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Science Letters:

A new class of negative refractive index transmission line*

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Abstract: We propose a new class of negative refractive index transmission line in which ideal operational amplifiers are applied to form the periodically loaded negative-impedance-converted inductors and capacitors. The phase response of the new transmission line is opposite to that of a positive refractive index conventional transmission line. Unlike the existing negative refractive index transmission line, the new negative refractive index transmission line is non-dispersive and thus can lead to many novel applications such as designing new broadband devices.

Key words: Negative refractive index (NRI), Transmission line (TL), Negative-impedance-converted, Non-dispersive
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INTRODUCTION

An artificial dielectric medium that exhibits simultaneously both negative electric permittivity and magnetic permeability, was first envisioned by Veselago (1968), who theoretically predicted that such a medium would have a negative refractive index (NRI). Such an artificial NRI medium was first realized experimentally at microwave frequencies (Shelby *et al*., 2001) using a volumetric structure with thin wire strips and split-ring resonators. A planar NRI medium was later realized by periodically loading a conventional transmission line (TL) with lumped-element series capacitors and shunt inductors (Iyer and Eleftheriades, 2002; Eleftheriades *et al*., 2002).

Using negative impedance converters (NICs) (Horowitz and Hill, 1993), we propose in this letter a new class of NRI TL. Unlike the existing NRI TL, the equivalent material parameters of the proposed NRI TL are non-dispersive and thus can lead to many interesting applications. A full-length article with an application example of a broadband power divider will be given in another research paper.

THEORY

Distributed network approach

Two-dimensional telegrapher's equations for the distributed structure of Fig.1 can be expressed as

$$\frac{\partial v_y}{\partial z}=-i_z Z,\quad \frac{\partial v_y}{\partial x}=-i_x Z,\quad \frac{\partial i_z}{\partial z}+\frac{\partial i_x}{\partial x}=-v_y Y,\quad (1)$$

where Z and Y are the per-unit-length series impedance and parallel admittance, respectively.

Comparing the above telegrapher's equations with Maxwell's equations in a homogeneous isotropic medium and mapping the field components of the transverse magnetic (TM_y) solution to the voltages and currents in the structure, we can obtain the following equivalent material parameters for the distributed TL structure:

$$\begin{cases} \mathrm{j}\omega\mu = Z \Rightarrow \mu = Z/(\mathrm{j}\omega), \\ \mathrm{j}\omega\varepsilon = Y \Rightarrow \varepsilon = Y/(\mathrm{j}\omega), \end{cases} \quad (2)$$

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where μ and ε are the equivalent permeability and permittivity of the structured material (i.e., the distributed TL structure), respectively.

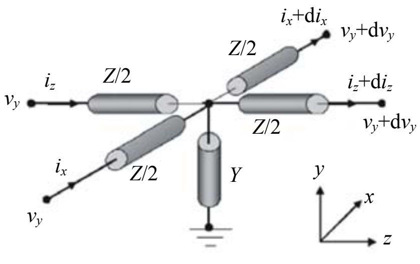

Fig.1 Unit cell for a two-dimensional distributed LC network (Eleftheriades *et al.*, 2002)

In the case of positive refractive index (PRI) material, Eq.(2) implies a medium of low-pass topology with

$$\begin{cases} \mathrm{j}\omega L = Z = \mathrm{j}\omega\mu \Rightarrow L = \mu(H/m), \\ \mathrm{j}\omega C = Y = \mathrm{j}\omega\varepsilon \Rightarrow C = \varepsilon(F/m), \end{cases} \quad (3)$$

where L and C are the per-unit-length capacitance and inductance of the distributed TL structure, respectively, both of which are positive real quantities. Eleftheriades *et al.*(2002) demonstrated an NRI TL by periodically loading a conventional TL with lumped-element series capacitors (C') and shunt inductors (L').

However, in such an approach the equivalent material parameters are highly dispersive (though negative) as one has $\mu=-1/(\omega^2 C')$ and $\varepsilon=-1/(\omega^2 L')$ (Eleftheriades *et al.*, 2002).

In this letter, we propose a new approach to the realization of NRI TL by using negative-impedance-converted LC network as follows:

$$\begin{cases} \mathrm{j}\omega\mu = Z = -\mathrm{j}\omega L'' \Rightarrow \mu = -L''(H/m), \\ \mathrm{j}\omega\varepsilon = Y = -\mathrm{j}\omega C'' \Rightarrow \varepsilon = -C''(F/m). \end{cases} \quad (4)$$

Eq.(4) shows that the permeability and permittivity of the equivalent material are negative and do not vary with frequency. The corresponding propagation constant is proportional to the frequency:

$$\beta = -\sqrt{-ZY} = -\omega\sqrt{L''C''}. \quad (5)$$

Furthermore, the relationship between the network characteristic impedance (Z_0) and the equivalent wave impedance (η) is preserved as

$$Z_0 = \sqrt{L''/C''} = \eta = \sqrt{\mu/\varepsilon}. \quad (6)$$

Negative-impedance-converted LC network

Here for simplicity we take the 1D unit cell as an example. The LC unit cells in PRI TL and NIC TL networks are shown in Fig.2a and Fig.2b, respectively.

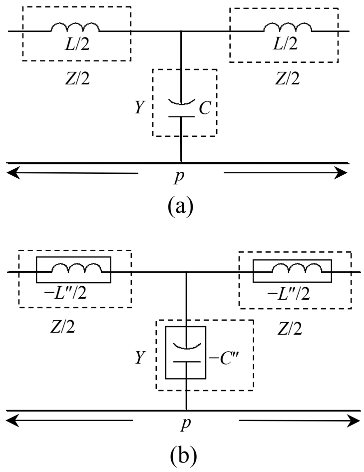

Fig.2 Unit cells in one-dimensional network of (a) positive refractive index transmission line and (b) negative impedance converter transmission line

By applying the periodic boundary conditions related to the Bloch-Floquet theorem to the LC unit cells (Collin, 1992; Pozar, 1998), we obtain the following dispersion relation:

$$\cos(\beta p) = 1 + ZY/2, \quad (7)$$

where p is the period, and the series impedance (Z) and shunt admittance (Y) of the unit cell are given by

$$\begin{cases} Z = \mathrm{j}\omega L, \quad Y = \mathrm{j}\omega C, & \text{for PRI TL}, \\ Z = -\mathrm{j}\omega L'', \quad Y = -\mathrm{j}\omega C'', & \text{for NIC TL}. \end{cases} \quad (8)$$

Since the electrical length of the unit cell is small, we can apply Taylor approximation $\cos(\beta p)\approx 1-(\beta p)^2/2$ to Eq.(7) and obtain

$$\beta = s\sqrt{-ZY}, \quad (9)$$

where

$$s = \begin{cases} +1, & \text{for PRI TL}, \\ -1, & \text{for NIC TL}. \end{cases}$$

NEGATIVE-IMPEDANCE-CONVERTED LC TRANSMISSION LINE MODEL

Fig.3 shows an ideal operational amplifier with differential input and differential output and its equivalent circuit (Carter and Brown, 2001), where

$$V_{out}=V_d\cdot Gain=(V_+-V_-)\cdot Gain. \tag{10}$$

By using *ABCD*-matrix analysis for the circuit shown in Fig.4, we can realize equivalent negative series impedance and negative parallel impedance if the operational amplifier is in its linear region.

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix}=\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} V_2 \\ I_2 \end{pmatrix}. \tag{11}$$

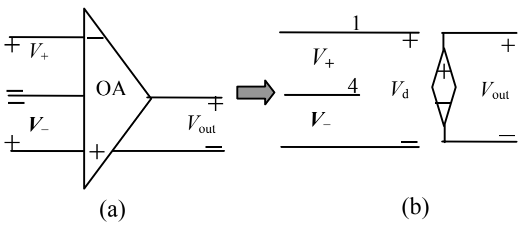

Fig.3 Operational amplifier (a) and its equivalent circuit (b)

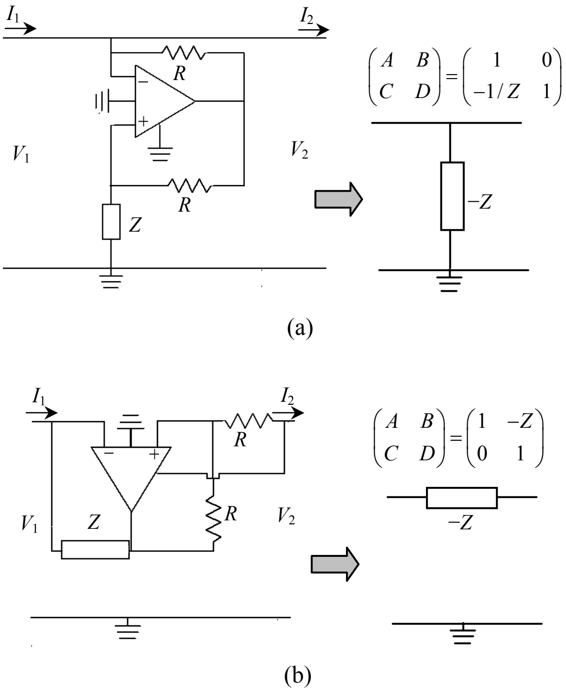

Fig.4 Negative impedance converters of series impedance (a) and parallel impedance (b)

If we replace the impedance Z with a capacitor (C), an inductance (L) or a resistance (R), we can obtain the corresponding negative capacitor, negative inductance or negative resistance.

By periodically connecting the negative series inductance and negative parallel capacitor, we can realize the active NIC LC transmission line as shown in Fig.5.

Our NIC TL structure is still of low pass topology, which means that it has the same amplitude response as the PRI TL in Fig.6a. However, its phase response is opposite to that of the PRI TL, as shown in Fig.6b.

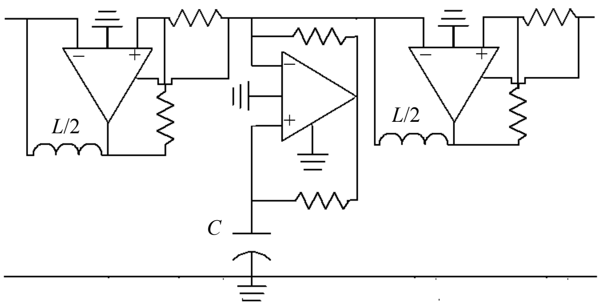

Fig.5 Unit cell of our negative impedance converter LC transmission line

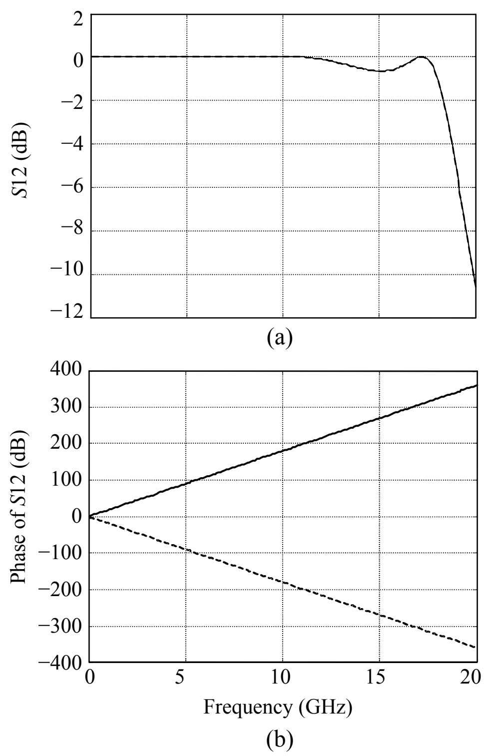

Fig.6 Amplitude responses (a) and phase responses (b) of NIC TL (solid line) and PRI TL (dashed line)

CONCLUSION

A new class of negative refractive index (NRI) transmission lines (TLs) has been proposed. Ideal operational amplifiers have been applied to form the required lumped elements of negative values, i.e., the negative-impedance-converted inductors and capacitors. The amplitude response of the new NRI TL is the same as the conventional TL, but its phase response is opposite to that of a conventional TL. Different from the existing NRI TL, the proposed NRI TL is non-dispersive and thus can lead to many interesting new applications such as designing some novel broadband devices.

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