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Topology evolutions of silhouettes*

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Abstract: We give the topology changing of the silhouette in 3D space while others study the projections in an image. Silhouettes play a crucial role in visualization, graphics and vision. This work focuses on the global behaviors of silhouettes, especially their topological evolutions, such as splitting, merging, appearing and disappearing. The dynamics of silhouettes are governed by the topology, the curvature of the surface, and the view point. In this paper, we work on a more theoretical level to give enumerative properties of the silhouette including: the integration of signed geodesic curvature along a silhouette is equal to the view cone angle; in elliptic regions, no silhouette can be contained in another one; in hyperbolic regions, if a silhouette is homotopic to a point, then it has at least 4 cusps; finally, critical events can only happen when the view point is on the aspect surfaces (ruled surface of the asymptotic lines of parabolic points with surface itself). We also introduce a method to visualize the evolution of silhouettes, especially all the critical events where the topologies of the silhouettes change. The results have broad applications in computer vision for recognition, graphics for rendering and visualization.

Key words: Topological change, Silhouette, Geodesic curvature, Cusp

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INTRODUCTION

Silhouettes refer to the locus of points on the surface where the view rays tangentially touch the surface. Projected silhouettes refer to the projection images of silhouettes.

Silhouettes play a crucial role in computer vision. Projected silhouettes convey rich geometric information about the original surface. For example, the inflection point of projected silhouettes corresponds to the parabolic point on the original surface. The curvature signs of points on the projected silhouettes are consistent with the Gaussian curvature sign of their pre-images on the surface. Silhouettes have been applied for surface reconstruction, pattern recognition and aspect graphs in computer vision.

Silhouettes are one of the major research focuses in non-photorealistic rendering in graphics. For human visual perception, silhouettes carry the most important shape information. Accurately and efficiently computing the silhouettes has attracted many researchers. In order to improve the efficiency of computing silhouettes, it is highly desirable to interpolate silhouettes from pre-computed ones when the view is moved between sampled views. If the topological structures of the pre-computed silhouettes are consistent, the interpolation is sensible and easy to perform. Therefore, it is critical to fully understand the topological evolutions of silhouettes when the view point is moved in space.

The major goal of this work is to study when, where, and how the topologies of the silhouettes will change, along with the global properties of the silhouettes. Silhouettes can shrink to a point and disappear, intersect each other either transversally or tangentially and reconnect. These critical events can

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only happen when the view point is on the ruled surface of asymptotic lines of parabolic points. Furthermore, we show that the integration of the geodesic curvature along a closed smooth silhouette is equals to the view cone angle.

In this work, we make the following contributions:

(1) Visualizing all possible topological changes of a silhouette.

(2) Developing a theorem of the relation of geodesic curvature of a silhouette and the view cone angle. Some global properties of silhouettes follow as corollaries.

(3) Introducing the concept of the aspect surface. All topological changes happen when the view is on the aspect surface. The silhouettes are homotopic to each other if two views can be connected by a curve which does not cross the aspect surface.

Previous works

1. Non-photorealistic rendering (NPR)

Finding silhouettes from a given object is a key ingredient in NPR. A comprehensive review of silhouettes for NPR is not the scope of this paper, and we refer the reader to the excellent books and survey papers, e.g. (Ebert and Rheingans, 2000; Gooch and Gooch, 2001; Strothotte and Schlechtweg, 2002; Isenberg *et al*., 2003) for details.

Usually, for NPR, silhouettes are rendered with a hybrid approach of object-space detection and image-space detection (Northrup and Markosian, 2000). The object-space silhouette detection is performed with simple $n(p)\cdot v(p)$ computation, where $n(p)$ and $v(p)$ are the normal direction and the view direction defined with a point p on the object, respectively. The silhouettes are detected with the contours of zero-points on the object (Hertzmann and Zorin, 2000). The image-space algorithms detect the visible portions of silhouette in the image-space, and generate meaningful strokes for stylization.

In contrast to previous work in computer graphics, we study the silhouette revolution on the object space with respect to view point changes. Our methods could be utilized to the animation with NPR scene, since the key problem here is to smoothly interpolate the silhouettes between coherent frames (Kalnins *et al*., 2003).

2. Silhouette for shape recognition and aspect graph

For 3D shape analysis and recognition from images (Laurentini, 1995; Ponce and Kriegman, 1989; Ponce *et al*., 1992; Forsyth, 1993; Boyer and Berger, 1995; 1997; Bottino and Laurentini, 2003; Lee *et al*., 2003), silhouettes are well studied in computer vision literature, e.g. (Marr, 1977; 1982; Forsyth and Ponce, 2002).

Since image-space silhouettes cannot convey sufficient information for non-trivial objects, several studies have been done on the connections between the evolution of image-space silhouettes and the 3D shape of objects. Pae and Ponce (2001) studied the structural changes of silhouette of algebraic surfaces.

The aspect graph presents the topological changes of silhouettes in the image-space with respect to view point changes. A node of the aspect graph corresponds to a topological configuration of image silhouettes of the object. The status changing from a node to another happens when a view point is at the critical position (Koenderink, 1984; Cyr and Kimia, 2001).

In contrast to the approaches in computer vision, we study the evolution of silhouettes on the object-space since we have exact 3D geometry in graphics applications. The topological changes of silhouettes on a surface are much rare than those of projected silhouettes in images, because projection introduces many singularities, e.g., the image of a smooth silhouette may contain several cusps (Cipolla and Zisserman, 1992; Cipolla *et al*., 1995; 1997). Moreover, we propose the novel concept of aspect surfaces, which is the locus of all the critical view points. By identifying the aspect surfaces, we can easily detect the topological changes of silhouettes for computer graphics and visualization applications.

3. Silhouette in singularity and catastrophe theory

Also, in mathematics, silhouette is analyzed as the projections of surfaces to planes from a visual perspective (Koenderink and van Doorn, 1976). All the critical events of projected silhouettes have been thoroughly classified in (McCrory, 1980; Landis, 1981; Platonova, 1981; Arnold, 1986) in the setting of singularity and catastrophe theory.

McCrory (1980) and Arnold (1986) gave us a classification of apparent contours of surfaces. Related work has been done by Platonova (1981) and Landis (1981).

LOCAL PROPERTIES OF SILHOUETTES

This section aims at visualizing all possible critical events for silhouettes. The local properties of silhouettes have been thoroughly studied in computer vision, singularity theory and catastrophe theory. The curvature of projected silhouettes and the normal curvature are strongly related to the Gaussian curvature. All the possible critical events of projected silhouettes have been completely categorized. We follow the categories as explained in (Arnold, 1986).

Preliminaries

The Gauss map $G:\Sigma\rightarrow\mathbb{S}^2$ maps a point $r(u,v)\in\Sigma$ to its normal $n(u,v)\in\mathbb{S}^2$.

Definition 1 The derivative map of the Gauss map $DG:T\Sigma\rightarrow T\mathbb{S}^2$ is called the Weingarten map,

$$\begin{cases} \mathrm{d}n=-W\mathrm{d}r, \\ \mathrm{d}r=r_u\mathrm{d}u+r_v\mathrm{d}v, \\ \mathrm{d}n=n_u\mathrm{d}u+n_v\mathrm{d}v. \end{cases}$$

A Weingarten map is a self-conjugate linear map, where the eigenvalues k_1, k_2 of W are called principal curvatures, and the eigen vectors $\boldsymbol{e}_1$, $\boldsymbol{e}_2$ are called principal directions. The product of k_1, k_2 are called the Gaussian curvature.

The local behavior of a silhouette is determined by the view direction and the Gaussian curvature at the touching point. We classify the points on a surface according to their Gaussian curvature and study the local behavior of silhouettes on each class respectively.

Definition 2 On a smooth surface with at least C^2 continuity, k_1, k_2 are principal curvatures. All points are classified as follows: (1) elliptic, $0<k_1\leq k_2$; (2) hyperbolic, $k_1<0<k_2$; (3) parabolic, $k_1k_2=0$. A special class of parabolic points is called flat, if both k_1 and k_2 are zero.

For general surfaces, the locus of parabolic points is a finite number of curves on the surface. Silhouettes on them are also curves. For special surfaces with flat regions, silhouettes may contain some flat regions, and they are difficult to analyze with classical differential geometry. In the following discussion, we assume the flat regions are with zero measures.

Definition 3 An asymptotic direction for a parabolic point is the principal direction with zero principal curvature. An asymptotic direction dr on a hyperbolic point satisfies <dr, Wdr>=0, where < , > is the inner product in E^3. Two tangent vectors dr_1, dr_2 are conjugate, if <dr_1, Wdr_2>=0.

Asymptotic directions play important roles in analyzing the local behavior of silhouettes.

Relation between curvatures

The curvature of the projected silhouettes and the normal curvature along the view direction are strongly related to the Gaussian curvature. Assume the orthographic projection map is π, a point p is on the silhouette γ, then $\pi(p)\in\pi(\gamma)$. The curvature of $\pi(p)$ is denoted as k_c. The view direction and the normal at p to the surface determine a plane, which intersects the surface at the sectional curve τ. The curvature of τ at p is denoted as k_r. According to (Koenderink, 1984), the following equation holds:

$$k_r(p)k_c(\pi(p))=K(p),$$

where $K(p)$ is the Gaussian curvature at p, as shown in Fig.1.

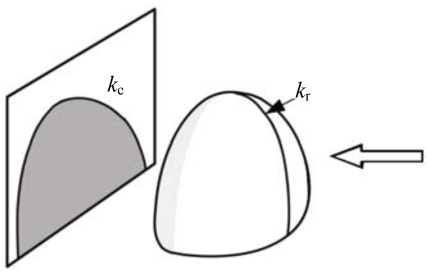


Fig.1 Curvature relation $k_r(p)k_c(\pi(p))=K(p)$
K is the Gaussian curvature at p; k_c is the curvature of $\pi(p)$ with π being an orthographic projection; k_r is the normal curvature of the view ray direction

At the cusps of the projected silhouette, the view direction is asymptotic to a hyperbolic point, k_r=0, therefore k_c is infinite. We will show that at those cusps, the geodesic curvature is also zero. Therefore, those cusps correspond to an inflection point on the silhouette on the view cone surface.

An important corollary is that the sign of the curvature of a point on a visible projected silhouette is consistent with the sign of the Gaussian curvature of its pre-image on the surface. Therefore, the projected silhouette of a convex surface must be convex.

Critical events

Catastrophe theory (Arnold, 1986) has indicated all possible critical events for projected silhouettes. The major approach to study the critical events is to classify points on the surface according to the maximum contact order of all tangency directions.

Definition 4 Suppose Σ is a generic smooth surface with position function $r(u,v)$. A point $p\in\Sigma$, a tangent direction $t\in T\Sigma(p)$ has contact of order n, if

$$\partial^k r(u,v)/\partial^k t=0,\ k=1,\ldots,n-1,$$

and

$$\partial^n r(u,v)/\partial^n t\neq 0.$$

All points on the surface can be classified according to their order of tangency as shown in Fig.2. The critical events corresponding to the special directions of each class are illustrated in Fig.3.

1. The elliptic domain, where all tangents are of order 2.

2. The hyperbolic domain, where each point has two asymptotic directions. The asymptotic directions have tangency order of 3. On the projected silhouettes, there will be a cusp as shown in Fig.3b.

3. The curve of parabolic points, with each point having one asymptotic direction. The lip event is

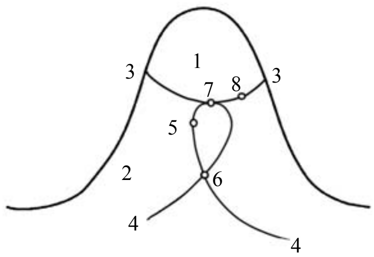


Fig.2 Classification of points on a generic surface
The class types are labelled numerically [adopted from (Arnold, 1986)]

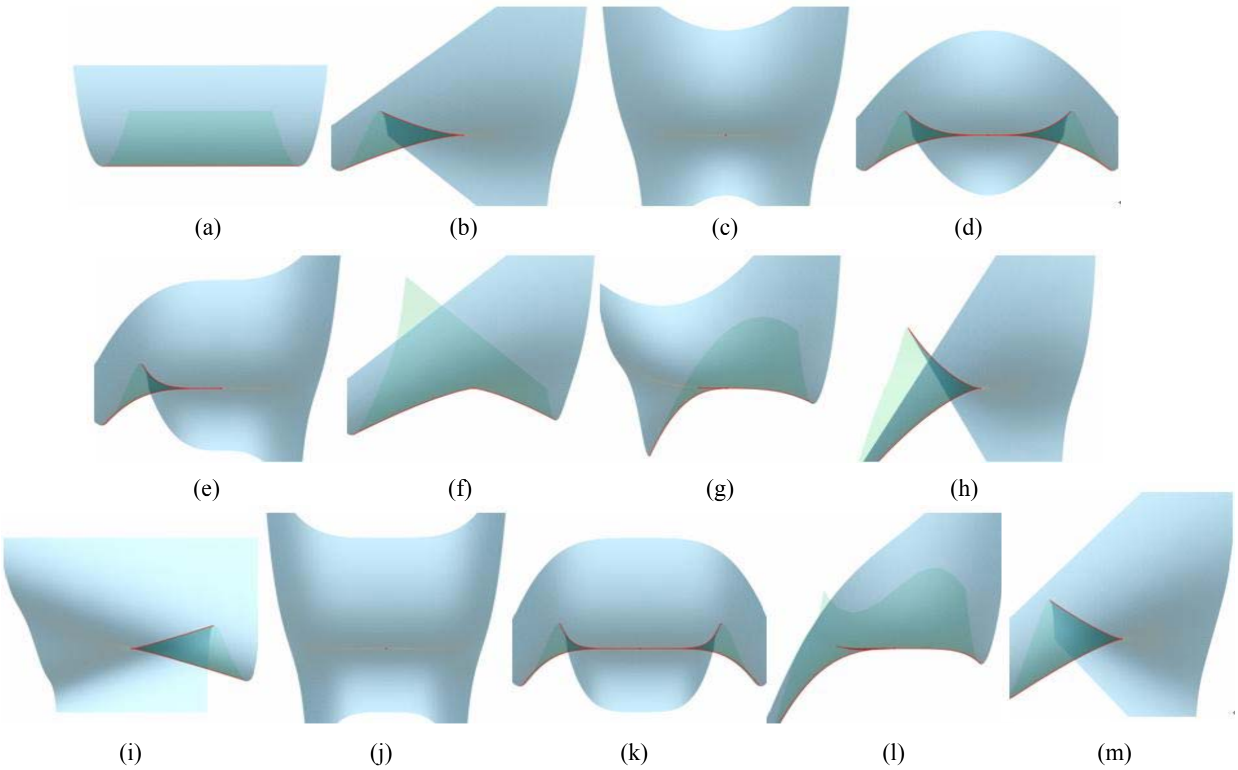


Fig.3 All possible critical events for generic surfaces with arbitrary views. (a) $z=x^2+y^2$; (b) $z=x^3+xy$; (c) $z=x^3+xy^2$; (d) $z=x^3-xy^2$; (e) $z=x^3+xy^3$; (f) $z=x^4+xy$; (g) $z=x^4+x^2y+xy^2$; (h) $z=x^5+x^3y+xy$; (i) $z=x^5+x^3y-xy$; (j) $z=x^3+xy^4$; (k) $z=x^3-xy^4$; (l) $z=x^4+x^2y+xy^3$; (m) $z=x^5+xy$

shown as Figs.3c and 3j, and the beak-to-beak event is shown as Figs.3d and 3k.

4. The envelops of asymptotic directions are called asymptotic curves. The inflections of the projection of the asymptotic curves onto the tangent plane form a flecnodal curve, where each point has a tangent of order 4 or greater. This is shown in Fig.3f.

5. The self-intersection point of a flecnodal curve, which is with two tangents of order 4.

6. The biflecnodes, which are the inflections of flecnodal curve, with a tangent of order 5. This is shown in Figs.3h, 3i and 3m.

7. The godrons, which are points of tangency of the parabolic curve and the flecnodal curve, with a tangent of order 4. This is shown in Figs.3g and 3l.

8. The gutter points on parabolic curves, which are stationary points of the asymptotic tangents, as shown in Fig.3e.

The topological changes happen when the view is along the asymptotic directions of some parabolic points: this case gives birth to the lip (Figs.3c and 3j) and beak-to-beak events (Figs.3d and 3k); the asymptotic directions along the parabolic curves and flecnodal curves give birth to the self-intersection of the silhouettes on the surface (Figs.3g and 3l). The other critical events change the topologies of the projected silhouettes, but preserve the topologies of the silhouettes on the surface.

GLOBAL PROPERTIES OF SILHOUETTE

This section aims at developing some theoretical results for the global properties of silhouettes without using any advanced machinery from singularity theory or catastrophe theory.

Aspect surface

In this subsection, we define the concept of aspect surface, and prove that all critical events for silhouettes happen when the view is on the aspect surface. If two views can be connected by a curve without intersecting the aspect surface, then the silhouettes are homotopic. The proofs use only local differential geometry.

Lemma 1 Suppose $r(u,v)$ is a generic smooth surface, with local parameters (u,v). The view point is v, and $r(s)$ is a silhouette. Then the tangent direction r is conjugate to the view ray direction $r-v$.

Proof According to the definition of silhouette, $<r-v, n>=0$, take the derivative on both sides, $<\dot{r}, \dot{n}>+<r-v,\dot{n}>=0$, therefore $<r-v,\dot{n}>=<r-v, W(\dot{r})>=0$, where W is the Weingarten map.

Lemma 2 Suppose Γ is a silhouette on a generic smooth surface Σ with a view point v, $v\notin\Sigma$. A point $p\in\Gamma$ is in one of the three cases: (1) p is elliptic, $K(p)>0$; (2) p is hyperbolic, $K(p)<0$; (3) p is parabolic, but the view direction $p-v$ is not along the asymptotic direction of p. Then in a neighborhood of p, the silhouette Γ is a 1D manifold.

Proof We take a special local parameterization, such that, at point p, $r_u=e_1$, $r_v=e_2$, where e_1, e_2 are principal directions. Then $n_u=-k_1e_1$, $n_v=-k_2e_2$.

The silhouette is the zero level set of the function

$$f(u,v)=<r(u,v)-v, n(u, v)>.$$

At the point p,

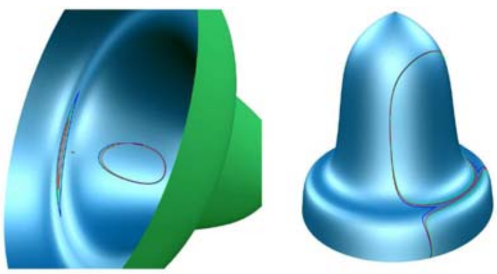
$$\partial f/\partial u=<r-v, n_u>,\quad \partial f/\partial v=<r-v, n_v>.$$

When $K(p)\neq 0$, since $r-v\neq 0$, $\partial f/\partial u$ and $\partial f/\partial v$ cannot be both zeros.

When $K(p)=0$, assume e_1 is the asymptotic direction and $r(p)-v$ is not along e_1, then $r(p)-v$ is not orthogonal to e_2, therefore $\partial f/\partial v$ is not zero.

From the implicit function theorem, we know that locally the solution of $f=0$ around p is a 1D submanifold.

When the view is along the asymptotic direction of a parabolic point, the silhouettes may not be a 1D manifold, thus the lip event or the beak-to-beak event may happen as shown in Fig.4, with the three colored curves showing the progress of topology changes of silhouettes.



(a) (b)

Fig.4 Critical events for a bell shape. (a) Lip event; (b) Beak-to-beak event

If the view point is on the surface, the following lemma depicts the topological change of the silhouettes. Fig.5 illustrates the evolution of the silhouettes when the view crosses the surface.

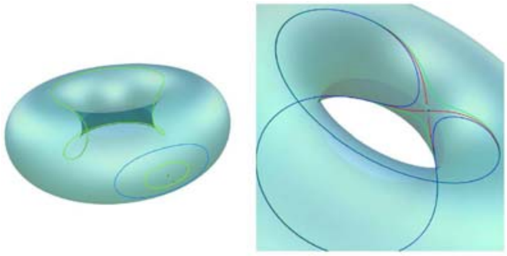


Fig.5 Topological changes when the view crosses the surface. The view point and its corresponding silhouette are marked with the same color. (a) View crosses an elliptic point; (b) View crosses a hyperbolic point

Lemma 3 Suppose Σ is a generic smooth surface, view point v crosses the surface along the normal direction through p from outside to inside, then

(1) if p is elliptic, then a closed silhouette will shrink to the point p and disappear;

(2) if p is hyperbolic, two silhouettes will intersect and reconnect, with the silhouettes being along the asymptotic directions of p.

Proof For elliptic and hyperbolic points, we can use quadratic models $(x, y, z(x, y))$ to locally approximate the surface, where

$$z=(k_1x^2+k_2y^2)/2.$$

By examining the silhouette evolutions when v crosses the surface along z direction, we can straightforwardly obtain the conclusion.

For parabolic points, one has to use higher order approximation model as described in Section 2. From Lemmas 2 and 3, it is obvious that the topological changes of silhouettes can only happen when the view points are either on the object surface itself or along asymptotic directions of parabolic points.

Definition 5 Suppose Σ is a generic smooth surface, $\gamma(s)$ is a parabolic curve, at each point $e(s)$ is the asymptotic direction of $\gamma(s)$. The following surface

$$\Gamma(s, t)=\gamma(s)+t\cdot e(s)$$

is called the aspect surface of $\gamma(s)$. The union of the aspect surfaces of all parabolic curves and the surface Σ itself is called the aspect surface of Σ, and denoted as $\Omega(\Sigma)$.

Theorem 1 Suppose Σ is a generic smooth surface. The topological changes of the silhouettes only happens when the view point v is on the aspect surface of Σ, i.e., $v \in \Omega(\Sigma)$.

Proof It is obvious from Lemmas 2 and 3.

Suppose Σ is a generic smooth surface, two views v_0 and v_1 are connected by a curve $v(t)$. Suppose the view curve does not intersect the aspect surface $\Omega(\Sigma)$, Γ_k is the silhouette for v_k, k=0, 1, then

$$\Gamma_k = \{\gamma_0^k, \gamma_1^k, ..., \gamma_n^k\},$$

such that γ_i^k (i=0,1,...n) are curve segments or loops, γ_i^0 is homotopic to γ_i^1.

Geodesic curvature relation

In this subsection, we discover a relation between the integration of the geodesic curvature of a silhouette and the view cone angle. To our surprise, this simple relation has not been discussed before. And most research focuses only on the extrinsic geometry.

This relation induces some corollaries, such as in an elliptic domain, no silhouette bounds another silhouette; in an hyperbolic domain, if a silhouette bounds a topological disk, the projected silhouette must have at least 4 cusps.

Definition 6 Suppose Σ is a generic smooth surface. A view v is at a generic position, if $v \notin \Omega(\Sigma)$. Otherwise, it is at a critical position.

First we define the view cone surface as shown in Fig.6.

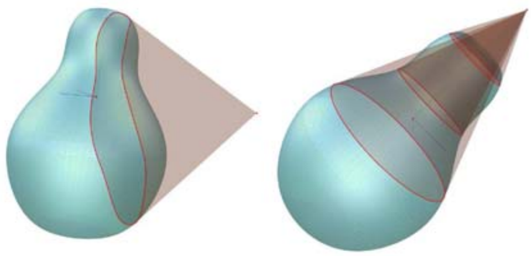


Fig.6 View cones for a pear shape

Definition 7 Suppose the view v is in a generic position for a generic smooth surface. The distance between v and Σ is finite. A connected component of the silhouette is $\gamma(s)$. The view cone surface $\Lambda(s, t)$ is defined as

$$\Lambda(s,t) = vt + \gamma(s)(1-t), \quad t \in [0,+\infty).$$

The view cone surface Λ is tangent to the original surface Σ at the silhouette γ. But the orientation of the silhouette with respect to the visible region of Σ and the orientation with respect to the view cone may not be consistent. More precisely, we define an orientation of γ, such that if one travels along γ with the orientation, the visible region on Σ is always on the left hand side of γ. Then we treat γ as a curve on the view cone Λ, sometimes the view point v is on the left hand side of γ, sometimes it is on the right hand side. Then we define the following orientation function:

Definition 8 Suppose Σ is a generic smooth surface, v is the generic view point, γ is a silhouette with consistent orientation of the visible region on Σ. The view cone surface is Λ. Then orientation function ϕ is defined as

$$\phi : \gamma \to \{+1, 0, -1\}, \phi(p) = \text{sign} < (v-p) \times \dot{\gamma}, n(p) >,$$

where $n(p)$ is the normal to the surface Σ.

It is obvious that if $\phi(p)$=0, then p must be a hyperbolic point with the view direction being along its asymptotic direction.

Lemma 4 The geodesic curvature of a point p on the silhouette is zero when p is hyperbolic and the view direction is along its asymptotic direction.

Proof At the asymptotic direction, the line of sight is tangent to the surface of order 3, therefore the curvature k of the silhouette at p is zero.

$$k^2 = k_n^2 + k_g^2,$$

therefore both k_n and k_g vanish.

The view cone surface $\Lambda(s, t)$ is a developable surface. One can slice $\Lambda(s, t)$ open along a straight line $\Lambda(0, t)$. The angle at the view point is called the cone angle at the view point.

Theorem 2 Suppose Σ is a generic smooth surface. $v \notin \Sigma$. A closed silhouette $\gamma(s)$ is smooth (without cusps on Σ), where s is the arc length; the cone angle at the view point is Φ, then

$$\int_\gamma k_g(s)\mathrm{d}s = \Phi.$$

Proof Fix the orientation of γ such that it is consistent with both Σ and Λ. Denote its geodesic curvature as $k_g(s)$. Because the view cone surface Λ is tangent to the original surface Σ at the curve γ, the geodesic curvature of γ on Λ equals $k_g(s)$.

On the view cone surface Γ, $\gamma(s)$ bounds a topological disk. We use Gauss-Bonnet theorem,

$$\int_\gamma k_g(s)\mathrm{d}s + K(v) = 2\pi,$$

where $K(v)$ is the discrete Gaussian curvature at the view point v, $K(v)=2\pi-\Phi$.

When the view point moves to infinity, then the view cone angle Φ approaches zero, and the integration of the signed geodesic curvature goes to zero.

In elliptic region, no silhouettes can be contained by another one.

Corollary 1 Suppose Σ is a generic smooth surface. The view v is at the generic position. If Σ is elliptic everywhere, γ is a silhouette bounding a topological disk, then γ does not contain any other silhouette in its visible region.

Proof Suppose γ_1 is contained in the visible region bounded by γ, then the view cone of γ_1 is contained in the view cone of γ.

Because γ is inside the elliptic region, the curvature of its projected image is positive everywhere on the image, therefore the projected image is a convex planar curve, as is the projected image of γ_1. The view cone angle Φ_1 of γ_1 is less than the view cone angle Φ of γ. Suppose γ and γ_1 bound a topological annulus D, the boundary of D is

$$\partial D = \gamma - \gamma_1.$$

According to the Gauss-Bonnet theorem,

$$\int_\gamma k_g\mathrm{d}s + \int_{\gamma_1} k_g\mathrm{d}s + \int_D K\mathrm{d}A = 0.$$

On the other hand, there is no hyperbolic point, the sign functions are constant, therefore $\int_\gamma k_g\mathrm{d}s = \Phi$,

$\int_{\gamma_1} k_g \mathrm{d}s = \Phi_1$; $\Phi_1 < \Phi$; $K>0$. Therefore the left hand side is positive. Contradiction.

In hyperbolic regions, if a closed silhouette bounds a topological disk, then the projected silhouettes must have cusps.

Corollary 2 Suppose the view v is in a generic position for a generic smooth surface Σ. If Σ is hyperbolic everywhere, a silhouette is $\gamma(s)$; if $\gamma(s)$ is closed and bounds a topological disk on Σ, then the projected silhouette $\pi(\gamma)$ must have at least 4 cusps.

Proof Consider the projected silhouettes $\pi(\gamma)$ on the image plane. According to Koenderink theorem, the curvature k of $\pi(\gamma)$ is negative everywhere. On the other hand, $\pi(\gamma)$ bounds a disk on the image plane, therefore

$$\int_{\pi(\gamma)} k \mathrm{d}s + \sum_{i=1}^{n} \alpha_i = 2\pi,$$

where $\alpha_i = \pi$ for each cusps.

Consider the sign function Φ, if the cusps separate the silhouette γ to even number of segments, the consecutive segments have different signs, therefore the number of cusps must be even. Therefore n is no less than 4. Fig.7 gives an example.

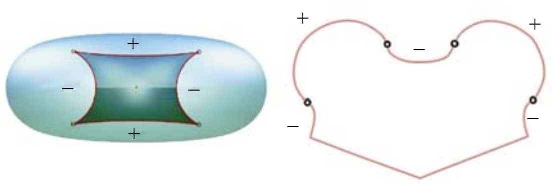


Fig.7 View cones angle for a torus. There are 4 cusps on the image, which correspond to 4 hyperbolic points, where the view rays are along their asymptotic directions. The geodesic curvatures are zeros at these points. The orientation sign function reverts at these points as well

EXPERIMENTAL RESULTS

We verified our theoretical results by tracing silhouettes on several models, represented as both triangular meshes and spline surfaces shown in Figs.8~10. The experimental results are consistent with our theorems and corollaries.

The silhouette tracing system is implemented in C++ on the windows platform with 3.6 GHz CPU and 3.0 G RAM. The silhouette tracing is efficient enough to allow the user to move views arbitrarily and visualize the evolution of the silhouettes in real time.

We computed the silhouettes, aspect surfaces for spline surfaces as shown in Fig.10. When the view point crosses the aspect surfaces, the topologies of silhouettes are changed. From Fig.10a to Fig.10b, a beak-to-beak event happens; from Fig.10b to Fig.10e, the silhouette disappears; from Fig.10e to Fig.10h, a lip event happens. If the view is moved without touching the aspect surface, the silhouettes are homotopic.

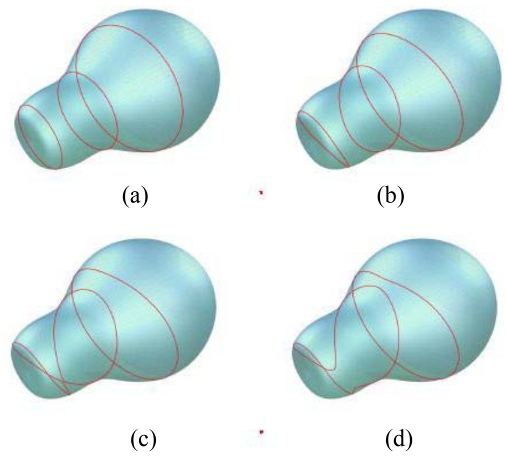


Fig.8 Beak-to-beak critical event step by step on a pear surface with the view point. (a) Step 1; (b) Step 2; (c) Step 3; (d) Step 4

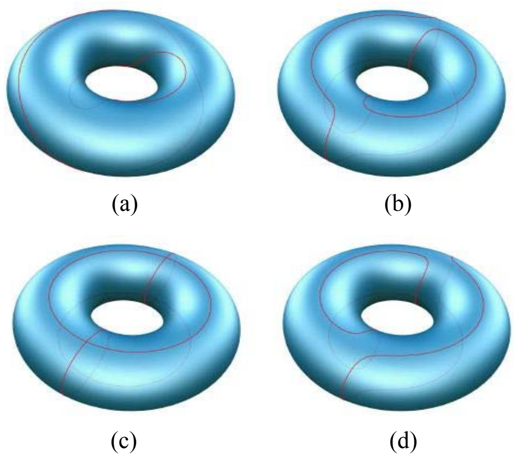


Fig.9 Beak-to-beak critical event step by step on a torus surface. (a) Step 1; (b) Step 2; (c) Step 3; (d) Step 4

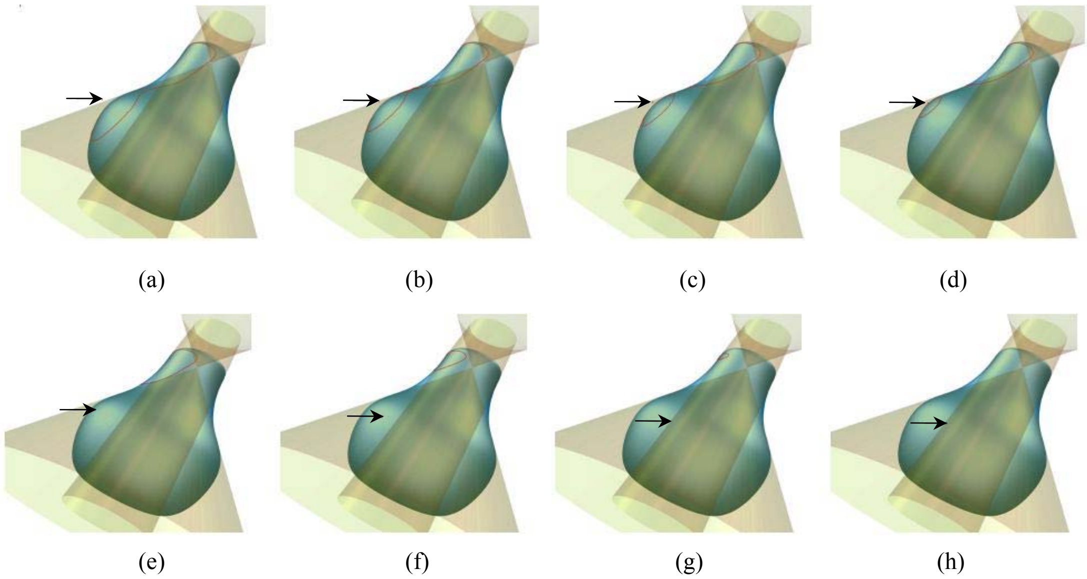


Fig.10 Evolution of silhouettes when view crosses the aspect surface of a pear shape. The arrow points to the view point. (a) View point close to the aspect surface; (b) A beak-to-beak event; (c) View point moving to the surface; (d) View point close to the surface; (e) View point on the surface; (f) View point close to the aspect surface; (g) A lip event; (h) View point on the aspect surface

CONCLUSION

This paper studies the global behavior of silhouettes. All the topological changes happen when the view is on the aspect surface. The typical topological changes are the lip event and the beak-to-beak event. Therefore, silhouettes of two views are homotopic if the views can be connected by a curve without crossing the aspect surface. The integration of a signed geodesic curvature along a silhouettes is equal to the view cone angle.

We also illustrate all possible critical events for the projected silhouettes. Finally, we built a real-time silhouette visualization.

In the future, we will design practical algorithms to interpolate silhouettes of different views for non-photorealistic rendering purposes. Furthermore, we will explore other global properties of silhouettes.

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