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A new neural network model for the feedback stabilization of nonlinear systems*

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Abstract: A new neural network model termed 'standard neural network model' (SNNM) is presented, and a state-feedback control law is then designed for the SNNM to stabilize the closed-loop system. The control design constraints are shown to be a set of linear matrix inequalities (LMIs), which can be easily solved by the MATLAB LMI Control Toolbox to determine the control law. Most recurrent neural networks (including the chaotic neural network) and nonlinear systems modeled by neural networks or Takagi and Sugeno (T-S) fuzzy models can be transformed into the SNNMs to be stabilization controllers synthesized in the framework of a unified SNNM. Finally, three numerical examples are provided to illustrate the design developed in this paper.

Key words: Standard neural network model (SNNM), Linear matrix inequality (LMI), Nonlinear control, Asymptotic stability, Chaotic cellular neural network, Takagi and Sugeno (T-S) fuzzy model
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INTRODUCTION

In recent years, there are many applications of fuzzy control (Tanaka and Wang, 2001; Ma and Feng, 2003; Zhou and Meng, 2005), neural control (Suykens *et al.*, 1996; Nørgaard *et al.*, 2000; Lin and Hsu, 2005), and fuzzy-neural control (Er and Gao, 2003; Lin and Hsu, 2004). Stability is one of the most important concepts concerning the properties of a control system. Many kinds of techniques of stability analysis for fuzzy control systems (Tanaka and Sugeno, 1992; Tanaka and Wang, 2001) and neural control systems (Tanaka, 1995; Hu and Wang, 2006; Zhang *et al.*, 2007) have been developed via a linear matrix inequality (LMI) approach. Based on the stability analysis, various approaches of stabilization and performance synthesis have been investigated extensively by using the LMI technique in recent years (Limanond and Si, 1998; Lin and Lin, 2001; Wu and Cai, 2004). However, we have noticed that, although some stability conditions and designed stabilization (or performance) controllers obtained in the above literature have explicit expressions and, therefore, are convenient to verify and obtain in practice, stability analysis and stabilization synthesis for fuzzy control systems and those for neural network control systems have been separately discussed. Tanaka (1995) studied the stability of intelligent control systems that consist not only of linear models but also of fuzzy models and neural network models. Tanaka represented both sigmoid multilayer perceptrons (MLPs) and Takagi and Sugeno (T-S) fuzzy models as linear difference inclusion (LDI) systems to be analyzed in a unified way. However, while the intelligent systems consist of recurrent neural networks (RNNs), they cannot be transformed into the LDIs by Tanaka's approaches. Suykens *et al.*(1996) introduced NL_q-systems to describe the dynamics of RNNs and neural network control systems, and analyzed the stability of equilibrium points for NL_q-

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systems. Based on stability results of NL_q-systems, some novel stabilization synthesis techniques for neural network control systems are proposed by Suykens *et al.*(1996). Although the NL_q theory is widely applied to the neuro-control field, the NL_q-systems cannot represent the fuzzy systems or fuzzy-neural systems. Recently, many references have provided a common neural network model to describe several well-known RNNs, such as Hopfield neural networks (Hopfield, 1984), cellular neural networks (CNNs) (Chua and Yang, 1988a; 1988b), bidirectional associative memory (BAM) networks (Kosko, 1988), analyzed its stability (Sun *et al.*, 2002; Zhang and Liu, 2005; Cao *et al.*, 2006) and designed stabilized controllers (Cao *et al.*, 2005; Liu, 2006). However, this model could not include the neural network with multiple hidden layers, to say nothing of the fuzzy systems and fuzzy-neural systems. We have advanced a delayed standard neural network model in (Liu, 2005; 2007), and gave its application to stability analysis of various types of RNNs and state-feedback controller synthesis of nonlinear systems modeled by neural networks. Nevertheless, we have not provided any application to fuzzy systems. In this paper, we will improve this general model and apply it to the fuzzy control.

At present, the controller synthesis approaches vary as the different intelligent systems. As far as we know, there are no unified methods to deal with the controller synthesis problem of these systems, which consist of RNNs, fuzzy models and forward multi-layer neural network models. Referring to (Liu, 2005; 2007), we advance a standard neural network model (SNNM) with inputs and outputs to describe these complicated intelligent control systems. If we have a unified design method of stabilization controller for the SNNM, the SNNM may be widely applied to more complicated nonlinear systems.

In this paper, the following notations are used. $\mathbb{R}^n$ denotes n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices; $\boldsymbol{I}$ denotes the identity matrix of an appropriate dimension; $\|\boldsymbol{x}\|$ denotes the Euclidean norm of the vector $\boldsymbol{x}$; $*$ denotes the symmetric part. The notations $\boldsymbol{X}>\boldsymbol{Y}$ and $\boldsymbol{X}\geq\boldsymbol{Y}$, where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are symmetric matrices of the same dimension, mean that the matrix $\boldsymbol{X}-\boldsymbol{Y}$ is positive definite and positive semi-definite, respectively. If $\boldsymbol{X}\in\mathbb{R}^p$ and $\boldsymbol{Y}\in\mathbb{R}^q$, $C(\boldsymbol{X}; \boldsymbol{Y})$ denotes the space of all continuous functions mapping $\mathbb{R}^p\rightarrow\mathbb{R}^q$.

STANDARD NEURAL NETWORK MODEL

Similar to the approaches in (Liu, 2005; 2007), we can synthesize controllers for the nonlinear system composed of neural network or T-S fuzzy model by transforming them into SNNMs. The SNNM represents a neural network model as the interconnection of a linear dynamic system and static nonlinear operators consisting of bounded activation functions. Here, we discuss only the continuous-time SNNM, since there are similar architecture and results for the corresponding discrete-time model (Liu, 2005). The continuous-time SNNM with inputs and outputs is shown in Fig.1. The block Φ is a block diagonal operator composed of nonlinear activation functions $\phi_i(\xi_i(t))$, which are typically continuous, differentiable, monotonically increasing, and bounded. The matrix $\boldsymbol{N}$ represents a linear mapping between the inputs and outputs of the integrator $\int$ in the continuous-time case (or time delay $z^{-1}\boldsymbol{I}$ in the discrete-time case) and the operator Φ. The vectors $\boldsymbol{\xi}(t)$ and $\boldsymbol{\phi}(\boldsymbol{\xi}(t))$ are the input and output of the nonlinear operator Φ, respectively. The vectors $\boldsymbol{u}(t)$ and $\boldsymbol{y}(t)$ are the input and output of the SNNM, respectively.

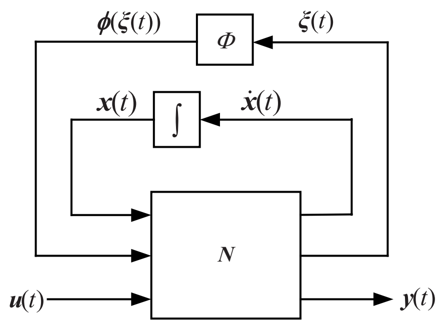

Fig.1 Continuous-time standard neural network model (SNNM) with inputs and outputs

If $\boldsymbol{N}$ in Fig.1 is partitioned as

$$\boldsymbol{N}=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}_p & \boldsymbol{B}_u\\ \boldsymbol{C}_q & \boldsymbol{D}_p & \boldsymbol{D}_{qu}\\ \boldsymbol{C}_y & \boldsymbol{D}_{yp} & \boldsymbol{D}_u\end{bmatrix}, \tag{1}$$

the input-output SNNM can be depicted as an input-output LDI,

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}_p\boldsymbol{\phi}(\boldsymbol{\xi}(t))+\boldsymbol{B}_u\boldsymbol{u}(t),\\ \boldsymbol{\xi}(t)=\boldsymbol{C}_q\boldsymbol{x}(t)+\boldsymbol{D}_p\boldsymbol{\phi}(\boldsymbol{\xi}(t))+\boldsymbol{D}_{qu}\boldsymbol{u}(t),\\ \boldsymbol{y}(t)=\boldsymbol{C}_y\boldsymbol{x}(t)+\boldsymbol{D}_{yp}\boldsymbol{\phi}(\boldsymbol{\xi}(t))+\boldsymbol{D}_u\boldsymbol{u}(t),\end{cases} \tag{2}$$

where $\boldsymbol{x}\in\mathbb{R}^n$ is the state vector, $\boldsymbol{A}\in\mathbb{R}^{n\times n}$, $\boldsymbol{B}_p\in\mathbb{R}^{n\times L}$, $\boldsymbol{B}_u\in\mathbb{R}^{n\times m}$, $\boldsymbol{C}_q\in\mathbb{R}^{L\times n}$, $\boldsymbol{C}_y\in\mathbb{R}^{l\times n}$, $\boldsymbol{D}_p\in\mathbb{R}^{L\times L}$, $\boldsymbol{D}_u\in\mathbb{R}^{l\times m}$, $\boldsymbol{D}_{qu}\in\mathbb{R}^{L\times m}$, and $\boldsymbol{D}_{yp}\in\mathbb{R}^{l\times L}$ are the corresponding state-space matrices, $\boldsymbol{\xi}\in\mathbb{R}^L$ is the input vector of the nonlinear operator Φ, $\boldsymbol{\phi}\in C(\mathbb{R}^L;\ \mathbb{R}^L)$ is the output vector of the nonlinear operator Φ satisfying $\boldsymbol{\phi}(\boldsymbol{0})=\boldsymbol{0}$, $\boldsymbol{u}\in\mathbb{R}^m$ is the input vector, $\boldsymbol{y}\in\mathbb{R}^l$ is the output vector, and $L\in\mathbb{R}$ is the number of nonlinear activation functions (i.e., the total number of neurons in the hidden layers and output layer of the neural network).

Firstly, we analyze the stability of the SNNM Eq.(2) at the equilibrium point, on which the inputs and outputs can be set to the zero vectors of appropriate dimensions. The autonomic SNNM can be described by

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}_p\boldsymbol{\phi}(\boldsymbol{\xi}(t)),\\ \boldsymbol{\xi}(t)=\boldsymbol{C}_q\boldsymbol{x}(t)+\boldsymbol{D}_p\boldsymbol{\phi}(\boldsymbol{\xi}(t)).\end{cases} \tag{3}$$

Since $\boldsymbol{x}=\boldsymbol{0}$, $\boldsymbol{\xi}=\boldsymbol{0}$ is a solution of Eq.(3), there exists at least one equilibrium point located at the origin (i.e., $\boldsymbol{x}_{\mathrm{eq}}=\boldsymbol{0}$, $\boldsymbol{\xi}_{\mathrm{eq}}=\boldsymbol{0}$). In this paper, we assume that the activation functions in the SNNM satisfy the sector conditions $\phi_i(\xi_i(t))/\xi_i(t)\in[q_i, h_i]$, i.e., $[\phi_i(\xi_i(t))-q_i\xi_i(t)]\cdot[\phi_i(\xi_i(t))-h_i\xi_i(t)]\le 0$, $h_i>q_i\ge 0$. For the autonomic SNNM Eq.(3), the global asymptotic stability can be judged by the following lemma [see Corollary 3 in (Liu, 2007)]:

Lemma 1 The origin of the SNNM Eq.(3) is globally asymptotically stable, if there exist a symmetric positive definite matrix $\boldsymbol{P}$, and a diagonal semi-positive definite matrix $\boldsymbol{T}$, such that the following LMI is satisfied:

$$\boldsymbol{G}=\begin{bmatrix}\begin{pmatrix}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}\\ -2\boldsymbol{C}_q^{\mathrm{T}}\boldsymbol{T}\boldsymbol{Q}\boldsymbol{H}\boldsymbol{C}_q\end{pmatrix} & \begin{pmatrix}\boldsymbol{P}\boldsymbol{B}_p-2\boldsymbol{C}_q^{\mathrm{T}}\boldsymbol{T}\boldsymbol{Q}\boldsymbol{H}\boldsymbol{D}_p\\ +\boldsymbol{C}_q^{\mathrm{T}}(\boldsymbol{Q}+\boldsymbol{H})\boldsymbol{T}\end{pmatrix}\\ * & \begin{pmatrix}-2\boldsymbol{D}_p^{\mathrm{T}}\boldsymbol{T}\boldsymbol{Q}\boldsymbol{H}\boldsymbol{D}_p-2\boldsymbol{T}+\boldsymbol{D}_p^{\mathrm{T}}\\ \cdot(\boldsymbol{Q}+\boldsymbol{H})\boldsymbol{T}+\boldsymbol{T}(\boldsymbol{Q}+\boldsymbol{H})\boldsymbol{D}_p\end{pmatrix}\end{bmatrix}<\boldsymbol{0}, \tag{4}$$

where $\boldsymbol{Q}=\mathrm{diag}\{q_1,q_2,\cdots,q_L\}$, $\boldsymbol{H}=\mathrm{diag}\{h_1,h_2,\cdots,h_L\}$.

The proof of Lemma 1 is given in the appendix.

FEEDBACK STABILIZATION OF SNNM

In this section, we will design a state-feedback controller for the SNNM Eq.(2) so that the overall closed-loop system is globally asymptotically stable. The controller is of the form

$$\boldsymbol{u}(t)=\boldsymbol{K}\boldsymbol{x}(t), \tag{5}$$

where $\boldsymbol{K}\in\mathbb{R}^{m\times n}$ is the feedback gain. The overall closed-loop system of the SNNM Eq.(2) and the feedback controller Eq.(5) is described by

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\tilde{\boldsymbol{A}}\boldsymbol{x}(t)+\boldsymbol{B}_p\boldsymbol{\phi}(\boldsymbol{\xi}(t)),\\ \boldsymbol{\xi}(t)=\tilde{\boldsymbol{C}}_q\boldsymbol{x}(t)+\boldsymbol{D}_p\boldsymbol{\phi}(\boldsymbol{\xi}(t)),\end{cases} \tag{6}$$

where $\tilde{\boldsymbol{A}}=\boldsymbol{A}+\boldsymbol{B}_u\boldsymbol{K}$, $\tilde{\boldsymbol{C}}_q=\boldsymbol{C}_q+\boldsymbol{D}_{qu}\boldsymbol{K}$.

Theorem 1 There exists a state-feedback control law $\boldsymbol{u}(t)=\boldsymbol{K}\boldsymbol{x}(t)$ such that the closed-loop system Eq.(6) is globally asymptotically stable provided that there exist a symmetric positive definite matrix $\boldsymbol{X}$, a matrix $\boldsymbol{Y}$, and a diagonal positive definite matrix $\boldsymbol{\Sigma}$ that satisfy the following LMI:

$$\begin{bmatrix}\begin{pmatrix}(\boldsymbol{A}\boldsymbol{X}+\boldsymbol{B}_u\boldsymbol{Y})^{\mathrm{T}}\\ +\boldsymbol{A}\boldsymbol{X}+\boldsymbol{B}_u\boldsymbol{Y}\end{pmatrix} & \begin{pmatrix}\boldsymbol{B}_p\boldsymbol{\Sigma}+(\boldsymbol{C}_q\boldsymbol{X}\\ +\boldsymbol{D}_{qu}\boldsymbol{Y})^{\mathrm{T}}(\boldsymbol{Q}+\boldsymbol{H})\end{pmatrix}\\ * & \begin{pmatrix}\boldsymbol{\Sigma}\boldsymbol{D}_p^{\mathrm{T}}(\boldsymbol{Q}+\boldsymbol{H})\\ +(\boldsymbol{Q}+\boldsymbol{H})\boldsymbol{D}_p\boldsymbol{\Sigma}-2\boldsymbol{\Sigma}\end{pmatrix}\end{bmatrix}<\boldsymbol{0}, \tag{7}$$

where $\boldsymbol{Q}=\mathrm{diag}\{q_1,q_2,\cdots,q_L\}$, $\boldsymbol{H}=\mathrm{diag}\{h_1,h_2,\cdots,h_L\}$. Furthermore, the feedback gain $\boldsymbol{K}$ is obtained as $\boldsymbol{K}=\boldsymbol{Y}\boldsymbol{X}^{-1}$.

Proof According to Lemma 1, if

$$\begin{bmatrix}\begin{pmatrix}\tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\tilde{\boldsymbol{A}}\\ -2\tilde{\boldsymbol{C}}_q^{\mathrm{T}}\boldsymbol{T}\boldsymbol{Q}\boldsymbol{H}\tilde{\boldsymbol{C}}_q\end{pmatrix} & \begin{pmatrix}\boldsymbol{P}\tilde{\boldsymbol{B}}_p-2\tilde{\boldsymbol{C}}_q^{\mathrm{T}}\boldsymbol{T}\boldsymbol{Q}\boldsymbol{H}\boldsymbol{D}_p\\ +\tilde{\boldsymbol{C}}_q^{\mathrm{T}}(\boldsymbol{Q}+\boldsymbol{H})\boldsymbol{T}\end{pmatrix}\\ * & \begin{pmatrix}-2\boldsymbol{D}_p^{\mathrm{T}}\boldsymbol{T}\boldsymbol{Q}\boldsymbol{H}\boldsymbol{D}_p-2\boldsymbol{T}+\boldsymbol{D}_p^{\mathrm{T}}\\ \cdot(\boldsymbol{Q}+\boldsymbol{H})\boldsymbol{T}+\boldsymbol{T}(\boldsymbol{Q}+\boldsymbol{H})\boldsymbol{D}_p\end{pmatrix}\end{bmatrix}<\boldsymbol{0} \tag{8}$$

holds, the origin of the closed-loop system Eq.(6) is globally asymptotically stable. Eq.(8) is equivalent to

$$\begin{bmatrix} \tilde{A}^{\mathrm{T}}P + P\tilde{A} & P\tilde{B}_p + \tilde{C}_q^{\mathrm{T}}(Q+H)T \\ * & D_p^{\mathrm{T}}(Q+H)T + T(Q+H)D_p - 2T \end{bmatrix} - 2\begin{bmatrix} \tilde{C}_q^{\mathrm{T}} \\ D_p^{\mathrm{T}} \end{bmatrix} TQH \begin{bmatrix} \tilde{C}_q & D_p \end{bmatrix} < \mathbf{0}. \tag{9}$$

Since $TQH \geq \mathbf{0}$, if

$$\begin{bmatrix} \tilde{A}^{\mathrm{T}}P + P\tilde{A} & P\tilde{B}_p + \tilde{C}_q^{\mathrm{T}}(Q+H)T \\ * & D_p^{\mathrm{T}}(Q+H)T + T(Q+H)D_p - 2T \end{bmatrix} < \mathbf{0} \tag{10}$$

holds, Eq.(9) also holds. Observing the structure of the parameters in Eq.(10), it is a nonlinear matrix inequality over P, T and K. Since there are no efficient algorithms and computing software to solve Eq.(10), we must convert it into an LMI which can be solved by the MATLAB LMI Control Toolbox (Gahinet *et al.*, 1995). Pre- and post-multiplying the left-hand side matrix of Eq.(10) by diag$\{P^{-1}, T^{-1}\}$, Eq.(10) is equivalent to

$$\begin{bmatrix} P^{-1}\tilde{A}^{\mathrm{T}} + \tilde{A}P^{-1} & B_pT^{-1} + P^{-1}\tilde{C}_q^{\mathrm{T}}(Q+H) \\ * & \begin{pmatrix} T^{-1}D_p^{\mathrm{T}}(Q+H) + \\ (Q+H)D_pT^{-1} - 2T^{-1} \end{pmatrix} \end{bmatrix} < \mathbf{0}. \tag{11}$$

Defining

$$X = P^{-1},\ Y = KX,\ \Sigma = T^{-1}, \tag{12}$$

we rewrite Eq.(11) as Eq.(7). The feedback gain K is derived from Eq.(12). The proof of Theorem 1 is thus complete.

ILLUSTRATIVE EXAMPLES

To apply Theorem 1 to synthesize the state-feedback controller to stabilize an intelligent control system, it is necessary to transform it into the SNNM. The following examples, synthesis of feedback stabilizing controllers for an SISO (single-input single-output) continuous-time nonlinear system modeled by the neural networks in (Lin and Lin, 2001), a chaotic CNN in (Cheng *et al.*, 2005), and an inverted pendulum system modeled by the T-S fuzzy model in (Ma and Feng, 2003), illustrate that the SNNM can be widely applied to the synthesis of control systems.

Controller synthesis for the nonlinear system modeled by the MLPs

We consider a simple example to illustrate our SNNM-based feedback control design algorithm and to demonstrate the closed-loop stability guarantee. In particular, consider the following SISO continuous-time nonlinear system (Lin and Lin, 2001):

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -0.25 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ f_1(x(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ f_2(u(t)) \end{bmatrix}, \tag{13}$$

where $f_1(x(t))=\exp(-(x_1(t)+x_2(t)))\cos(x_1(t)+x_2(t))-1$, and $f_2(u(t))=1.5\cos^2(u(t))\sin^2(u(t))$.

The state-feedback controller Eq.(5) is adopted to guarantee the closed-loop stability. Two MLPs are used to approximate the nonlinear terms $f_1(x(t))$ and $f_2(u(t))$ in (Lin and Lin, 2001), respectively. The MLPs are represented as

$$\begin{cases} f_1(x(t)) = \tanh(W_2 \tanh(W_1 x(t))), \\ f_2(u(t)) = \tanh(V_2 \tanh(V_1 u(t))), \end{cases} \tag{14}$$

where

$$W_1 = \begin{bmatrix} -0.59 & -0.14\times10^{-1} \\ 0.40\times10^{-2} & 0.10\times10^{-3} \\ 0.18\times10^{-1} & 0.40\times10^{-3} \end{bmatrix},$$

$$W_2 = \begin{bmatrix} -0.30\times10^{-2} & 0.12\times10^{-5} & 0.10\times10^{-3} \end{bmatrix},$$

$$V_1 = \begin{bmatrix} -0.27\times10^{-12} \\ -0.31\times10^{-2} \end{bmatrix}, V_2 = \begin{bmatrix} 0.27\times10^{-12} & 0.31\times10^{-2} \end{bmatrix}.$$

The system Eq.(13) approximated by MLPs can be converted into the following form:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -0.25 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \tanh(W_2 \tanh(W_1 x(t))) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ \tanh(V_2 \tanh(V_1 u(t))) \end{bmatrix}. \tag{15}$$

We transform the system Eq.(15) into the SNNM Eq.(2), where

$$A = \begin{bmatrix} 0 & 1 \\ -0.25 & -1 \end{bmatrix}, B_p = \begin{bmatrix} 0 & 0 & \mathbf{0}_{1\times3} & \mathbf{0}_{1\times2} \\ 1 & 1 & \mathbf{0}_{1\times3} & \mathbf{0}_{1\times2} \end{bmatrix}, B_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\boldsymbol{C}_q=\begin{bmatrix}\boldsymbol{0}_{1\times2}\\ \boldsymbol{0}_{1\times2}\\ \boldsymbol{W}_1\\ \boldsymbol{0}_{2\times2}\end{bmatrix},\ \boldsymbol{D}_p=\begin{bmatrix}0 & 0 & \boldsymbol{W}_2 & \boldsymbol{0}_{1\times2}\\ 0 & 0 & \boldsymbol{0}_{1\times3} & \boldsymbol{V}_2\\ \boldsymbol{0}_{3\times1} & \boldsymbol{0}_{3\times1} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times2}\\ \boldsymbol{0}_{2\times1} & \boldsymbol{0}_{2\times1} & \boldsymbol{0}_{2\times3} & \boldsymbol{0}_{2\times2}\end{bmatrix},$$

$$\boldsymbol{D}_{qu}=\begin{bmatrix}0\\0\\ \boldsymbol{0}_{3\times1}\\ \boldsymbol{V}_1\end{bmatrix},\ \boldsymbol{\phi}(\boldsymbol{\xi}(t))=\tanh(\boldsymbol{\xi}(t)),\ \boldsymbol{Q}=\boldsymbol{0}_{7\times7},\ \boldsymbol{H}=\boldsymbol{I}_{7\times7}.$$

According to Theorem 1, solving the LMI Eq.(7) by the MATLAB LMI Control Toolbox (Gahinet *et al.*, 1995), we obtain the solutions of LMI Eq.(7) and the feedback gain as

$$\boldsymbol{X}=\begin{bmatrix}13.3112 & -6.8002\\ -6.8002 & 22.5778\end{bmatrix},\ \boldsymbol{Y}=[-26.0499\ \ 9.6402],$$

$$\boldsymbol{\Sigma}=\mathrm{diag}\{9.6321,\ 9.6321,\ 13.4849,\ 13.4849,\ 13.4849,\ 13.4849,\ 13.4849\},$$

$$\boldsymbol{K}=\boldsymbol{Y}\boldsymbol{X}^{-1}=[-2.0551\ \ -0.1920].$$

When the state-feedback law $u(t)=\boldsymbol{Kx}(t)$, which is shown in Fig.2a, is put on the unforced nonlinear system Eq.(13), the state trajectories of the closed-loop system converge to zeroes asymptotically, as shown in Fig.2b.

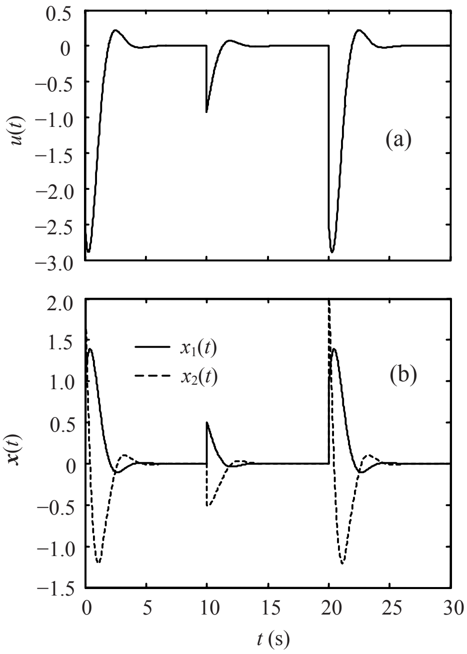

Fig.2 Control signal $u(t)$ (a) and state response of $x_1(t)$ and $x_2(t)$ (b) of the closed-loop system where the states are initialized arbitrarily at t=0, 10, and 20 s, respectively

Feedback control of a chaotic CNN

Now, we consider a chaotic CNN as follows (Cheng *et al.*, 2005):

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix}=-\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix}+\begin{bmatrix}1.25 & -3.2 & -3.2\\ -3.2 & 1.1 & -4.4\\ -3.2 & 4.3 & 1\end{bmatrix}\begin{bmatrix}f(x_1(t))\\ f(x_2(t))\\ f(x_3(t))\end{bmatrix},\tag{16}$$

where $f(x_i)=(|x_i+1|-|x_i-1|)/2$, i=1, 2, 3. As shown in Fig.3, the CNN Eq.(16) possesses chaotic behavior. We convert the CNN Eq.(16) into the autonomic SNNM Eq.(3), where

$$\boldsymbol{x}=[x_1(t)\ \ x_2(t)\ \ x_3(t)]^{\mathrm{T}},\ \boldsymbol{A}=\mathrm{diag}\{-1,\ -1,\ -1\},$$

$$\boldsymbol{B}_p=\begin{bmatrix}1.25 & -3.2 & -3.2\\ -3.2 & 1.1 & -4.4\\ -3.2 & 4.3 & 1\end{bmatrix},\ \boldsymbol{C}_q=\boldsymbol{H}=\boldsymbol{I}_{3\times3},$$

$$\boldsymbol{D}_p=\boldsymbol{Q}=\boldsymbol{0}_{3\times3},\ \phi_i(x_i)=f_i(x_i),\ i=1, 2, 3.$$

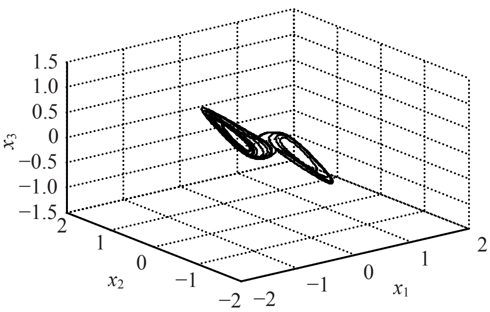

Fig.3 Chaotic behavior of the CNN Eq.(16) with the initial condition $[x_1(0)\ x_2(0)\ x_3(0)]^{\mathrm{T}}$=[0.1 0.1 0.1]$^{\mathrm{T}}$

In order to globally stabilize the chaotic CNN Eq.(16), we introduce an external control term $\boldsymbol{u}(t)$ into the CNN Eq.(16), which yields the control system of the form

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\boldsymbol{Ax}(t)+\boldsymbol{B}_p\boldsymbol{\phi}(\boldsymbol{\xi}(t))+\boldsymbol{u}(t),\\ \boldsymbol{\xi}(t)=\boldsymbol{C}_q\boldsymbol{x}(t),\end{cases}\tag{17}$$

and the controller Eq.(5) is adopted to ensure the closed-loop asymptotic stability, where $\boldsymbol{u}(t)\in\mathbb{R}^3$, $\boldsymbol{K}\in\mathbb{R}^{3\times3}$. According to Theorem 1, solving the LMI Eq.(7), we obtain the solutions of LMI Eq.(7) and the feedback gain as

$$\boldsymbol{X}=\begin{bmatrix}15.5308 & 5.8887 & 5.7071\\ 5.8887 & 12.2036 & 2.1672\\ 5.7071 & 2.1672 & 12.3470\end{bmatrix},$$

$$\boldsymbol{Y}=10^3\times\begin{bmatrix}-0.0334 & -5.2789 & -1.0604\\ 5.3156 & -0.0531 & 0.6104\\ 1.1350 & -0.6298 & -0.0562\end{bmatrix},$$

$$\boldsymbol{\Sigma}=\text{diag}\{7.0462, 4.7962, 5.2362\},$$

$$\boldsymbol{K}=\begin{bmatrix}235.6111 & -528.1283 & -102.0912\\ 469.0305 & -207.4213 & -130.9581\\ 130.3390 & -106.3042 & -46.1362\end{bmatrix}.$$

Under the state-feedback control law Eq.(5), which is shown in Fig.4a, the state response of the closed-loop system of the chaotic CNN Eq.(16) is shown in Fig.4b, where the state is initialized arbitrarily.

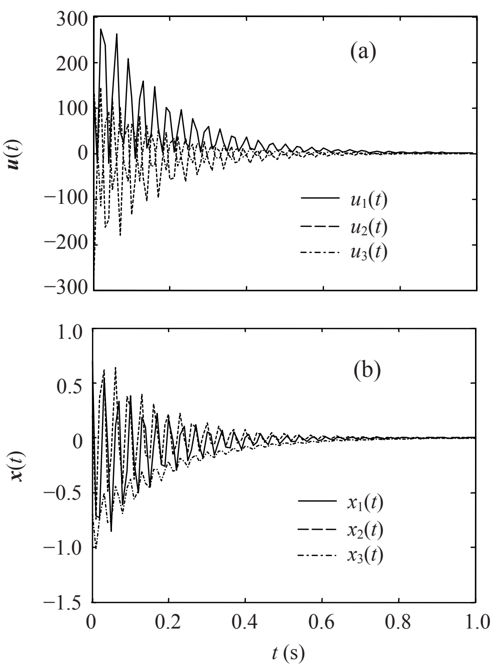

Fig.4 Control signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ (a) and state trajectories of $x_1(t)$, $x_2(t)$ and $x_3(t)$ (b) of the closed-loop system of the chaotic CNN Eq.(16)

Feedback controller synthesis for the nonlinear system approximated by the T-S fuzzy model

To further illustrate the generality of the SNNM, we consider the following problem of balancing an inverted pendulum on a cart. The equations of motion for the pendulum are

$$\begin{cases}\dot{x}_1(t)=x_2(t),\\ \dot{x}_2(t)=[g\sin(x_1(t))-amlx_2^2(t)\sin(2x_1(t))/2\\ \quad -a\cos(x_1(t))u(t)]\cdot[4l/3-aml\cos^2(x_1(t))]^{-1},\end{cases}\tag{18}$$

where $x_1(t)$ denotes the angle of the pendulum from the vertical, and $x_2(t)$ is the angular velocity. g=9.8 m/s^2 is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, and $u(t)$ is the force applied to the cart. $a=1/(m+M)$. We choose m=2.0 kg, M=8.0 kg, $2l$=1.0 m in the simulation. The state-feedback controller Eq.(5) is adopted to stabilize the closed-loop system. The following continuous-time T-S fuzzy model is used to express the nonlinear system Eq.(18) (Ma and Feng, 2003):

$$\dot{\boldsymbol{x}}(t)=\sum_{i=1}^{2}\mu_i(t)[\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}_iu(t)],\tag{19}$$

where the state $\boldsymbol{x}(t)=[x_1(t)\ x_2(t)]^{\mathrm{T}}$, the membership functions $\mu_i(t)$ (i=1, 2) satisfy:

$$0\le\mu_1(t)=\left(1-\frac{1}{1+\exp(-7(x_1(t)-\pi/4))}\right)\cdot\frac{1}{1+\exp(-7(x_1(t)+\pi/4))}\le 1,$$

$$0\le\mu_2(t)=1-\mu_1(t)\le 1,$$

and the state matrices are as follows:

$$\boldsymbol{A}_1=\begin{bmatrix}0 & 1\\ 17.2941 & 0\end{bmatrix},\ \boldsymbol{B}_1=\begin{bmatrix}0\\ -0.1765\end{bmatrix},$$

$$\boldsymbol{A}_2=\begin{bmatrix}0 & 1\\ 9.3600 & 0\end{bmatrix},\ \boldsymbol{B}_2=\begin{bmatrix}0\\ -0.0052\end{bmatrix}.$$

We write the T-S model Eq.(19) as

$$\begin{cases}\dot{x}_1(t)=x_2(t),\\ \dot{x}_2(t)=\mu_1(t)(7.9341x_1(t)-0.1713u(t))\\ \qquad +9.3600x_1(t)-0.0052u(t).\end{cases}\tag{20}$$

If we introduce $\phi(\xi(t))=\mu_1(t)\xi(t)$, the T-S model Eq.(20) is equivalent to

$$\begin{cases}\dot{x}_1(t)=x_2(t),\\ \dot{x}_2(t)=\phi(7.9341x_1(t)-0.1713u(t))\\ \qquad +9.3600x_1(t)-0.0052u(t),\end{cases}\tag{21}$$

where $\phi(\cdot)$ satisfies the sector condition, and the sector bound is [0, 1]. We transform the model Eq.(21) into the SNNM Eq.(2), where

$$\boldsymbol{A}=\begin{bmatrix}0 & 1\\ 9.3600 & 0\end{bmatrix},\ \boldsymbol{B}_p=\begin{bmatrix}0\\1\end{bmatrix},\ \boldsymbol{B}_u=\begin{bmatrix}0\\-0.0052\end{bmatrix},$$

$$\boldsymbol{C}_q=[7.9341\quad 0],\ D_p=0,\ D_{qu}=-0.1713,\ Q=0,\ H=1.$$

According to Theorem 1, we solve the LMI Eq.(7) and obtain the solutions of LMI Eq.(7) and the feedback gain as

$$\boldsymbol{X}=\begin{bmatrix}0.3775 & -1.1233\\ -1.1233 & 3.4261\end{bmatrix},\ \boldsymbol{Y}=[19.3102\quad 13.1053],$$

$$\Sigma=11.4413,\ \boldsymbol{K}=\boldsymbol{YX}^{-1}=[2549.2\quad 839.6].$$

Fig.5a shows the state-feedback control law $u(t)$; Fig.5b shows the response for the angle and angular velocity of the pendulum system Eq.(18) under $u(t)$. In Fig.5b, the initial condition $x_1(0)$=1.1345 rad and $x_2(0)$=0 rad/s. It can be seen that the angle and the angular velocity converge to zeroes asymptotically.

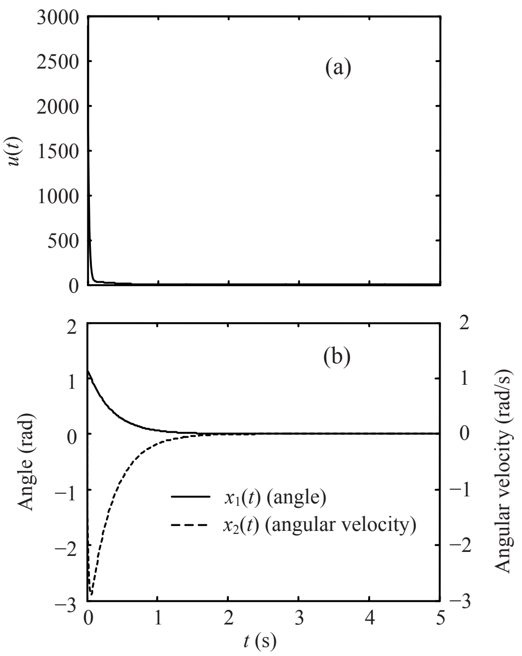

Fig.5 (a) Control law $u(t)$ on the pendulum system Eq.(18); (b) $x_1(t)$ (angle) and $x_2(t)$ (angular velocity) response of the pendulum system Eq.(18)

CONCLUSION

In this paper, we studied a control design algorithm for a class of intelligent control systems that consist not only of fuzzy models but also of forward (or recurrent) neural network models. Central to our design are the introduction of the SNNM, which interconnects a linear dynamic system with static nonlinear operators composed of bounded activation functions, and the transformation of the intelligent control system to the SNNM. A state-feedback controller has been designed for the SNNM such that the closed-loop system is globally asymptotically stable. The resulting design inequalities are a set of LMIs which can be solved by the MATLAB LMI Control Toolbox (Gahinet *et al.*, 1995) to determine the control laws. The design approach can be extended to synthesize any nonlinear control systems as long as their equations can be transformed into the SNNM. Here, it is worth noting that there are no unified ways about how to convert the non-SNNM into the SNNM, but generally state transformation is applied.

References

Boyd, S.P., Ghaoui, L.E., Feron, E., Balakrishnan, V., 1994. Linear Matrix Inequalities in System and Control Theory. SIAM, Philadelphia, PA.

Cao, J., Li, H.X., Ho, D.W.C., 2005. Synchronization criteria of Lur'e systems with time-delay feedback control. *Chaos Solit. Fract.*, **23**(4):1285-1298. [doi:10.1016/j.chaos.2004.06.025]

Cao, J., Yuan, K., Li, H.X., 2006. Global asymptotical stability of recurrent neural networks with multiple discrete delays and distributed delays. *IEEE Trans. on Neural Networks*, **17**(6):1646-1651. [doi:10.1109/TNN.2006.881488]

Cheng, C.J., Liao, T.L., Hwang, C.C., 2005. Exponential synchronization of a class of chaotic neural networks. *Chaos Solit. Fract.*, **24**(1):197-206. [doi:10.1016/j.chaos.2004.09.022]

Chua, L.O., Yang, L., 1988a. Cellular neural networks: theory. *IEEE Trans. on Circuits Syst.*, **35**(10):1257-1272. [doi:10.1109/31.7600]

Chua, L.O., Yang, L., 1988b. Cellular neural networks: applications. *IEEE Trans. on Circuits Syst.*, **35**(10): 1273-1290. [doi:10.1109/31.7601]

Er, M.J., Gao, Y., 2003. Robust adaptive control of robot manipulators using generalized fuzzy neural networks. *IEEE Trans. on Ind. Electr.*, **50**(3):620-628. [doi:10.1109/TIE.2003.812454]

Gahinet, P., Nemirovski, A., Laub, A.J., Chilali, M., 1995. LMI Control Toolbox for Use with Matlab. The MATH Works, Inc., Natick, MA.

Hopfield, J.J., 1984. Neurons with graded response have collective computational properties like those of two-state neurons. *Proc. Natl. Acad. Sci. USA*, **81**(10):3088-3092. [doi:10.1073/pnas.81.10.3088]

Hu, S., Wang, J., 2006. Global robust stability of a class of discrete-time interval neural networks. *IEEE Trans. on Circuits Syst.-I: Regul. Papers*, **53**(1):129-138. [doi:10.1109/TCSI.2005.854288]

Kosko, B., 1988. Adaptive bi-directional associative memories. *IEEE Trans. on Syst. Man Cybern.*, **18**(1):49-60. [doi:10.1109/21.87054]

Limanond, S., Si, J., 1998. Neural-network-based control design: an LMI approach. *IEEE Trans. on Neural Networks*, **9**(6):1422-1429. [doi:10.1109/72.728392]

Lin, C.L., Lin, T.Y., 2001. An H_∞ design approach for neural net-based control schemes. *IEEE Trans. on Automatic Control*, **46**(10):1599-1605. [doi:10.1109/9.956056]

Lin, C.M., Hsu, C.F., 2004. Supervisory recurrent fuzzy neural network control of wing rock for slender delta wings. *IEEE Trans. on Fuzzy Syst.*, **12**(5):733-742. [doi:10.1109/TFUZZ.2004.834803]

Lin, C.M., Hsu, C.F., 2005. Recurrent-neural-network-based adaptive-backstepping control for induction servomotors. *IEEE Trans. on Ind. Electr.*, **52**(6):1677-1684. [doi:10.1109/TIE.2005.858704]

Liu, M., 2005. Discrete-time delayed standard neural network model and its application. *Sci. China Ser. E-Inf. Sci.*, **35**(10):1031-1048.

Liu, M., 2006. Interval standard neural network models for nonlinear systems. *J. Zhejiang Univ. Sci. A*, **7**(4):530-538. [doi:10.1631/jzus.2006.A0530]

Liu, M., 2007. Delayed standard neural network models for control systems. *IEEE Trans. on Neural Networks*, **18**(5):1376-1391. [doi:10.1109/TNN.2007.894084]

Ma, J., Feng, G., 2003. An approach to H_∞ control of fuzzy dynamic systems. *Fuzzy Sets Syst.*, **137**(3):367-386. [doi:10.1016/S0165-0114(02)00281-6]

Nørgaard, M., Ravn, O., Poulsen, N.K., Hansen, L.K., 2000. Neural Networks for Modeling and Control of Dynamic Systems. Springer-Verlag, London, Berlin, Heidelberg.

Sun, C., Zhang, K., Fei, S., Feng, C.B., 2002. On exponential stability of delayed neural networks with a general class of activation functions. *Phys. Lett. A*, **298**(2-3):122-132. [doi:10.1016/S0375-9601(02)00471-1]

Suykens, J.A.K., Vandewalle, J.P.L., de Moor, B.L.R., 1996. Artificial Neural Networks for Modeling and Control of Non-linear Systems. Kluwer Academic Publishers, Boston, Dordrecht, London.

Tanaka, K., 1995. Stability and stabilizability of fuzzy-neural-linear control systems. *IEEE Trans. on Fuzzy Syst.*, **3**(4):438-447. [doi:10.1109/91.481952]

Tanaka, K., Sugeno, M., 1992. Stability and design of fuzzy control systems. *Fuzzy Sets Syst.*, **45**(2):135-156. [doi:10.1016/0165-0114(92)90113-I]

Tanaka, K., Wang, H.O., 2001. Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach. John Wiley & Sons, Hoboken, New Jersey, USA.

Wu, H.N., Cai, K.Y., 2004. H_2 guaranteed cost fuzzy control for uncertain nonlinear systems via linear matrix inequalities. *Fuzzy Sets Syst.*, **148**(3):411-429. [doi:10.1016/j.fss.2003.11.015]

Zhang, H., Wang, Z., Liu, D., 2007. Robust exponential stability of recurrent neural networks with multiple time-varying delays. *IEEE Trans. on Circuits Syst.-II: Expr. Briefs*, **54**(8):730-734. [doi:10.1109/TCSII.2007.896799]

Zhang, S., Liu, M., 2005. Stability analysis of discrete-time BAM neural networks based on standard neural network models. *J. Zhejiang Univ. Sci.*, **6A**(7):689-696. [doi:10.1631/jzus.2005.A0689]

Zhou, W.N., Meng, P.Y., 2005. LMI-based H_∞ Control Design with Regional Stability Constraint for T-S Fuzzy System. Proc. Int. Conf. on Machine Learning and Cybernetics, Guangzhou, China, **2**:868-873. [doi:10.1109/ICMLC.2005.1527065]

APPENDIX: PROOF OF LEMMA 1

For simplicity, we denote $\boldsymbol{x}(t)$ as $\boldsymbol{x}$, $\xi_i(t)$ as ξ_i, $\phi_i(\xi_i(t))$ as ϕ_i, and $\boldsymbol{\phi}(\boldsymbol{\xi}(t))$ as $\boldsymbol{\phi}$. For the SNNM Eq.(3), we construct the Lyapunov function

$$V(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x},$$

where $\boldsymbol{P}>\boldsymbol{0}$. Thus, $\forall \boldsymbol{x}\neq\boldsymbol{0}$, $V(\boldsymbol{x})>0$; $V(\boldsymbol{x})=0$ iff $\boldsymbol{x}=\boldsymbol{0}$. The derivative of $V(\boldsymbol{x})$ along the solution of Eq.(3) is

$$\frac{\mathrm{d}V(\boldsymbol{x})}{\mathrm{d}t} = 2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{A}\boldsymbol{x}+\boldsymbol{B}_p\boldsymbol{\phi}) = \boldsymbol{x}^{\mathrm{T}}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A})\boldsymbol{x} + \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}_p\boldsymbol{\phi}+\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{B}_p^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x} = \begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{\phi}\end{bmatrix}^{\mathrm{T}}\boldsymbol{R}_0\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{\phi}\end{bmatrix},$$

where $\boldsymbol{R}_0=\begin{bmatrix}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A} & \boldsymbol{P}\boldsymbol{B}_p\\ * & \boldsymbol{0}\end{bmatrix}$. The sector conditions $(\phi_i-q_i\xi_i)(\phi_i-h_i\xi_i)\le 0$ can be rewritten as follows:

$$(\phi_i-q_i\boldsymbol{C}_{q,i}\boldsymbol{x}-q_i\boldsymbol{D}_{p,i}\boldsymbol{\phi})(\phi_i-h_i\boldsymbol{C}_{q,i}\boldsymbol{x}-h_i\boldsymbol{D}_{p,i}\boldsymbol{\phi})\le 0,$$

which is equivalent to

$$\begin{aligned}&2\phi_i^2-2\phi_i(q_i+h_i)\boldsymbol{C}_{q,i}\boldsymbol{x}-2\phi_i(q_i+h_i)\boldsymbol{D}_{p,i}\boldsymbol{\phi}\\&+2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{C}_{q,i}^{\mathrm{T}}q_ih_i\boldsymbol{C}_{q,i}\boldsymbol{x}+2\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{D}_{p,i}^{\mathrm{T}}q_ih_i\boldsymbol{D}_{p,i}\boldsymbol{\phi}\\&+2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{C}_{q,i}^{\mathrm{T}}q_ih_i\boldsymbol{D}_{p,i}\boldsymbol{\phi}+2\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{D}_{p,i}^{\mathrm{T}}q_ih_i\boldsymbol{C}_{q,i}\boldsymbol{x}\le 0,\end{aligned} \tag{A1}$$

where $\boldsymbol{C}_{q,i}$ and $\boldsymbol{D}_{p,i}$ denote the ith row of $\boldsymbol{C}_q$ and $\boldsymbol{D}_p$, respectively. We rewrite Eq.(A1) in matrix notation as follows:

$$\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{\phi}\end{bmatrix}^{\mathrm{T}}\boldsymbol{R}_i\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{\phi}\end{bmatrix}+\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{\phi}\end{bmatrix}^{\mathrm{T}}\bar{\boldsymbol{R}}_i\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{\phi}\end{bmatrix}\le 0,$$

where

$$\boldsymbol{\phi}=[\phi_1,\ \cdots,\ \phi_{i-1},\ \phi_i,\ \phi_{i+1},\ \cdots,\ \phi_L\ ]^{\mathrm{T}},$$

$$\boldsymbol{R}_i=\begin{bmatrix}
\boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & -\boldsymbol{C}_{q,i}^{\mathrm{T}}s_i & \boldsymbol{0} & \cdots & \boldsymbol{0}\\
\boldsymbol{0} & 0 & \cdots & 0 & -d_{p,i,1}s_i & 0 & \cdots & 0\\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\
\boldsymbol{0} & 0 & \cdots & 0 & -d_{p,i,i-1}s_i & 0 & \cdots & 0\\
-s_i\boldsymbol{C}_{q,i} & -s_i d_{p,i,1} & \cdots & -s_i d_{p,i,i-1} & 2-2s_i d_{p,i,i} & s_i d_{p,i,i+1} & \cdots & s_i d_{p,i,L}\\
\boldsymbol{0} & 0 & \cdots & 0 & -d_{p,i,i+1}s_i & 0 & \cdots & 0\\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots\\
\boldsymbol{0} & 0 & 0 & 0 & -d_{p,i,L}s_i & 0 & \cdots & 0
\end{bmatrix},$$

$$\bar{\boldsymbol{R}}_i=\begin{bmatrix}2\boldsymbol{C}_{q,i}^{\mathrm{T}}q_ih_i\boldsymbol{C}_{q,i} & 2\boldsymbol{C}_{q,i}^{\mathrm{T}}q_ih_i\boldsymbol{D}_{p,i}\\ 2\boldsymbol{D}_{p,i}^{\mathrm{T}}q_ih_i\boldsymbol{C}_{q,i} & 2\boldsymbol{D}_{p,i}^{\mathrm{T}}q_ih_i\boldsymbol{D}_{p,i}\end{bmatrix},$$

$s_i=q_i+h_i$, $d_{p,i,j}$ is the entry of the matrix $\boldsymbol{D}_p$ at the ith row and jth column. By the S-procedure (Boyd *et al.*, 1994), if there exist $\tau_i\ge 0$ (i=1, 2, …, L), such that the following inequality

$$\boldsymbol{R}_0-\sum_{i=1}^{L}\tau_i(\boldsymbol{R}_i+\bar{\boldsymbol{R}}_i)=\begin{bmatrix}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A} & \boldsymbol{P}\boldsymbol{B}_p\\ * & \boldsymbol{0}\end{bmatrix}-\begin{bmatrix}2\boldsymbol{C}_q^{\mathrm{T}}\boldsymbol{T}\boldsymbol{Q}\boldsymbol{H}\boldsymbol{C}_q & \begin{pmatrix}2\boldsymbol{C}_q^{\mathrm{T}}\boldsymbol{T}\boldsymbol{Q}\boldsymbol{H}\boldsymbol{D}_p\\ -\boldsymbol{C}_q^{\mathrm{T}}(\boldsymbol{Q}+\boldsymbol{H})\boldsymbol{T}\end{pmatrix}\\ * & \begin{pmatrix}2\boldsymbol{D}_p^{\mathrm{T}}\boldsymbol{T}\boldsymbol{Q}\boldsymbol{H}\boldsymbol{D}_p+2\boldsymbol{T}-\boldsymbol{D}_p^{\mathrm{T}}\\ \cdot(\boldsymbol{Q}+\boldsymbol{H})\boldsymbol{T}-\boldsymbol{T}(\boldsymbol{Q}+\boldsymbol{H})\boldsymbol{D}_p\end{pmatrix}\end{bmatrix}=\boldsymbol{G}<\boldsymbol{0}$$

holds, where $\boldsymbol{T}=\mathrm{diag}\{\tau_1,\tau_2,\cdots,\tau_L\}$, and $\boldsymbol{T}\ge\boldsymbol{0}$, then $\boldsymbol{R}_0<\boldsymbol{0}$, that is, $\forall\boldsymbol{x}\ne\boldsymbol{0}$, $\mathrm{d}V(\boldsymbol{x})/\mathrm{d}t<0$; $\mathrm{d}V(\boldsymbol{x})/\mathrm{d}t=0$ iff $\boldsymbol{x}=\boldsymbol{0}$. Therefore, if the LMI Eq.(4) holds, the origin of the autonomic SNNM Eq.(3) is globally asymptotically stable. This completes the proof.