



Iterative learning control of SOFC based on ARX identification model*

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Received July 30, 2007; revision accepted Oct. 8, 2007

Abstract: This paper presents an application of iterative learning control (ILC) technique to the voltage control of solid oxide fuel cell (SOFC) stack. To meet the demands of the control system design, an autoregressive model with exogenous input (ARX) is established. Firstly, by regulating the variation of the hydrogen flow rate proportional to that of the current, the fuel utilization of the SOFC is kept within its admissible range. Then, based on the ARX model, three kinds of ILC controllers, i.e. P-, PI- and PD-type are designed to keep the voltage at a desired level. Simulation results demonstrate the potential of the ARX model applied to the control of the SOFC, and prove the excellence of the ILC controllers for the voltage control of the SOFC.

Key words: Autoregressive model with exogenous input (ARX), Iterative learning control (ILC), Solid oxide fuel cell (SOFC), Identification

doi:10.1631/jzus.2007.A1921

Document code: A

CLC number: TP273; TM911.4

INTRODUCTION

Distributed generation (DG) is a promising technology that can be used to address some of the technical as well as environmental concerns in power systems. As a kind of high-temperature fuel cell, solid oxide fuel cell (SOFC) presents an attractive option for the DG technology because it is modular, efficient and environmentally friendly. Unlike other types of fuel cells, SOFC is entirely solid state with no liquid components. And it usually works at a high temperature, in the range of 800~1000 °C to reach the electrolytes ionic conductivity requirement (Qi *et al.*, 2005).

SOFC is a dynamic device which will affect the dynamic behavior of the power system to which it is connected. In order to analyze such behavior, accurate dynamic models of the SOFC are required. Until now, several nonlinear dynamic models of the SOFC have been proposed (Hall and Colclaser, 1999; Pa-

dullés *et al.*, 2000; Zhu and Tomsovic, 2002; Sedghisigarchi and Feliachi, 2004; Kandepu *et al.*, 2007; Udagawa *et al.*, 2007). Most of these models emphasize the detailed description of cell internal processes, such as component material balance, energy balance and electrochemical kinetics, etc. These models are very useful to analyze the transient characteristics of the SOFC, but they are too complicated to be used for control system design. To meet the demands of developing valid control strategies, Jurado (2004; 2006) presented the identification models of the SOFC. But one of the most important cell performance variables, fuel utilization, has not been examined when they explored the SOFC dynamic response after disturbances.

System identification is a process which constructs a mathematical model by input and output data for a dynamic system under testing. It is usually applied in the case when theoretical modeling is too complex. Although the nonlinear nature of an SOFC must be recognized, in many cases a linearized system representation allows for a more efficient means of analysis. By far, an autoregressive model with

* Project (No. 2006AA05Z148) supported by the Hi-Tech Research and Development Program (863) of China

exogenous input (ARX) is the most widely applied linear dynamic model (Ghaffari *et al.*, 2007). And it is also among the most common and robust model types used by engineers to characterize linear systems (Ljung, 1999). So in this paper, an ARX model of the SOFC suited for control system design is established firstly, in which operating issues about the fuel utilization are considered specifically.

Iterative learning control (ILC), firstly introduced by Arimoto *et al.* (1984), is well known with its ability to determine a control input iteratively so that the tracking of a given reference signal or the output trajectory over a fixed time interval is possible. Until now, ILC has been successfully applied in many fields, such as robotic manipulators, chemical batch processes, etc. However, the concrete study of ILC schemes design for the SOFC has not been found in prior literature.

SOFC generates DC power from fuel and oxidant via an electrochemical process. Any changes in the load circuit or its power demand cause the changes of the SOFC operating conditions. As a result, the output voltage of the SOFC will have a high fluctuation. Furthermore, if these changes result in the fuel of the SOFC to be overused or underused, the FC will be damaged permanently. In this study, the fuel utilization of the SOFC is kept within its admissible range by regulating the variation of the hydrogen flow rate proportional to that of the current. Then, in order to keep the DC voltage at a desired level, ILC schemes based on the ARX model are developed.

THEORY FOR SOFC DYNAMIC MODEL

In the followings, some knowledge about the nonlinear dynamic model of the SOFC and fuel utilization is briefly introduced.

SOFC dynamic model

The nonlinear dynamic model of the SOFC stack adopted in this paper is shown in Fig.1 (Padullés *et al.*, 2000). This model is based on the following assumptions:

- (1) The gases are ideal.
- (2) Stack is fed with hydrogen and oxygen (fuel processor dynamics is not included).
- (3) The temperature is stable at all time.

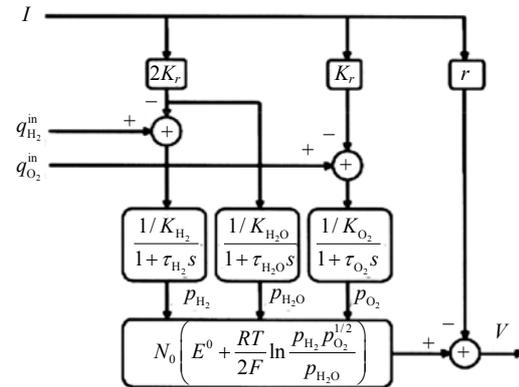


Fig.1 Dynamic model of the solid oxide fuel cell (SOFC) stack

(4) The ratio of pressures between the interior and exterior of the channel is large enough to consider the orifice choked (Padullés *et al.*, 2000; Sedghisgarchi and Feliachi, 2004).

DC current in the electrical circuit across two electrodes is generated due to the released electrons ($2e$) at the anode. Applying Nernst's equation and Ohm's law (taking into account ohmic losses), the stack output voltage could be modeled as follows (Padullés *et al.*, 2000; Li *et al.*, 2007):

$$V = N_0 \left(E^0 + \frac{RT}{2F} \ln \frac{p_{H_2} p_{O_2}^{1/2}}{p_{H_2O}} \right) - rI, \quad (1)$$

where

$$p_{H_2}(s) = \frac{1/K_{H_2}}{1 + \tau_{H_2}s} (q_{H_2}^{in} - 2K_r I), \quad (2)$$

$$p_{O_2}(s) = \frac{1/K_{O_2}}{1 + \tau_{O_2}s} (q_{O_2}^{in} - K_r I), \quad (3)$$

$$p_{H_2O}(s) = \frac{1/K_{H_2O}}{1 + \tau_{H_2O}s} \cdot 2K_r I, \quad (4)$$

where, I , E^0 , V , T and p_i are stack current, standard reversible cell potential, stack output voltage, operating temperature and partial pressure of the i th reactant, respectively. $q_{H_2}^{in}$ and $q_{O_2}^{in}$ are input hydrogen flow and oxygen flow, respectively; K_{H_2} , K_{O_2} and K_{H_2O} are valve molar constants for hydrogen, oxygen and water, respectively; τ_{H_2} , τ_{O_2} and τ_{H_2O} are response times for hydrogen flow, oxygen flow and water flow, respectively; r is ohmic resistance.

Fuel utilization

Fuel utilization is one of the most important variables affecting the performance of FC, which is defined as

$$u = \frac{q_{H_2}^{in} - q_{H_2}^{out}}{q_{H_2}^{in}} = \frac{q_{H_2}^r}{q_{H_2}^{in}} = \frac{N_0 I}{2Fq_{H_2}^{in}} \quad (5)$$

For protecting SOFC stack, the desired range of fuel utilization is set between 0.7 and 0.9.

In order to keep the fuel utilization within its admissible range and to protect the stack, the following constraint is imposed on utilization (Sedghisigarchi and Feliachi, 2006)

$$\Delta u = 0, \quad (6)$$

where

$$\Delta u = \begin{cases} u - u_{max}, & u > u_{max}, \\ 0, & u_{min} < u < u_{max}, \\ u_{min} - u, & u < u_{min}. \end{cases} \quad (7)$$

By linearizing Eq.(5), it becomes

$$\Delta u = \frac{N_0}{2Fq_{H_2}^{in^0}} \Delta I - \frac{N_0 I^0}{2F(q_{H_2}^{in^0})^2} \Delta q_{H_2}^{in}, \quad (8)$$

where, I^0 is nominal stack current, $q_{H_2}^{in^0}$ is nominal input hydrogen flow.

Since ΔI generally arises from the external load change, only $\Delta q_{H_2}^{in}$ can be regulated to make the constraint $\Delta u=0$. Therefore, we may keep u within its admissible range by adjusting the input hydrogen flow rate according to

$$\Delta q_{H_2}^{in} = \frac{q_{H_2}^{in^0}}{I^0} \Delta I. \quad (9)$$

ARX IDENTIFICATION MODEL

Among all kinds of parametric models, the ARX model should be selected firstly because of its simplicity. The ARX model has the form:

$$y(k) + a_1 y(k-1) + \dots + a_{n_a} y(k-n_a) = b_1 u(k-n_k) + b_2 u(k-n_k-1) + \dots + b_{n_b} u(k-n_k-n_b+1) + \varepsilon(k), \quad (10)$$

or in the compact form of

$$A(q^{-1})y(k) = B(q^{-1})u(k-n_k) + \varepsilon(k), \quad (11)$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_{n_a} q^{-n_a}, \quad (12)$$

$$B(q^{-1}) = b_1 + b_2 q^{-1} + \dots + b_{n_b} q^{-n_b+1}, \quad (13)$$

where $y(k)$ and $u(k)$ are the process output and input at time step k ; q^{-1} is the delay operator, e.g., $q^{-1}u(k)=u(k-1)$; $\varepsilon(k)$ is a white noise signal; n_k is the pure time-delay of the model, n_a the model order of the observed state (also called the number of poles), n_b the model order of the control signal (also called the number of zeros); a_i and b_j are model parameters, with $i=1, \dots, n_a$ and $j=1, \dots, n_b$.

From Eq.(10), it can be seen that the ARX model describes the relationship among input signals, output signals and noise through a linear difference equation, where the output at a given time step k is computed as a linear combination of past outputs and past inputs. During the identification, an important step, which precedes the parameter estimation of the ARX model shown in Eq.(10), is to determine the model structure, which is entirely defined by the three integers n_a , n_b and n_k . In this paper, the optimal ARX model order is selected by minimizing Akaike information criterion (AIC). Parameter estimation via least-squares (LS) with an ARX model structure is perhaps the most widely used approach to system identification. Hence, after the model structure is determined, the standard LS method is used for online parameter identification of the ARX model.

ITERATIVE LEARNING CONTROL

An ILC is a kind of control algorithm which is capable of tracking a desired trajectory perfectly in a period. This algorithm searches an optimal input based on previous responses by iterations. The basic configuration of ILC is illustrated in Fig.2 (Gopinath and Kar, 2004), where, $u_i(k)$ denotes the input signal during the i th iteration applied to the system, $y_i(k)$ the output trajectory, and $y_d(k)$ the desired trajectory.

These signals are stored in the memory until the iteration is completed, at which time they are processed offline by the ILC algorithm. The learning controller is nothing but the ILC algorithm compares $y_d(k)$ with $y_i(k)$ and adds the updated term(s) to $u_i(k)$ to produce $u_{i+1}(k)$, the refined input signal given to the system for the $(i+1)$ th iteration. The input signal $u_i(k)$ keeps updating until the required error goal is reached.

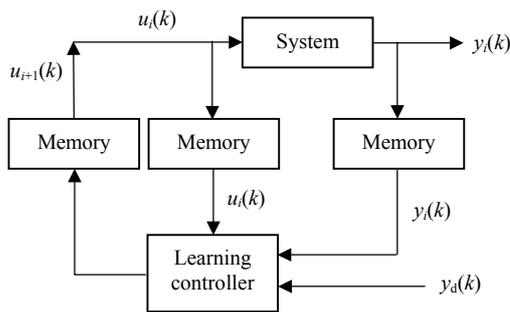


Fig.2 Basic iterative learning control configuration

Consider a discrete-time, linear, time-invariant system in the form using Z-transfer function:

$$Y(z) = H(z)U(z) = (h_m z^{-m} + h_{m+1} z^{-(m+1)} + h_{m+2} z^{-(m+2)} + \dots)U(z), \quad (14)$$

where m is the relative degree of the system, z^{-1} is the standard delay operator in time, and h_i are the standard Markov parameters of the system $H(z)$. Assume $m=1$. If the "supervectors" (Moore *et al.*, 2005) are defined as follows:

$$U_i = [u_i(0), u_i(1), \dots, u_i(N-1)]^T, \quad (15)$$

$$Y_i = [y_i(1), y_i(2), \dots, y_i(N)]^T, \quad (16)$$

$$Y_d = [y_d(1), y_d(2), \dots, y_d(N)]^T, \quad (17)$$

and

$$E_i = [e_i(1), e_i(2), \dots, e_i(N)]^T, \quad (18)$$

then Eq.(14) can be written as

$$Y_i = H_p U_i, \quad (19)$$

where H_p is a matrix of rank N whose elements are the Markov parameters of the plant. And

$$H_p = \begin{bmatrix} h_1 & 0 & 0 & \dots & 0 \\ h_2 & h_1 & 0 & \dots & 0 \\ h_3 & h_2 & h_1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_N & h_{N-1} & h_{N-2} & \dots & h_1 \end{bmatrix}. \quad (20)$$

For this system, the learning controller's goal is to derive an optimal input $u^*(k)$ for $k \in [0, N-1]$ by evaluating the error $e_i(k) = y_d(k) - y_i(k)$, where N denotes the iteration length. Generally, $u^*(k)$ can be obtained by choosing an appropriate ILC algorithm.

Until now, there have been a lot of ILC algorithms presented in the area of control systems (Sugie and Ono, 1991; Fang and Chow, 1998; Wang, 1998; Chow *et al.*, 2000; Li *et al.*, 2005). In terms of how to use the tracking error signal of previous iteration to form the control signal of current iteration, ILC updating schemes can be classified as P-, PI-, PD-, and PID-type, etc. For Eq.(14), these four kinds of ILC laws can be written as

P-type:
$$u_{i+1}(k) = u_i(k) + K_p e_i(k), \quad (21)$$

PI-type:
$$u_{i+1}(k) = u_i(k) + K_p e_i(k) + K_I \sum_{j=1}^k e_i(j), \quad (22)$$

PD-type:
$$u_{i+1}(k) = u_i(k) + K_p e_i(k) + K_D (e_i(k) - e_i(k-1)), \quad (23)$$

PID-type:
$$u_{i+1}(k) = u_i(k) + K_p e_i(k) + K_I \sum_{j=1}^k e_i(j) + K_D (e_i(k) - e_i(k-1)), \quad (24)$$

where K_p , K_I and K_D are PID learning gains. As we know, one important method for designing an iterative learning law of a control input is based on optimality criteria (Fang *et al.*, 2005). In order to develop P-, PI- and PD-type ILC controllers based on the ARX model, we firstly obtain their respective learning gains K_p , K_I and K_D by solving the following optimization problem

$$J^* = \min \|h_e\|_2^2. \quad (25)$$

Now we give some knowledge about h_e . To simplify the presentation, we introduce the operator F to map the vector $h = [h_1, h_2, \dots, h_N]^T$ to a lower trian-

gular Toeplitz matrix H_p , i.e., $H_p=F(h)$. Using supervector representation, the vector expressions of Eqs.(21)~(23) can be written. Combining each of the vector expressions with Eq.(19), we will have $E_{i+1}=F(h_e)E_i$ for the P-, PI- and PD-type ILC schemes respectively. Thus, by solving Eq.(25) we will obtain the learning gains K_p for the P-type ILC scheme, K_p and K_I for the PI-type ILC scheme, and K_p and K_D for the PD-type ILC scheme respectively. Finally, the iterative learning laws according to Eqs.(21)~(23) can be built.

RESULTS

To establish a valid ARX model, we choose the hydrogen flow rate, oxygen flow rate, operating temperature, and stack current as the model inputs, while the voltage as the output. As introduced before, in order to keep the fuel utilization within its admissible range, the amount of fuel flow rate is controlled according to the stack current. The decrease of the current will decrease the fuel flow rate.

For identification of the ARX model, the white-box model of the SOFC described above is excited with band-limited white noise around the nominal value of the hydrogen flow rate, while all the other inputs are kept constant in their nominal values. And the nominal operating conditions of the SOFC are given in Table 1 (Sedghisigarchi and Feliachi, 2004).

Table 1 SOFC operating point data

Parameter	Value
Number of single cells in the stack	384
Operating temperature (K)	1273
Nominal stack current (A)	300
Fuel utilization	0.7~0.9
Nominal input hydrogen flow (mol/s)	1.2
Standard reversible cell potential (V)	0.935
Valve molar constant for hydrogen (mol/(s·atm))	0.843

In the followings, it is assumed that the load resistor has the following variation. The SOFC is operating at its rated operation point initially. At 150 s, a load disturbance causes a step change of the stack current from 300 A to 240 A. In this situation, the variation of hydrogen flow rate is depicted in Fig.3. In order to establish the ARX model, a record of 600

experimental samples of the output voltage due to this change of hydrogen flow rate is collected. Based on these data, we firstly choose the appropriate ranges of n_a, n_b and n_k as $n_a=1\sim 10, n_b=1\sim 20$ and $n_k=0\sim 5$, thus the optimum ARX model structure can be determined using AIC criteria. Then, the ARX model parameters are estimated by using the standard LS algorithm. As a result, the voltage response of the SOFC due to the change in the hydrogen flow rate is shown in Fig.4. From Fig.4, we can see that the ARX model output voltage follows the actual voltage very well with a fitting precision of 99.96%. This indicates that the ARX identification model established is accurate and valid. Based on this ARX model, we can go along the ILC schemes design in the followings.

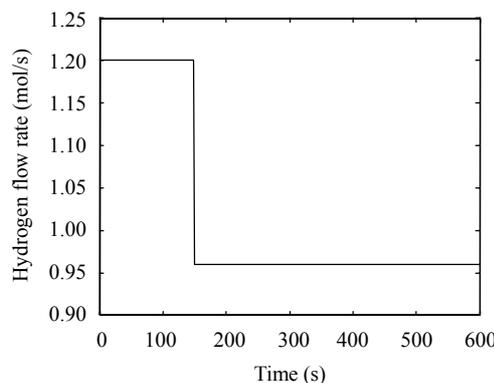


Fig.3 Variation of the hydrogen flow rate due to a step change of the current from 300 A to 240 A

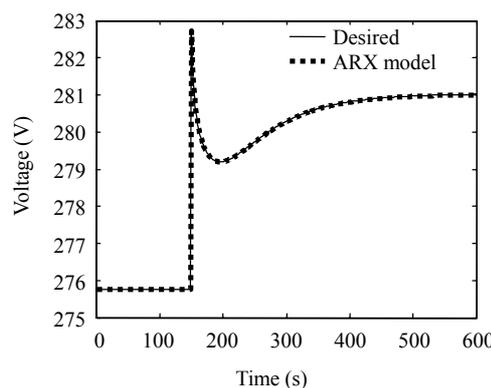


Fig.4 Voltage response of the SOFC due to the change of hydrogen flow rate

After obtaining the learning gains K_p for the P-type ILC controller, K_p and K_I for the PI-type ILC controller, and K_p and K_D for the PD-type ILC controller, the optimal input $u^*(k)$ for each of the three ILC controllers can be acquired according to the re-

quirement of the minimal tracking error or the range of learning iterations. The focus of the following simulations is to compare the performance of these ILC controllers in voltage control.

Figs.5~7 show the output voltage response due to the change of hydrogen flow rate based on these three ILC controllers. Comparing the responses, one will notice that the minimal iterative times needed for the P-, PI- and PD-type ILC controllers to obtain better voltage tracking effects are respectively 23, 80 and 22. And at these iterations, the output voltage can track the desired value change very well. Compared with the PI-type ILC controller, it can be seen that fewer iterations are needed for the P- and PD-type controllers to obtain good tracking effects. But then, voltage tracking response time for the PI-type ILC controller is much less than that for the others.

For comparison, voltage tracking response based on the PID-type ILC scheme is also given in Fig.8. Comparing this response with that resulted from the

foregoing three situations, it is worth noting that there is an extremely high voltage fluctuation for the PID-type ILC scheme before it enters the steady state again. So we realize that maybe PID-type ILC scheme is unsuitable for solving the voltage tracking problem in this study.

CONCLUSION

To meet the demands of developing valid control strategies, this paper proposes an ARX model of the SOFC stack. It is shown that the ARX model is more attractive in that it avoids using complicated differential equations to describe the stack, and that the input-output characteristics can be achieved quickly by ARX estimation. The performance of our proposed ARX modeling approach has been tested, and simulation results show that the ARX model has a higher fitting accuracy of 99.96%, which indicates that it is

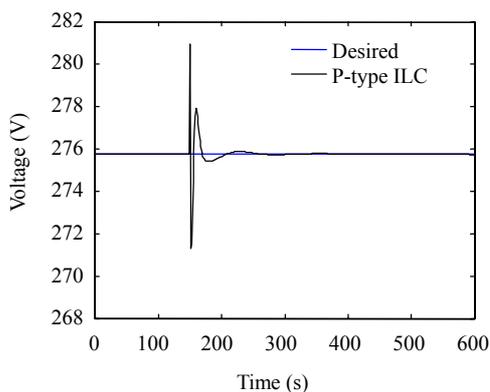


Fig.5 Output voltage response using P-type ILC controller at the 23rd iteration

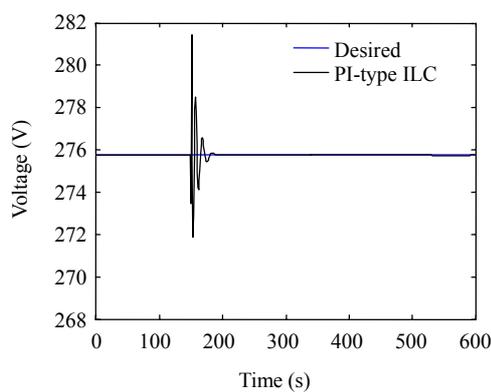


Fig.6 Output voltage response using PI-type ILC controller at the 80th iteration

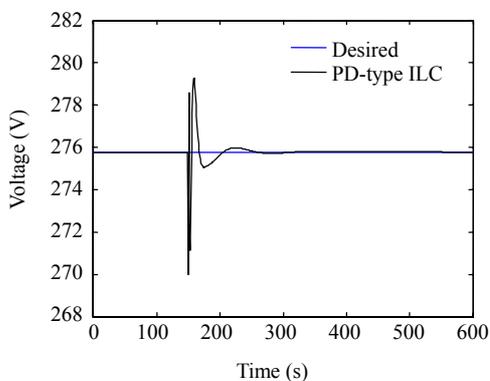


Fig.7 Output voltage response using PD-type ILC controller at the 22nd iteration

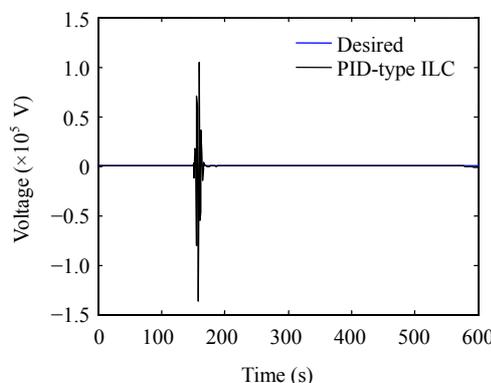


Fig.8 Output voltage response using PID-type ILC controller at the 217th iteration

feasible to establish the SOFC model by using ARX identification technology.

Based on the ARX model, this work has described the P-, PI- and PD-type ILC controller design for the voltage control of the SOFC. By simulation study and comparison, the excellence of these three ILC controllers for voltage control of the SOFC is proved.

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