + \boldsymbol{K}_{\text{cap}}\dot{\boldsymbol{T}} = \overline{\boldsymbol{F}}_Q, \tag{8}$$

where $\boldsymbol{K}_{\text{comb}}$ is elemental conduction and/or convection matrix, $\boldsymbol{K}_{\text{cap}}$ is elemental capacitance matrix and $\overline{\boldsymbol{F}}_Q$ is force vector. Eq.(8) must be solved for the variation of temperature in space and time domain to obtain the temperature distribution across the thickness of the beam.

Beam finite element formulation

The finite element idealization for the simply supported beam subjected to heat source on one side and insulated on other side is shown in Fig.5. The weak form of the governing equation Eq.(1) is as follows:

$$EI\left(N^T\frac{\partial^3 v}{\partial x^3}\bigg|_0^l-\frac{\partial N^T}{\partial x}\frac{\partial^2 v}{\partial x^2}\bigg|_0^l+\int_0^l\frac{\partial^2 v}{\partial x^2}\frac{\partial^2 N^T}{\partial x^2}\mathrm{d}x\right)$$
$$+\int_0^l N^T\rho A\frac{\partial^2 v}{\partial t^2}\mathrm{d}x-N^T\frac{\partial M_\mathrm{T}}{\partial x}\bigg|_0^l+\int_0^l\frac{\partial M_\mathrm{T}}{\partial x}\frac{\partial N^T}{\partial x}\mathrm{d}x=0, \quad (9)$$

where, the first term refers to shear force, the second term refers to moment, the third term will give the stiffness matrix, the fourth term will yield the mass matrix, the fifth term gives the shear force and the last term will be zero as there is no change in thermal moment along the length of the beam. N^T is the weight function. Hermite shape functions are used to develop the various finite element matrices. In the standard Galerkin's method, weight functions are chosen as the shape functions. Transverse displacement field would be expressed in terms of cubic Hermite shape functions and nodal displacement as follows: $v(x,t)=\sum_{i=1}^{4}N_i v_i$. After obtaining the time dependent temperature distribution across the beam thickness, force vector $\boldsymbol{F}_\mathrm{T}$ is evaluated which will contain the thermal moment M_T only. Subsequently, static equation $\boldsymbol{Kv}=\boldsymbol{F}_\mathrm{T}$ is solved. The displacement thus obtained at time t is termed as the static displacement $\boldsymbol{v}_\mathrm{st}$. Newmark's method is used to solve the second order equation of motion involving the time dependent forcing function

$$\boldsymbol{M}\ddot{\boldsymbol{v}}+\boldsymbol{K}\boldsymbol{v}=\boldsymbol{F}_\mathrm{T}. \quad (10)$$

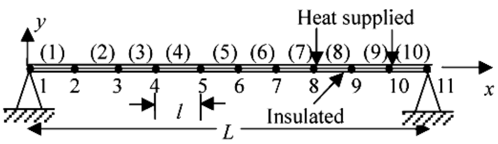

Fig.5 Finite element idealization of beam for structural analysis

The displacement obtained by solving Eq.(10) is termed as dynamic displacement $\boldsymbol{v}_\mathrm{dyn}$. From the dynamic displacement vector, the displacement for the central element of the beam is extracted to calculate the thermal moment at the centre of the beam and is termed as dynamic thermal moment:

$$(\boldsymbol{M}_\mathrm{TD})^\mathrm{e}=(\boldsymbol{K})^\mathrm{e}(\boldsymbol{v}_\mathrm{dyn})^\mathrm{e}, \quad (11)$$

where, superscript e refers to elemental solution. Hence, the dynamic thermal moment at the centre of the beam is given as:

$$M_\mathrm{T\,dyn}=M_\mathrm{TD}^\mathrm{node}\pm M_\mathrm{T\,st}, \quad (12)$$

where $M_\mathrm{T\,st}=M_\mathrm{T}$. It is to be noted that the structural damping is ignored and that material properties are independent of temperature.

RESULTS AND DISCUSSION

Numerical exercises are presented for thin beams with different boundary conditions like simply supported (SS), clamped simply supported (CS) and clamped free (CF) for the analysis of dynamic response and dynamic thermal moment when the beam is subjected to internal heating with heat transfer boundary conditions as insulation and convection heat loss occurring due to constant heat transfer coefficient and forced convection caused by transverse motion of beam. The slenderness ratios of the beam considered for the study are 88 and 165. Length of the beam is 0.254 m and has unit width. The evaluation of the temperature distribution across the cross section of the beam has been validated with the close form solution given by Boley (1956) or Carslaw and Jaeger (1959). The finite element approach for the analysis of the dynamic response of beam subjected to thermal boundary conditions has been validated with the results reported by Boley (1956) and Manolis and Beskos (1980) for the simply supported beam.

Validation

The thermal structural data for the validation of the FEM formulation to analyze thermally induced vibration are reproduced below from (Boley, 1956): b=1 m, L=0.254 m, k=201.87 W/(m·K), α=22.0×10^{-6} /°C, ρ=2700 kg/m^3, c_p=869.38 J/(kg·°C), Q=1.63×10^6 W/m^2, E=73.5×10^9 Pa, and G=26.0×10^9 Pa and other data are listed in Table 1. Fig.6 shows good agreement of the FE and close form solution (Boley, 1956) for temperature distribution across the thickness of the beam with surface heating in the form of step heat input and opposite surface insulated.

Table 1 Geometric and time data for validation problem

B	L/h	H (m)	t (s)
0	25400	0.000010	0.000002
1	165	0.001544	0.04
∞	10	0.025400	10.0

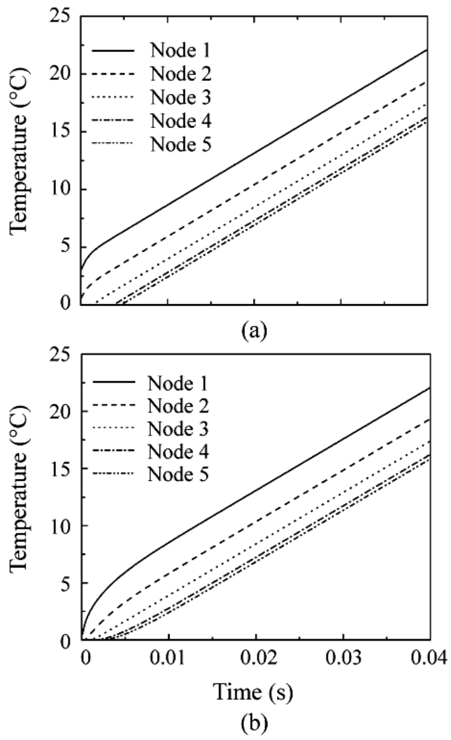

Fig.6 Close form solution (a) and FEM solution (b) of temperature variation across the thickness of the beam as referred from (Boley, 1956) for B=1

The non-dimensional plots of dynamic mid-span deflection (Fig.7a) and mid-span thermal moment (Fig.7b) for various values of B were obtained and compared with the results given by Boley (1956) and Manolis and Beskos (1980). It was found that the trends of the results are in good agreement for simply supported beam subjected to rapid heating. The variation of the ratio of maximum dynamic mid-span deflection to maximum static mid-span deflection with the thickness of a rectangular simply supported aluminium beam was also studied and it was inferred that, in order to avoid dynamic oscillations due to heating it is preferable to have higher thickness of the beam. The thermal structural data for aluminium beam provided by Manolis and Beskos (1980) are the same as those given by Boley (1956) except the length of the beam was taken to be equal to L=1 m and thickness of the beam was h=0.00385 m.

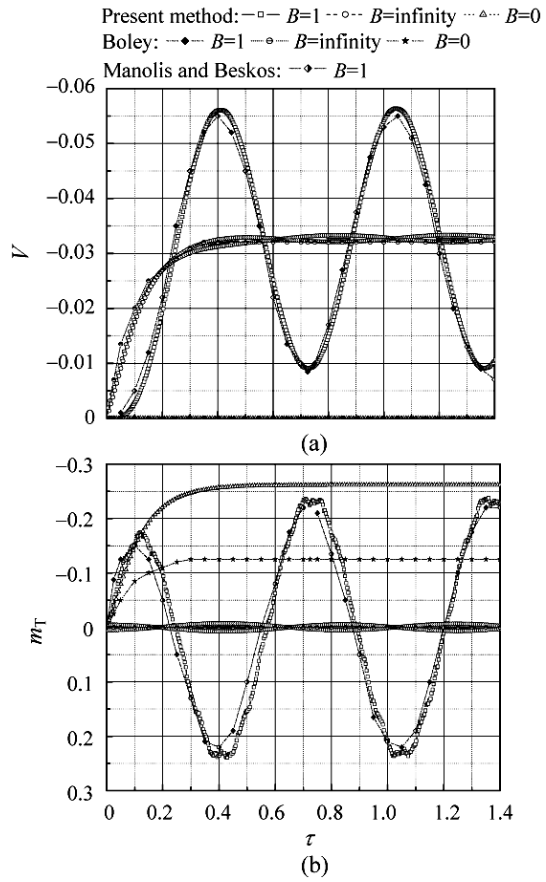

Fig.7 Non-dimensional dynamic mid-span deflection (a) and thermal moment (b) of simply supported beam for various values of B

Effect of natural convection on thermally induced vibrations of internally heated beams

The beam is subjected to internal heating of Q= 10.63×10^6 W/m^2. A beam with L/h=165 is considered for the analysis. Other data remain unchanged. Fig.8 shows the temperature variation across the thickness of the simply supported beam subjected to internal heating and undergoing convective heat loss with heat transfer coefficient h_c=20 W/(m^2·K). It can be seen from the figure that with the passage of time the temperature increases linearly and that the temperature variation across the thickness of the beam is almost negligible.

Fig.9 shows the corresponding dynamic midspan thermal moment. The trend of the thermal moment is the same as the one shown in Fig.8 for simply supported beam subjected to step heating on one side and insulated on the other side, but the amplitude of non-dimensional dynamic mid-span thermal moment is considerably less than that shown in Fig.7b.

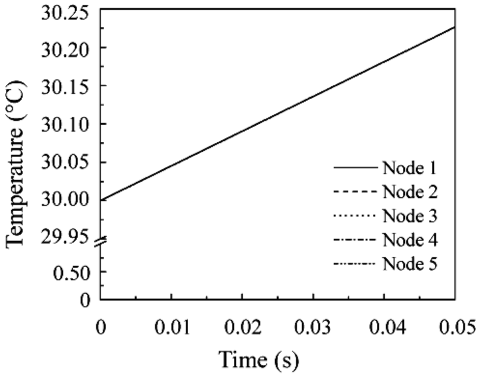

Fig.8 FEM solution of temperature variation across the thickness of the simply supported beam subjected to internal heating for h_c=20 W/(m²·K)

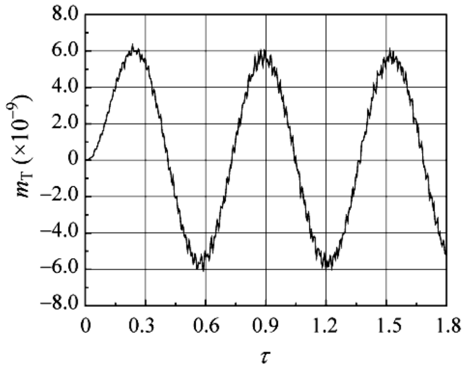

Fig.9 Non-dimensional dynamic mid-span thermal moment of simply supported beam subjected to internal heating for h_c=20 W/(m²·K)

Fig.10 shows the non-dimensional dynamic mid-span deflection of simply supported beam subjected to internal heating for various values of convective heat transfer coefficient (h_c). The convective heat transfer coefficient of zero i.e. h_c=0 refers to the beam which is insulated on both sides and as seen from Fig.10 the vibration amplitude is zero, the same is true as seen in Fig.12 and Fig.14 for CS and CF beam respectively. As the convective heat transfer coefficient is increased, during the initial time period, the non-dimensional displacement is almost equal to zero but later on there is increase in the amplitude of the non-dimensional dynamic displacement showing the oscillatory trend about some mean position (not shown).

The thermal moment for clamped-simply supported beam illustrated in Fig.11 shows a linear increase in its amplitude with slight oscillatory trend as the time progresses. Referring to Fig.12, as the convective heat transfer coefficient is increased the non dimensional displacement is equal to zero during the initial time period, but later on there is increase in the amplitude of the dynamic displacement and shows an oscillatory trend about some mean position (i.e. thermal static deflection and is not shown).

In case of clamped free beam, the dynamic ther-

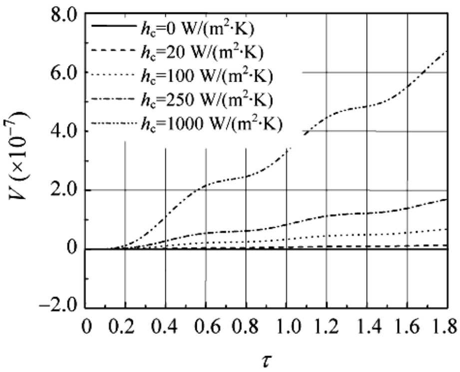

Fig.10 Non-dimensional dynamic mid-span deflection of simply supported beam subjected to internal heating for various values of convective heat transfer coefficient (h_c)

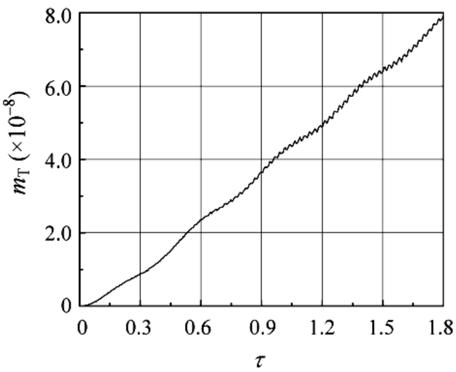

Fig.11 Non-dimensional dynamic mid-span thermal moment for clamped-simply supported beam subjected to internal heating for h_c=20 W/(m²·K)

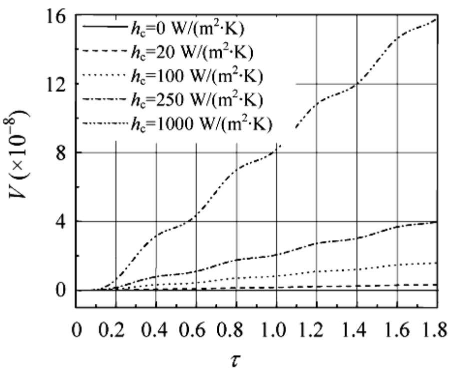

Fig.12 Non-dimensional dynamic mid-span deflection of clamped-simply supported beam subjected to internal heating for various values of convective heat transfer coefficient (h_c)

mal moment at the free end is equal to zero due to free expansion of the beam as shown in Fig.13. The dynamic deflection of clamped-free beam at the free end depends on the magnitude of the heat transfer coefficient. As illustrated in Fig.14, when the convective heat transfer coefficient is increased, there is gradual increase in the amplitude of the dynamic displacement at the free end.

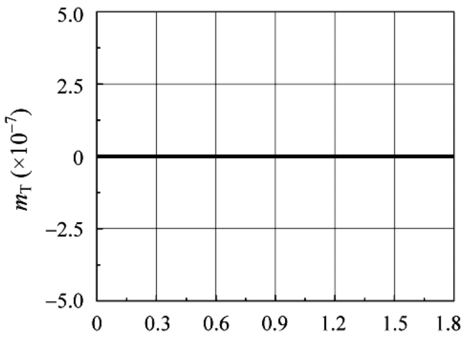

Fig.13 Non-dimensional dynamic thermal moment at free end for clamped-free beam subjected to internal for h_c=20 W/(m^2·K)

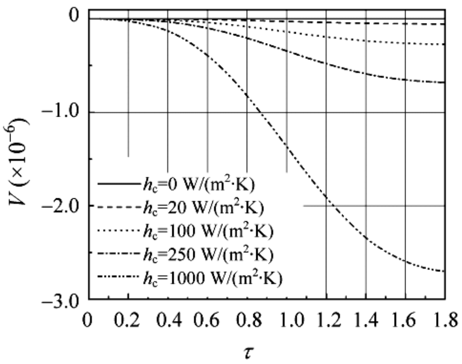

Fig.14 Non-dimensional dynamic deflection at free end of clamped-free beam subjected to internal heating for various values of convective heat transfer coefficient (h_c)

Effect of varying convection on thermally induced vibrations of internally heated beams

Numerical investigations were carried out for internally heated simply supported beam with forced convection on one surface and opposite surface being insulated. The variation of convection along the length of the beam is chosen to be a function of the transverse displacement of the beam. This assumption on the convection heat transfer coefficient will lead to minimum convection towards the simply supported ends and will increase toward the centre of the beam to a maximum value. This characteristic variation of heat transfer coefficient is based on the physical interpretation that, when the beam executes upward motion from the mean position, this will result in displacing the air upwards and it is reasonable to assume the heat transfer coefficient to be proportional to the velocity of the beam and the displacement vector. As the beam executes downward motion, the convection coefficient is assumed to decrease in proportion to the velocity and displacement vector under the circumstances that the air currents put in motion previously need finite time to change their direction. This decrease in convection coefficient is assumed to take place until the beam attains the mean position and for subsequent downward motion of the beam a constant natural convection is assumed to prevail. From the maximum downward position, as the beam executes upward motion until mean position, again the convection heat transfer coefficient is assumed to remain constant. The natural convective heat transfer coefficient is taken to be equal to 20 W/(m^2·K) which has been obtained experimentally under laboratory conditions. Thus, the spatial and time variation of heat transfer coefficient can be represented as follows:

$$h(x,t) = h_c + h(t)v(x,t)/v_{max}, \quad 0 < v \le v_{max} \text{ and } v_{max} \le v < 0, \tag{13}$$

$$h(x,t) = h_c,\ 0 < v \le -v_{max} \text{ and } -v_{max} \le v < 0, \tag{14}$$

where, h_c is natural convective heat transfer coefficient, $v(x,t)$ is transverse displacement of beam, v_{max} is maximum displaced position of beam, x is position along x-axis. Approximating the velocity of air equals velocity of oscillating beam, the Reynolds number is computed. Using this Reynolds number Re, the Nusselt number Nu is found which helps in finding the convective coefficient of heat transfer $h(t)$. The correlation given by Zhukauskas for computation of Nusslet number for flow over a circular cylinder in cross flow is,

$$Nu = CRe^m Pr^n (Pr/Pr_s)^{1/4}, \tag{15a}$$

and the empirical correlation given by Hilpert is

$$h(t) = Nuk_{air}/L, \tag{15b}$$

where C is constant (C is 0.75 for Pr<40, C is 0.51 for

Pr>40), *m* is constant (*m*=0.4 for *Pr*<40, *m*=0.5 for *Pr*>40), *Pr* is Prandlt number evaluated at ambient temperature T_∞, *n*=0.36 for *Pr*≤10 and *n*=0.37 for *Pr*>10, Pr_s=Prandlt number at instantaneous temperature T_s. Eqs.(15a) and (15b) are referred to from (Incropera and DeWitt, 2002). The Prandlt number at instantaneous temperature can be obtained from the table of thermodynamic properties of air as referred to from (Incropera and DeWitt, 2002). The Prandlt number is given for every 50 °C temperature difference, starting from temperature of 27 °C (300 K). To obtain the Prandlt number at intermediate temperatures the third degree polynomial fit is carried out for the thermodynamic properties of air (Incropera and DeWitt, 2002), with instantaneous temperature, T_s, as the variable which is given as:

$$Pr_s = 0.84071 - 6.8066\times10^{-4} T_s + 8.796\times10^{-7} T_s^2 - 2.9261\times10^{-10} T_s^3. \quad (16)$$

Pr_s is used in Eq.(15a) to evaluate the Nusselt number. The above expression is also used to evaluate Prandlt number at ambient temperature.

Accounting for forced convection arising due to motion of the beam, Fig.15 shows the temperature variation across the thickness of the simply supported beam. The slenderness ratio of the beam is 165. The internal heating was 1000.63×10^6 W/m^3. It was found that the temperature increases as time progresses. However the temperature at various points (nodes) across the thickness does not vary during the initial time period. But as the time progresses a small temperature difference is found to occur between various nodes across the thickness of the beam. The static displacement monotonously increases but the dynamic displacement continuously oscillates about the static displacement with increase in amplitude with respect to time as illustrated in Fig.16. The static thermal moment has exponential characteristics which can be observed for a small fraction of time during the initial stage and subsequently it increases linearly. The dynamic thermal moment continuously oscillates about zero with increase in amplitude with respect to time as illustrated in Fig.17. In case of beam subjected to varying convection, the amplitude of dynamic displacement continuously increases with time, however, when constant convection is considered, the oscillations are steady.

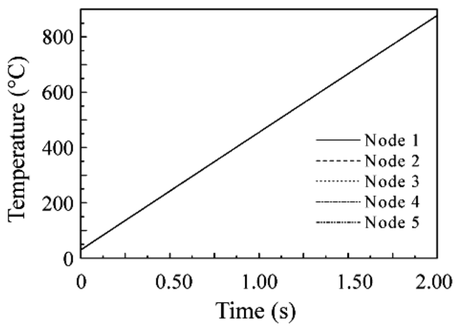

Fig.15 Temperature variation across the thickness of the simply supported beam with internal heating and varying heat transfer coefficient

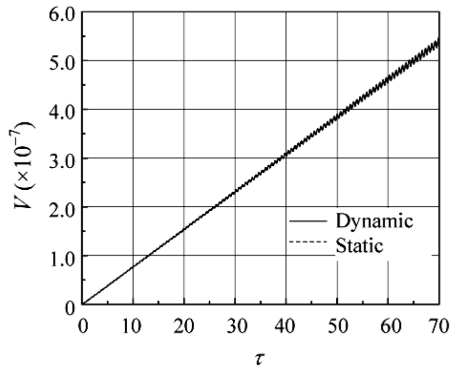

Fig.16 Non-dimensional dynamic and static mid-span deflection of the simply supported beam with internal heating and varying heat transfer coefficient

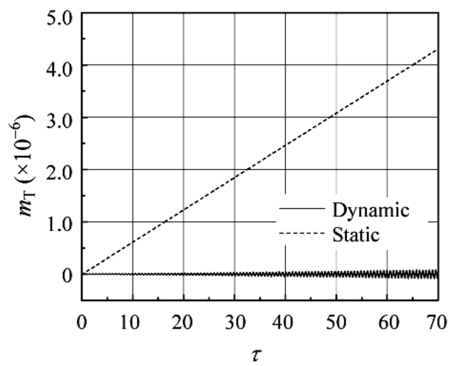

Fig.17 Non-dimensional dynamic and static mid-span thermal moment of the simply supported beam with internal heating and varying heat transfer coefficient

For *L*/*h*=165, the characteristic time of the heat transfer problem is equal to the characteristic time of vibration problem. In case of *L*/*h*=200, the characteristic thermal time is less than characteristic time of vibration problem. Thus the magnitude of thermal oscillations is higher in case of *L*/*h*=200 when compared to *L*/*h*=165, as shown in Fig.18. As the thickness of the beam increases, the characteristic time of the heat transfer problem increases. Hence, when L/h decreases, i.e. 125, 96 and 88, it was observed that there were no thermally induced oscillations, however there exists static thermal deflection.

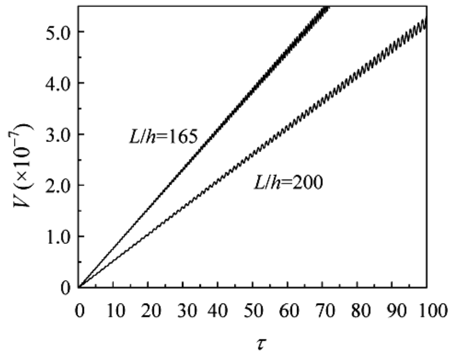

Fig.18 Comparison of non-dimensional dynamic and static mid-span deflection of the simply supported beam with internal heating and forced convection for *L*/*h*=165 and 200

CONCLUSION

A theoretical analysis was presented on the thermally induced vibrations of beams under various heat transfer and structural boundary conditions subjected to internal heating. The major observations for the case of constant convection boundary condition are: (1) With the passage of time the temperature increases linearly and the temperature variation across the thickness of the beam is almost negligible; (2) As the convective heat transfer coefficient is increased there is increase in the amplitude of the non-dimensional dynamic displacement; (3) The dynamic displacement has lower amplitude in case of clamped simply supported beam as compared to the other two; (4) The trends of the non-dimensional dynamic thermal moment for SS beam is the same as the one shown for SS beam with step heating and insulated boundary condition, but for CS beam the dynamic thermal moment linearly increases with time and the dynamic thermal moment for the CF beam at the free end is zero. It was also observed that when the convective heat transfer coefficient is a function of beam motion, the amplitude of dynamic displacement continuously increases with time, however, when constant convection is considered, the oscillations are steady. Finally, irrespective of the type of heat transfer and structural boundary condition the vibrations occurred in the first mode.

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