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Optimum weighting-coefficient-pair in inter-carrier interference self-cancellation scheme of OFDM system*

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Abstract: As one of the most important components of the wideband wireless access technique, orthogonal frequency division multiplexing (OFDM) has a high usage rate of spectrum and combats inter-symbol interference (ISI) in multi-path fading channel. However, when there are frequency offsets during the signal transmission, the inter-carrier interference (ICI) is introduced, which significantly degrades the performance. The existing ICI self-cancellation schemes such as PCC-OFDM are not optimum to minimize the interference considering both noise and ICI. In this paper, a new metric named SINR (signal-to-interference-and-noise ratio) is proposed. We discuss the optimization issue when a constant frequency offset exists and in time-varying channels. The optimum weighting-coefficient-pair (OWCP) is obtained, which maximizes SINR theoretically through the alternant iteration algorithm. Simulations show that the performance of OWCP-OFDM is better than that of PCC-OFDM, especially when the frequency offset is large. Although the ICI self-cancellation scheme suffers bandwidth inefficiency, from the simulation results we can also see that the performance of OWCP-OFDM is much better than that of the standard OFDM systems with the same bandwidth efficiency when a frequency offset exists. Moreover, since the redundant modulation provides the capability to suppress ICI as well as a receiving SNR gain, it can be considered as exchanging the bandwidth for SNR.

Key words: Orthogonal frequency division multiplexing (OFDM), Frequency offset, Inter-carrier interference (ICI), Polynomial cancellation coding (PCC), Optimum weighting-coefficient-pair (OWCP)

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INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising technique for high data-rate transmission (Cimini, 1985). In a classical OFDM system, the entire bandwidth is divided into many orthogonal subcarriers and information symbols are transmitted in parallel over these subcarriers by using computationally efficient fast Fourier transform (FFT), then a cycle prefix is inserted to combat inter-symbol interference (ISI). Hence the OFDM technology has a strong ability to cope with frequency-selective channels. Unfortunately, one of the main drawbacks of OFDM is its sensitivity to the frequency offset, which comes from local oscillators between the transmitter and the receiver, and the Doppler frequency shift in wireless channels. The frequency offset induces a loss of the orthogonality among subcarries so that inter-carrier interference (ICI) is caused, which would significantly degrade the system performance (Russell and Stuber, 1995; Robertson and Kaiser, 1999a; 1999b; Li and Cimini, 2001).

Several approaches to reducing ICI have been developed, such as time-domain windowing (Li and Stette, 1995; Muschallik, 1996), frequency-domain equalization (Ahn and Lee, 1993; Al-Dhahir, 1996), and the ICI self-cancellation scheme (Armstrong, 1999; Zhao and Haggman, 2001). Among these approaches the ICI self-cancellation scheme can achieve outstanding performance with an easy implementation. This paper focuses on the further research of the ICI self-cancellation scheme.

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The main idea of the ICI self-cancellation scheme is 'to modulate one data symbol onto a group of subcarriers with predefined weighting coefficients' (Zhao and Haggman, 2001). The ICI can be significantly suppressed by this operation because the adjacent subcarriers suffer the similar ICI. When the weighting coefficients satisfy the polynomial $(1-D)^L$, the scheme is called polynomial cancellation coding (PCC), where D denotes one subcarrier delay in the discrete frequency domain and L is the group length (Zhao and Haggman, 2001). However, no paper shows that the polynomial is the optimal weight coefficient to minimize the interference, and as a matter of fact it is not the optimum one according to the results shown in this paper.

Therefore, this paper focuses on designing the optimum weighting coefficients. Firstly, we introduce a new metric called signal-to-interference-and-noise ratio (SINR) to evaluate the degree that both the noise and the ICI affect the desired signal. Then we find the optimum weighting-coefficient-pair (OWCP) which can maximize SINR in theory. The reason to propose the notation 'weighting-coefficient-pair' is that the weighting coefficients in the transmitter and receiver might be different in the process of maximizing SINR.

The remainder of this paper is organized as follows. In Section 2, the system model of the ICI self-cancellation scheme is described and the SINR expression is derived. With the expression, we obtain the OWCP based on maximizing SINR in Section 3. Section 4 shows some numerical and simulation results. Finally, this paper is concluded in Section 5.

Notation: Superscripts 'T', 'H' and '*' stand for transpose, Hermitian, and conjugate, respectively. $E[\,]$ denotes expectation with the random variables. '∗' denotes the linear convolution operation. $(\tilde{S}_{s,s'})_{t,t'}$ denotes the (t, t')th entry of the matrix $\tilde{\boldsymbol{S}}_{s,s'}$.

SYSTEM MODEL AND SINR ANALYSIS

Consider an OFDM system using the ICI self-cancellation scheme as shown in Fig.1. Notice that we use different weighting coefficients in the transmitter and the receiver.

In the transmitter, the bit data are firstly modulated to P QPSK symbols $\{X_k \mid k=0, 1, \ldots, P-1\}$ for each OFDM symbol. In order to suppress the ICI, we insert a module of ICI cancellation modulation before FFT. The procedure of the ICI cancellation modulation is shown in Fig.2a in detail. Each QPSK symbol is spread by the weighting coefficients $\{c_t \mid t=0, 1, \ldots, Q-1\}$. Then we have N symbols $S_{kq+t}=X_k c_t$ as the frequency subcarrier data, where $N=P\times Q$. Convert them into the time domain by the IFFT of N points, and then the signal can be written as

$$x_n = \frac{1}{N}\sum_{k=0}^{P-1}\sum_{t=0}^{Q-1} S_{kQ+t}\mathrm{e}^{\mathrm{j}2\pi n(kQ+t)/N} = \frac{1}{N}\sum_{k=0}^{P-1} X_k \sum_{t=0}^{Q-1} c_t \mathrm{e}^{\mathrm{j}2\pi n(kQ+t)/N}, \tag{1}$$

where superscript n denotes the time index, k denotes the index of the original QPSK symbols, and t is the index of weighting coefficients in the transmitter.

After S/P conversion and insertion of the cyclic prefix, the signal is sent out. Here, we omit the ADC, DAC and RF modules in the transceiver for simplification.

In the receiver, the received sample signal is y_n. Suppose that the maximal delay of the multi-path channel is no longer than the cyclic prefix, then the inter-symbol interference can be ignored. After removing the cyclic prefix and FFT, we have

$$R_{mQ+s} = \sum_{n=0}^{N-1} y_n \mathrm{e}^{-\mathrm{j}2\pi n(mQ+s)/N}, \tag{2}$$

where $s=0, 1, \ldots, Q-1$; $m=0, 1, \ldots, P-1$.

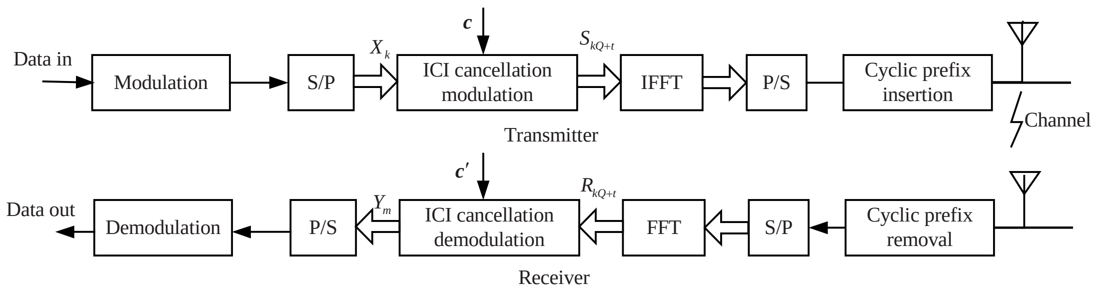
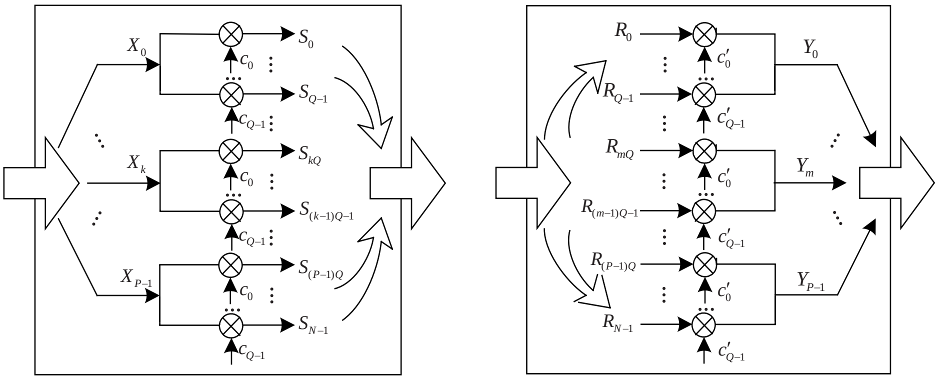


Fig.1 System model of the inter-carrier interference (ICI) self-cancellation scheme



(a) (b)

Fig.2 Module of ICI cancellation modulation (a) and demodulation (b)

Then the ICI cancellation demodulation is implemented with the receiving weighting coefficients $\{c'_m \mid m=0, 1, \ldots, Q-1\}$, as shown in Fig.2b. We obtain the sending symbols

$$Y_m=\sum_{s=0}^{Q-1}c'_sR_{mQ+s}=\sum_{s=0}^{Q-1}c'_s\sum_{n=0}^{N-1}y_n\mathrm{e}^{-\mathrm{j}2\pi n(mQ+s)/N}. \qquad (3)$$

Next, we discuss two-channel situations to see that the sending and receiving weighting coefficients can suppress the ICI if they are chosen well. One situation is the carrier-frequency offset only and the other is the time-varying multi-path channel.

When there is the carrier-frequency offset only, the received signal is given by

$$y_n=x_n\mathrm{e}^{\mathrm{j}2\pi\Delta fT_sn/N}+w_n, \qquad (4)$$

where w_n denotes the additive white Gaussian noise (AWGN), Δf is the carrier-frequency offset of the local oscillators between the transmitter and the receiver, ΔfT_s is called the normalized carrier-frequency offset, and T_s denotes the symbol duration.

Then, Eq.(3) can be rewritten as

$$Y_m=\frac{1}{N}\sum_{s=0}^{Q-1}\sum_{k=0}^{P-1}\sum_{t=0}^{Q-1}\sum_{n=0}^{N-1}\Big(X_kc_tc'_s\mathrm{e}^{\mathrm{j}2\pi\Delta fTn/N}\mathrm{e}^{\mathrm{j}2\pi n(kQ+t)/N}\cdot\mathrm{e}^{-\mathrm{j}2\pi n(mQ+s)/N}\Big)+\sum_{s=0}^{Q-1}c'_s\sum_{n=0}^{N-1}w_n\mathrm{e}^{-\mathrm{j}2\pi n(mQ+s)/N}. \qquad (5)$$

For a time-varying channel, the received signal is given by

$$y_n=x_n*h_n+w_n. \qquad (6)$$

Assume a time-variant impulse response of L multi-paths,

$$h(n,\tau)=\sum_{l=0}^{L-1}h_{n,l}\delta(\tau-\tau_l), \qquad (7)$$

where τ_l is the delay of the lth path, $h_{n,l}$ is a zero-mean complex Gaussian random variable which denotes the time-variant complex path gain of the lth path in the nth instant. $h_{n,l}$ is an f_d-limited complex Gaussian process that is independent of different paths, where f_d is the Doppler frequency, and $\Delta\varepsilon=f_dT_s$ is called the normalized Doppler frequency offset.

Then, Eq.(3) can be rewritten as

$$Y_m=\frac{1}{N}\sum_{s=0}^{Q-1}\sum_{n=0}^{N-1}\sum_{k=0}^{P-1}\sum_{t=0}^{Q-1}\sum_{l=0}^{L-1}\Big(c'_sc_tX_kh_{n,l}\mathrm{e}^{-\mathrm{j}2\pi kl/N}\mathrm{e}^{-\mathrm{j}2\pi n(mQ+s)/N}\cdot\mathrm{e}^{\mathrm{j}2\pi n(kQ+t)/N}\Big)+\sum_{s=0}^{Q-1}c'_s\sum_{n=0}^{N-1}w_n\mathrm{e}^{-\mathrm{j}2\pi n(mQ+s)/N}. \qquad (8)$$

Eqs.(5) and (8) can be rewritten equivalently as

$$Y_m=Y_m^{\mathrm{S}}+Y_m^{\mathrm{I}}+W_m, \qquad (9)$$

where

$$W_m=\sum_{s=0}^{Q-1}c'_sw_{mQ+s},\quad Y_m^{\mathrm{S}}=\sum_{s=0}^{Q-1}c'_s\sum_{t=0}^{Q-1}X_mc_ta_{t-s},$$

$$Y_m^{\mathrm{I}}=\sum_{\substack{p=0\\p\neq m}}^{P-1}\sum_{s=0}^{Q-1}c'_s\sum_{t=0}^{Q-1}X_mc_ta_{(p-m)Q+t-s}.$$

When only the carrier-frequency offset exists,

$$a_r=\frac{1}{N}\sum_{n=0}^{N-1}\mathrm{e}^{\mathrm{j}2\pi n\Delta\varepsilon/N}\mathrm{e}^{\mathrm{j}2\pi nr/N}. \qquad (10)$$

And for the time-varying Rayleigh channel,

$$a_r = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{L-1} h_{n,l} e^{-j2\pi(kQ+t)l/N} e^{-j2\pi nr/N}. \quad (11)$$

The first item in the right-hand side of Eq.(9) denotes the desired signal and the last two are considered as interference items. $Y_m^{\rm I}$ is the ICI due to the Doppler frequency or carrier-frequency offset. W_m is the white noise. Here, we define *SINR* to evaluate the effect of interference $Y_m^{\rm I}$ and W_m on a useful signal,

$$SINR = \frac{E[|Y_m^{\rm S}|^2]}{E[|Y_m^{\rm I}|^2] + E[|W_m|^2]} = \frac{P_{\rm S}}{P_{\rm ICI} + P_{\rm N}}. \quad (12)$$

Assuming the sending signals $\{X_k \mid k=0, 1, \ldots, P-1\}$ fulfill the independence condition and have zero means, we have

$$E[X_p X_q^*] = \begin{cases} \sigma_x^2, & p = q, \\ 0, & p \neq q. \end{cases} \quad (13)$$

Thus, the power of signal, ICI and noise can be expressed as

$$P_{\rm S} = \sigma_x^2 \sum_{s=0}^{Q-1}\sum_{s'=0}^{Q-1} c'_s (c'_{s'})^* \sum_{t=0}^{Q-1}\sum_{t'=0}^{Q-1} c_t c_{t'}^* E[a_{t-s} a_{t'-s'}^*], \quad (14)$$

$$P_{\rm ICI} = \sigma_x^2 \sum_{s=0}^{Q-1}\sum_{s'=0}^{Q-1} c'_s (c'_{s'})^* \sum_{t=0}^{Q-1}\sum_{t'=0}^{Q-1}\Bigg(c_t c_{t'}^* \cdot \sum_{p'=0, p'\neq m}^{P-1} E[a_{(p-m)Q+t-s} a_{(p'-m)Q+t'-s'}^*]\Bigg), \quad (15)$$

$$P_{\rm N} = \sigma_n^2 \sum_{s=0}^{Q-1}\sum_{s'=0}^{Q-1} c'_s (c'_{s'})^*. \quad (16)$$

Next, we give the expression of the expectation items in Eqs.(14) and (15).

When there is the frequency offset only,

$$\begin{aligned} &E[a_{s-t} a_{s'-t'}^*] \\ &= \frac{1}{N^2}\sum_{n=0}^{N-1} e^{j2\pi n\Delta\varepsilon/N} e^{-j2\pi n(s-t)/N} \sum_{n'=0}^{N-1} e^{-j2\pi n'\Delta\varepsilon/N} e^{j2\pi n'(s'-t')/N} \\ &= \frac{\sin(\pi(s-t+\Delta\varepsilon))\sin(\pi(s'-t'+\Delta\varepsilon))}{\sin(\pi(s-t+\Delta\varepsilon)/N)\sin(\pi(s'-t'+\Delta\varepsilon)/N)} \\ &\cdot \frac{1}{N^2} e^{j\pi(1-1/N)[(s-t)-(s'-t')]}. \end{aligned} \quad (17)$$

For the WSSUS (wide-sense stationary uncorrelated scattering) channel, using the Jake's model, the cross-correlation function between $h_{n',l}$ and $h_{n',l'}$ can be found as

$$E[h_{n,l} h_{n',l'}^*] = E_l J_0(2\pi f_{\rm d}(n-n')/N)\delta(l-l'). \quad (18)$$

Hence, the expectation item can be expressed as

$$\begin{aligned} E[a_{s-t} a_{s'-t'}^*] &= \frac{1}{N^2} E\Bigg[\sum_{n=0}^{N-1}\sum_{l=0}^{L-1} h_{n,l} e^{-j2\pi tl/N} e^{-j2\pi n(s-t)/N} \\ &\quad \cdot \sum_{n'=0}^{N-1}\sum_{l'=0}^{L-1} h_{n',l'}^* e^{j2\pi t'l'/N} e^{j2\pi n'(s'-t')/N}\Bigg] \\ &= \frac{1}{N^2}\sum_{n=0}^{N-1}\sum_{l=0}^{L-1}\sum_{n'=0}^{N-1}[E_l J_0(2\pi f_{\rm d}(n-n')/N) \\ &\quad \cdot e^{-j2\pi l(t-t')/N} e^{-j2\pi[n(s-t)-n'(s'-t')]/N}]. \end{aligned} \quad (19)$$

OPTIMUM WEIGHTING-COEFFICIENT-PAIR

In this section, the vector expression of the SINR is derived, and we design the optimum sending weighting coefficients $\boldsymbol{c}=[c_0, c_1, \ldots, c_{Q-1}]^{\rm T}$ and receiving weighting coefficients $\boldsymbol{c}' = [c'_0, c'_1, \ldots, c'_{Q-1}]^{\rm T}$ by maximizing the SINR.

The power of signal can be written as

$$P_{\rm S} = \sigma_x^2 (\boldsymbol{c}')^{\rm H} \hat{\boldsymbol{S}} \boldsymbol{c}' = \sigma_x^2 \boldsymbol{c}^{\rm H} \hat{\boldsymbol{S}}' \boldsymbol{c}, \quad (20)$$

where

$$\hat{\boldsymbol{S}} = \begin{bmatrix} \boldsymbol{c}^{\rm H}\tilde{\boldsymbol{S}}_{0,0}\boldsymbol{c} & \cdots & \boldsymbol{c}^{\rm H}\tilde{\boldsymbol{S}}_{0,Q-1}\boldsymbol{c} \\ \vdots & & \vdots \\ \boldsymbol{c}^{\rm H}\tilde{\boldsymbol{S}}_{Q-1,0}\boldsymbol{c} & \cdots & \boldsymbol{c}^{\rm H}\tilde{\boldsymbol{S}}_{Q-1,Q-1}\boldsymbol{c} \end{bmatrix},$$

$$\hat{\boldsymbol{S}}' = \begin{bmatrix} (\boldsymbol{c}')^{\rm H}\tilde{\boldsymbol{S}}'_{0,0}\boldsymbol{c}' & \cdots & \boldsymbol{c}^{\rm H}\tilde{\boldsymbol{S}}'_{0,Q-1}\boldsymbol{c}' \\ \vdots & & \vdots \\ (\boldsymbol{c}')^{\rm H}\tilde{\boldsymbol{S}}'_{Q-1,0}\boldsymbol{c}' & \cdots & \boldsymbol{c}^{\rm H}\tilde{\boldsymbol{S}}'_{Q-1,Q-1}\boldsymbol{c}' \end{bmatrix}.$$

The element of $\tilde{\boldsymbol{S}}_{s,s'}$ is $(\tilde{S}_{s,s'})_{t,t'} = E[a_{s-t} a_{s'-t'}^*]$. The element of $\tilde{\boldsymbol{S}}'_{s,s'}$ is $(\tilde{S}_{t,t'})_{s,s'} = E[a_{s-t} a_{s'-t'}^*]$.

The power of ICI can be written as

$$P_{\rm ICI} = \sigma_x^2 (\boldsymbol{c}')^{\rm H} \hat{\boldsymbol{I}} \boldsymbol{c}' = \sigma_x^2 \boldsymbol{c}^{\rm H} \hat{\boldsymbol{I}}' \boldsymbol{c}, \quad (21)$$

where

$$\hat{\boldsymbol{I}} = \begin{bmatrix} \boldsymbol{c}^{\rm H}\tilde{\boldsymbol{I}}_{0,0}\boldsymbol{c} & \cdots & \boldsymbol{c}^{\rm H}\tilde{\boldsymbol{I}}_{0,Q-1}\boldsymbol{c} \\ \vdots & & \vdots \\ \boldsymbol{c}^{\rm H}\tilde{\boldsymbol{I}}_{Q-1,0}\boldsymbol{c} & \cdots & \boldsymbol{c}^{\rm H}\tilde{\boldsymbol{I}}_{Q-1,Q-1}\boldsymbol{c} \end{bmatrix},$$

$$\hat{\boldsymbol{I}}' = \begin{bmatrix} (\boldsymbol{c}')^{\rm H}\tilde{\boldsymbol{I}}'_{0,0}\boldsymbol{c}' & \cdots & (\boldsymbol{c}')^{\rm H}\tilde{\boldsymbol{I}}'_{0,Q-1}\boldsymbol{c}' \\ \vdots & & \vdots \\ (\boldsymbol{c}')^{\rm H}\tilde{\boldsymbol{I}}'_{Q-1,0}\boldsymbol{c}' & \cdots & (\boldsymbol{c}')^{\rm H}\tilde{\boldsymbol{I}}'_{Q-1,Q-1}\boldsymbol{c}' \end{bmatrix}.$$

The element of matrix $\tilde{\boldsymbol{I}}_{s,s'}$ is $(\tilde{I}_{s,s'})_{t,t'} = E\left(\sum_{p=0,\, p\neq m}^{P-1} a_{(p-m)Q+s-t} a^*_{(p-m)Q+s'+t'}\right)$, and the element of $\tilde{\boldsymbol{I}}'_{s,s'}$ is $(\tilde{I}'_{s,s'})_{t,t'} = E\left(\sum_{p=0,\, p\neq m}^{P-1} a_{(p-m)Q+s-t} a^*_{(p-m)Q+s'+t'}\right)$.

The power of noise is

$$P_{\rm N} = \sigma_n^2 (\boldsymbol{c}')^{\rm H} \boldsymbol{I}_Q \boldsymbol{c}', \tag{22}$$

where $\boldsymbol{I}_Q$ is the $Q\times Q$ identity matrix, and σ_n^2 is the channel noise variance.

Therefore, *SINR* can be rewritten in vector form as

$$SINR = \frac{P_{\rm S}}{P_{\rm ICI} + P_{\rm N}} = \frac{\sigma_x^2 (\boldsymbol{c}')^{\rm H} \hat{\boldsymbol{S}}' \boldsymbol{c}'}{\sigma_x^2 (\boldsymbol{c}')^{\rm H} \hat{\boldsymbol{I}} \boldsymbol{c}' + \sigma_n^2 (\boldsymbol{c}')^{\rm H} \boldsymbol{I}_Q \boldsymbol{c}'} = \frac{(\boldsymbol{c}')^{\rm H} \hat{\boldsymbol{S}}' \boldsymbol{c}'}{(\boldsymbol{c}')^{\rm H} \hat{\boldsymbol{I}} \boldsymbol{c}' + (\boldsymbol{c}')^{\rm H} \boldsymbol{I}_Q \boldsymbol{c}' / SNR}, \tag{23}$$

where $SNR = \sigma_x^2 / \sigma_n^2$ is the signal-to-noise ratio. The OWCP $\boldsymbol{c}$ and $\boldsymbol{c}'$ can be optimized by maximizing *SINR* in Eq.(23),

$$\max_{c_i, c_i'} SINR, \quad \text{s.t.} \quad \boldsymbol{c}^{\rm H}\boldsymbol{c} = Q \text{ and } (\boldsymbol{c}')^{\rm H}\boldsymbol{c}' = Q. \tag{24}$$

To avoid the dividing operation, Eq.(24) can be transformed to the function below with factor γ involved:

$$E(\gamma) = \min_{\boldsymbol{c}(\gamma), \boldsymbol{c}'(\gamma)} \left(\gamma (P_{\rm ICI} + P_{\rm N})/Q - (1-\gamma)P_{\rm S} + C\right),\ \gamma \in [0,1],$$
$$\text{s.t.} \quad \boldsymbol{c}^{\rm H}\boldsymbol{c} = 1,\ (\boldsymbol{c}')^{\rm H}\boldsymbol{c}' = 1, \tag{25}$$
$$\hat{\boldsymbol{c}}, \hat{\boldsymbol{c}}' = \arg \max_{\{\boldsymbol{c}, \boldsymbol{c}'\} \subseteq \{\boldsymbol{c}(\gamma), \boldsymbol{c}'(\gamma)\}} SINR,$$

where C is a constant and we will see its function below. First, we find $E(\gamma)$ and the corresponding $\{\boldsymbol{c}, \boldsymbol{c}'\}$ that make the sum of the weighted numerator and denominator minimal when γ varies among [0, 1]. Then choose $\{\boldsymbol{c}, \boldsymbol{c}'\}$ from all the $\{\boldsymbol{c}(\gamma), \boldsymbol{c}'(\gamma)\}$ pairs that can maximize *SINR*.

The remaining issue is how to calculate $E(\gamma)$ and $\{\boldsymbol{c}(\gamma), \boldsymbol{c}'(\gamma)\}$ under each γ. Here, the alternant iteration algorithm is introduced for solving it.

Rewrite $E(\gamma)$ with two equivalent expressions:

$$E(\gamma) = \min_{\boldsymbol{c}(\gamma), \boldsymbol{c}'(\gamma)} \left\{ (\boldsymbol{c}')^{\rm H} (\boldsymbol{R}(\gamma) + C\boldsymbol{I}_Q) \boldsymbol{c}' \right\} = \min_{\boldsymbol{c}(\gamma), \boldsymbol{c}'(\gamma)} \left\{ \boldsymbol{c}^{\rm H} (\boldsymbol{R}'(\gamma) + C\boldsymbol{I}_Q) \boldsymbol{c} \right\}, \tag{26a}$$

$$\begin{cases} \boldsymbol{R}(\gamma) = \sigma_x^2 \left\{ \gamma \hat{\boldsymbol{I}} + \dfrac{1}{SNR \cdot Q} \boldsymbol{I}_Q - (1-\gamma)\hat{\boldsymbol{S}} \right\}, \\ \boldsymbol{R}'(\gamma) = \sigma_x^2 \left\{ \gamma \hat{\boldsymbol{I}}' + \dfrac{1}{SNR \cdot Q} \boldsymbol{I}_Q - (1-\gamma)\hat{\boldsymbol{S}}' \right\}. \end{cases} \tag{26b}$$

C is a large enough constant to make sure that $\boldsymbol{R}(\gamma)+C\boldsymbol{I}_Q$ and $\boldsymbol{R}'(\gamma)+C\boldsymbol{I}_Q$ are positive definite Hermitian matrices. The main idea of the alternant iteration algorithm is to fix one of $\boldsymbol{c}$ and $\boldsymbol{c}'$, to look for the value of the other one that minimizes the quadratic form in Eq.(24), and then to switch the positions of $\boldsymbol{c}$ and $\boldsymbol{c}'$, and do the same work until the convergence is reached (see the convergence proof in Appendix A).

The detailed calculation procedure is described as follows:

(1) Select an arbitrary vector $\boldsymbol{c}_i(\gamma)$, and then initialize $\boldsymbol{R}_i(\gamma)$ according to Eq.(26a). Set i=0, and select a small enough value δ>0.

(2) Calculate the smallest eigenvalue of $\boldsymbol{R}(\gamma)+C\boldsymbol{I}_Q$ as $E_{i+1}(\gamma)$, and let $\boldsymbol{c}'_{i+1}(\gamma)$ be the corresponding eigenvector. $\boldsymbol{c}'_{i+1}(\gamma)$ makes $(\boldsymbol{c}')^{\rm H}(\boldsymbol{R}(\gamma)+C\boldsymbol{I}_Q)\boldsymbol{c}'$ minimum (see Appendix B).

(3) Substituting $\boldsymbol{c}'_{i+1}(\gamma)$ into Eq.(26b), obtain $R'_{i+1}(\gamma)$.

(4) Calculate the smallest eigenvalue of $\boldsymbol{R}'(\gamma)+C\boldsymbol{I}_Q$ as $E_{i+2}(\gamma)$, and let $\boldsymbol{c}_{i+2}(\gamma)$ be the corresponding eigenvector.

(5) Substituting $\boldsymbol{c}_{i+2}(\gamma)$ into Eq.(26a), obtain $\boldsymbol{R}_{i+2}(\gamma)$.

(6) If $|E_{i+2}(\gamma)-E_{i+1}(\gamma)|<\delta$, go to (8).

(7) Let $i+1\rightarrow i$, go to (2).

(8) Save $\boldsymbol{c}_{i+2}(\gamma)$ as $\boldsymbol{c}(\gamma)$ and $\boldsymbol{c}'_{i+1}(\gamma)$ as $\boldsymbol{c}'(\gamma)$. Stop.

The above conclusion can be achieved when the channel state information (CSI), such as SNR, channel paths L, the average gain $E(l)$ of the lth path, and the Doppler frequency $f_{\rm d}$, is exactly known.

In practical wireless communications, it is usually difficult to obtain CSI. General results could be obtained by supposing a probability density function of the unknown CSI and taking the integral operation of Eq.(17) or Eq.(19).

NUMERICAL AND SIMULATION RESULTS

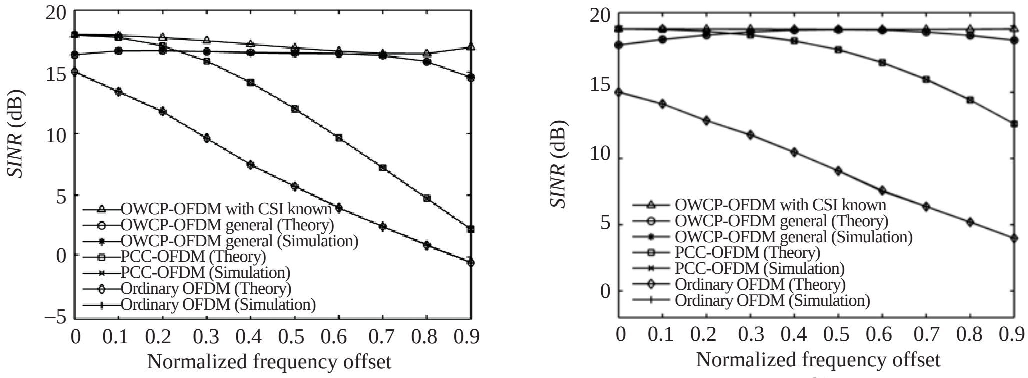
In this section, we present some numerical and simulation results to show the performance improvement of OWCP-OFDM compared to PCC-OFDM and ordinary OFDM.

Numerical results

To examine the performance improvement, we compare the SINR of OWCP-OFDM, PCC-OFDM and ordinary OFDM. To be fair, the ordinary OFDM in the simulation has the same bandwidth efficiency as others according to the sequence length.

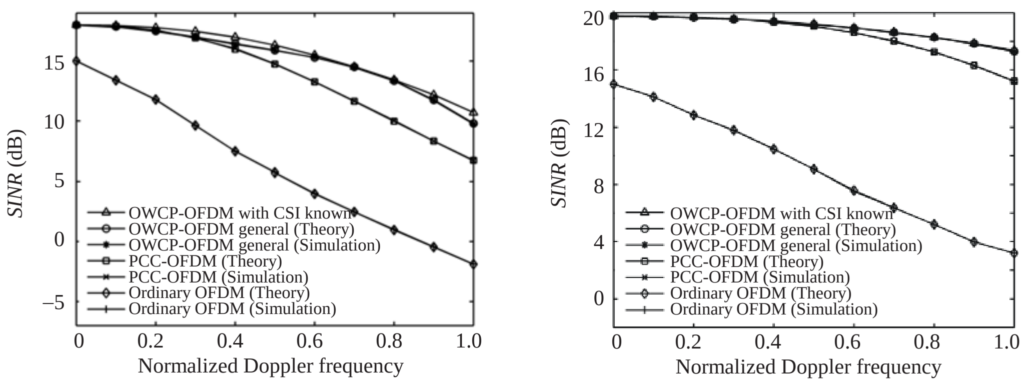
Figs.3a and 3b present the comparison of SINR when the length of the group Q is 2 and 3, respectively, with a constant frequency offset. The SNR is set to 15 dB. From the figure, the SINR of OWCP-OFDM far surpasses the one of ordinary OFDM by 5~15 dB when the CSI is perfectly known. And when there is no frequency offset, a receiving SNR gain can be clearly seen. However, in practical systems the frequency offset should be unknown. Thus we assume it is uniformly distributed among (0, 1). Then the OWCP-OFDM is obtained in a general situation. Fig.3 shows that the SINR of the general one has less than 1 dB loss compared to the one with CSI known. It can be clearly seen that the SINR of the PCC scheme falls when the normalized frequency offset gets larger, especially when Q is small. As the length of the group increases, the ability of ICI cancellation gets stronger.

Fig.4 shows the comparison in a time-varying multi-path Rayleigh channel, which is four-path and follows negative exponential distribution. The similar conclusions can be summarized through the contrast of the two figures. But the SINR improvement is less evident compared to Fig.3. General OWCP-OFDM is derived by assuming that the normalized Doppler frequency has a uniform distribution between 0 and 1.



(a) (b)

Fig.3 Comparison of *SINR* due to the carrier frequency offset
(a) Group length Q=2; (b) Group length Q=3



(a) (b)

Fig.4 Comparison of *SINR* due to the Doppler frequency
(a) Group length Q=2; (b) Group length Q=3

Simulation results

Here, our simulation results for the performance of OWCP-OFDM via Monte Carlo simulation are presented. The number of the subcarriers, N, is chosen to be 256, and the length of the cyclic prefix is 32. The QPSK constellation is adopted.

The relationship between symbol-error rate (SER) and SNR is shown in Fig.5 with a constant frequency offset. From the figure, the performance matches the numerical results shown in Fig.4 on the whole. In Fig.5, the error floor of ordinary OFDM is clearly seen. Both OWCP-OFDM and PCC-OFDM can eliminate the error floor of ordinary OFDM as shown in Fig.5a with $\Delta\varepsilon$=0.3. There is more than 1.77 dB SINR improvement when Q changes from 2 to 3, because an extra ICI cancellation gain is provided. However, in Fig.5b when $\Delta\varepsilon$=0.8 and Q=2 the error floor appears in the PCC scheme, whereas the OWCP-OFDM still has a satisfactory performance.

Fig.6 shows *SER* vs *SNR* under the time-varying multi-path Rayleigh channel. The channel condition is the same as that in simulation. From Fig.6, we can draw a similar conclusion that OWCP-OFDM has a better ability to eliminate error-floor than PCC-OFDM, especially when the normalized frequency offset gets larger.

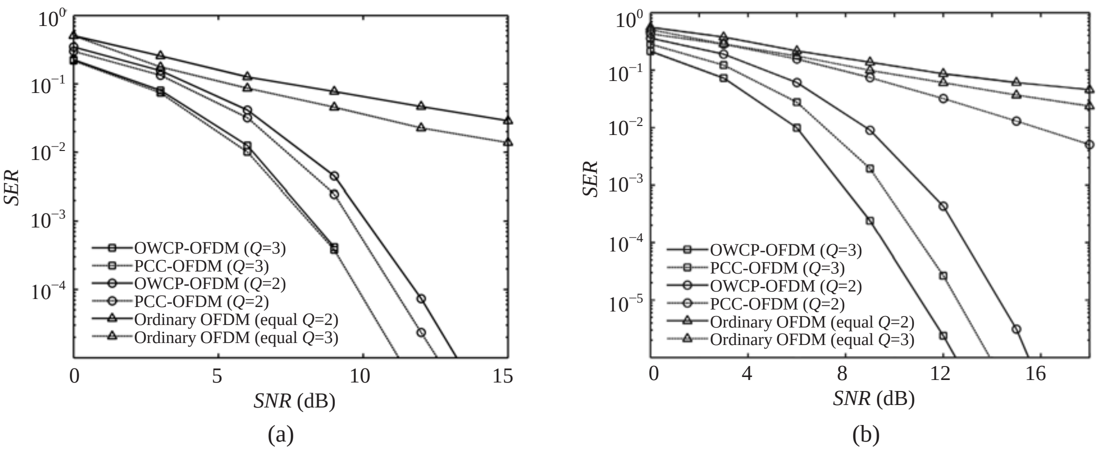


Fig.5 *SER* vs *SNR* with a constant frequency offset
(a) Normalized frequency offset ΔfT_s=0.3; (b) Normalized frequency offset ΔfT_s=0.8

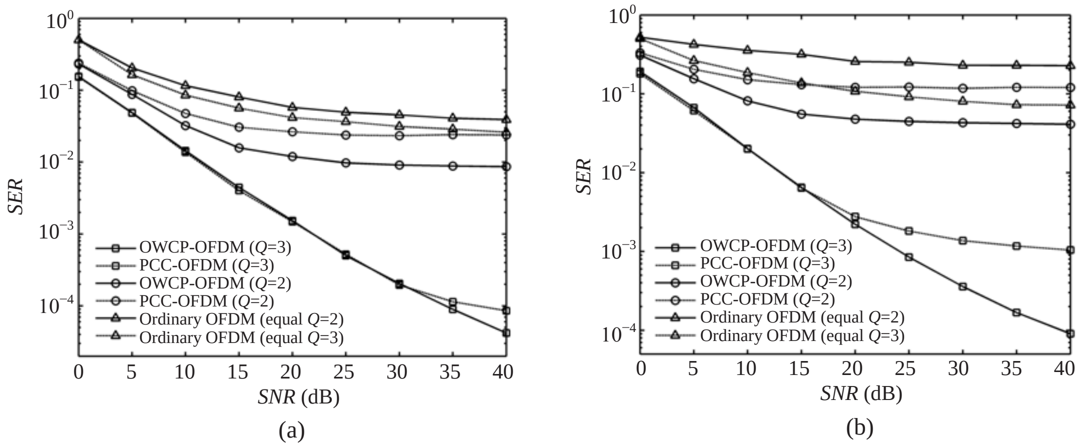


Fig.6 *SER* vs *SNR* in the time-varying multi-path Rayleigh channel
(a) Normalized Doppler frequency $\Delta\varepsilon$=0.3; (b) Normalized Doppler frequency $\Delta\varepsilon$=0.8

CONCLUSION

In this paper, we find the optimum weighting-coefficient-pair to maximize SINR in theory when frequency offsets exist and in time-varying channels. From the simulation results, we can make sure that this OWCP-OFDM can supply a steady gain of SINR compared to the conventional method. A problem of the self-cancellation scheme is the reduction of the available bandwidth, but through the simulation we can conclude that OWCP-OFDM has a much better performance than the traditional OFDM system with the same bandwidth efficiency. The complexity of the proposed algorithm is derived from the alternative iteration and the eigenvalue calculation, which is a burden on a real-time system. However, because the weighting-coefficient pair in the general situation also has a satisfactory performance, the OWCP can be designed and calculated by a computer in advance. Then the transceiver will suffer no extra burden of complexity. Therefore, the OWCP-OFDM system can meet the demand of a high-speed broadband wireless system perfectly.

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APPENDIX A: PROOF OF THE CONVERGENCE OF ALTERNANT ITERATION ALGORITHM

We define a target function

$$T(\boldsymbol{c},\boldsymbol{c}')=(\boldsymbol{c}')^{\mathrm{H}}(\boldsymbol{R}(\gamma)+C\boldsymbol{I}_Q)\boldsymbol{c}'=\boldsymbol{c}^{\mathrm{H}}(\boldsymbol{R}'(\gamma)+C\boldsymbol{I}_Q)\boldsymbol{c}, \quad \text{(A1)}$$

where $\boldsymbol{R}(\gamma)$ and $\boldsymbol{R}'(\gamma)$ are defined in Eq.(26b).

The Hessen matrix $\boldsymbol{H}$ of $T(\boldsymbol{c}, \boldsymbol{c}')$ is achieved by

$$\boldsymbol{H}=\begin{bmatrix} \frac{\partial}{\partial \boldsymbol{c}}\left(\frac{\partial T(\boldsymbol{c},\boldsymbol{c}')}{\partial \boldsymbol{c}}\right)^{\mathrm{H}} & \frac{\partial}{\partial \boldsymbol{c}'}\left(\frac{\partial T(\boldsymbol{c},\boldsymbol{c}')}{\partial \boldsymbol{c}}\right)^{\mathrm{H}} \\ \frac{\partial}{\partial \boldsymbol{c}}\left(\frac{\partial T(\boldsymbol{c},\boldsymbol{c}')}{\partial \boldsymbol{c}'}\right)^{\mathrm{H}} & \frac{\partial}{\partial \boldsymbol{c}'}\left(\frac{\partial T(\boldsymbol{c},\boldsymbol{c}')}{\partial \boldsymbol{c}'}\right)^{\mathrm{H}} \end{bmatrix} =\begin{bmatrix} \boldsymbol{H}_{11} & \boldsymbol{H}_{12} \\ \boldsymbol{H}_{21} & \boldsymbol{H}_{22} \end{bmatrix}+C\boldsymbol{I}_{2Q}, \quad \text{(A2)}$$

where $\boldsymbol{H}_{11}=4\boldsymbol{R}'$, $\boldsymbol{H}_{22}=4\boldsymbol{R}$. Because $\tilde{\boldsymbol{I}}'_{s,s'}=\tilde{\boldsymbol{I}}^{\mathrm{H}}_{s',s}$ and $\tilde{\boldsymbol{S}}'_{s,s'}=(\tilde{\boldsymbol{S}}'_{s',s})^{\mathrm{H}}$, we have $\boldsymbol{H}_{11}=\boldsymbol{H}_{11}^{\mathrm{H}}$, $\boldsymbol{H}_{22}=\boldsymbol{H}_{22}^{\mathrm{H}}$, and $\boldsymbol{H}_{21}=\boldsymbol{H}_{12}^{\mathrm{H}}$. Therefore, $\boldsymbol{H}$ is a Hermitian matrix.

We denote the SVD of $\boldsymbol{H}$ as $\boldsymbol{H}=\boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{\mathrm{H}}$, where $\boldsymbol{U}$ is a matrix with orthonormal columns and $\boldsymbol{\Lambda}$ is a diagonal matrix containing the singular values $\lambda_0\geq\lambda_1\geq\ldots\geq\lambda_{2Q-1}$ on its diagonal. Since we are dealing with a Hermitian matrix, the λ_ks are also eigenvalues.

Therefore, if constant C is large enough, the λ_ks are all positive. $\boldsymbol{H}$ is a positive definite matrix.

For the ith and (i+1)th iterations in the alternant iteration process,

$$\begin{cases} E_i(\gamma)=(\boldsymbol{c}'_{i+1})^{\mathrm{H}}(\boldsymbol{R}_i(\gamma)+C\boldsymbol{I}_Q)\boldsymbol{c}'_{i+1}, \\ E_{i+1}(\gamma)=\boldsymbol{c}^{\mathrm{H}}_{i+2}(\boldsymbol{R}'_{i+1}(\gamma)+C\boldsymbol{I}_Q)\boldsymbol{c}_{i+2}, \end{cases} \quad \text{(A3)}$$

where $\boldsymbol{R}_i(\gamma)$ and $\boldsymbol{R}'_{i+1}(\gamma)$ are obtained by substituting $\boldsymbol{c}_i$ and $\boldsymbol{c}'_{i+1}$, respectively. According to Appendix B, it can be concluded that $E_{i+1}(\gamma)\leq E_i(\gamma)$, which respectively means the minimum value in each iteration is no larger than the minimum value in the last iteration. Since we have proved that the Hessen matrix of $T(\boldsymbol{c}, \boldsymbol{c}')$ is a positive definite matrix, the equal sign can be taken away, and then we obtain $E_{i+1}(\gamma)<E_i(\gamma)$. Therefore, the $\{E_i(\gamma)\}$ is strictly decreasing.

Now we will show that the range of $\{E_i(\gamma)\}$ is limited. From Eq.(26) we know that it can be written as

$$(\boldsymbol{c}')^{\mathrm{H}}(\boldsymbol{R}(\gamma)+C\boldsymbol{I})\boldsymbol{c}' = (\boldsymbol{c}')^{\mathrm{H}}(\boldsymbol{I}_Q\otimes\boldsymbol{c}^{\mathrm{H}})\boldsymbol{T}(\boldsymbol{I}_Q\otimes\boldsymbol{c})\boldsymbol{c}'+C. \quad \text{(A4)}$$

Here we define a $Q^2\times Q^2$ Hermitian matrix

$$\boldsymbol{T}=\begin{bmatrix}\tilde{\boldsymbol{R}}(\gamma)_{0,0} & \cdots & \tilde{\boldsymbol{R}}(\gamma)_{0,Q-1}\\ \vdots & & \vdots\\ \tilde{\boldsymbol{R}}(\gamma)_{Q-1,0} & \cdots & \tilde{\boldsymbol{R}}(\gamma)_{Q-1,Q-1}\end{bmatrix},$$

where $\tilde{\boldsymbol{R}}(\gamma)=\sigma_x^2\left\{\gamma\tilde{\boldsymbol{I}}+\dfrac{1}{SNR\cdot Q}\boldsymbol{I}_Q-(1-\gamma)\tilde{\boldsymbol{S}}\right\}$.

In Eq.(A4) '⊗' denotes the Kronecker product. When constant C is large enough, matrix $\boldsymbol{T}$ will be positive definite. Thus we denote the SVD of $\boldsymbol{T}$,

$$\boldsymbol{T}=\boldsymbol{U}^{\mathrm{H}}\boldsymbol{\Lambda}\boldsymbol{U}, \quad \text{(A5)}$$

where $\boldsymbol{U}$ is a matrix with orthonormal columns and $\boldsymbol{\Lambda}$ is a diagonal matrix containing the singular values $\lambda'_0\geq\lambda'_1\geq\ldots\geq\lambda'_{Q^2-1}>0$ on its diagonal. Then Eq.(A4) can be rewritten as

$$(\boldsymbol{c}')^{\mathrm{H}}\boldsymbol{R}(\gamma)\boldsymbol{c}'=(\boldsymbol{c}')^{\mathrm{H}}(\boldsymbol{I}_Q\otimes\boldsymbol{c}^{\mathrm{H}})\boldsymbol{U}^{\mathrm{H}}\boldsymbol{\Lambda}\boldsymbol{U}(\boldsymbol{I}_Q\otimes\boldsymbol{c})\boldsymbol{c}'. \quad \text{(A6)}$$

We notice that $\boldsymbol{U}(\boldsymbol{I}_Q\otimes\boldsymbol{c})\boldsymbol{c}'$ is a $Q^2\times1$ vector and for every $Q^2\times1$ vector $\boldsymbol{k}$,

$$\begin{cases}\boldsymbol{k}^{\mathrm{H}}\boldsymbol{\Lambda}\boldsymbol{k}\leq\boldsymbol{k}^{\mathrm{H}}\mathrm{diag}\{\lambda'_0,\ldots,\lambda'_0\}\boldsymbol{k}, & \forall\boldsymbol{k}\in C,\\ \boldsymbol{k}^{\mathrm{H}}\boldsymbol{\Lambda}\boldsymbol{k}\geq\boldsymbol{k}^{\mathrm{H}}\mathrm{diag}\{\lambda'_{Q^2-1},\ldots,\lambda'_{Q^2-1}\}\boldsymbol{k}, & \forall\boldsymbol{k}\in C,\end{cases} \quad \text{(A7)}$$

and because

$$(\boldsymbol{c}')^{\mathrm{H}}(\boldsymbol{I}_Q\otimes\boldsymbol{c}^{\mathrm{H}})\boldsymbol{U}^{\mathrm{H}}\boldsymbol{U}(\boldsymbol{I}_Q\otimes\boldsymbol{c})\boldsymbol{c}'=1, \quad \text{(A8)}$$

thus we have

$$\lambda'_{Q^2-1}\leq(\boldsymbol{c}')^{\mathrm{H}}(\boldsymbol{R}(\gamma)+C\boldsymbol{I})\boldsymbol{c}'\leq\lambda'_0. \quad \text{(A9)}$$

Therefore, $\{E_i(\gamma)\}$ is limitary and strictly decreasing, which means its limit exists and it is convergent.

APPENDIX B

Let λ be the corresponding Lagrange multiplier value,

$$P(\boldsymbol{c}',\lambda)=(\boldsymbol{c}')^{\mathrm{H}}\tilde{\boldsymbol{R}}(\gamma)\boldsymbol{c}'+\lambda(1-(\boldsymbol{c}')^{\mathrm{H}}\boldsymbol{c}'), \quad \text{(B1)}$$

where $\tilde{\boldsymbol{R}}(\gamma)=\boldsymbol{R}(\gamma)+C\boldsymbol{I}$ can be a positive definite matrix if C is large enough. Thus the minimum of $P(\boldsymbol{c}', \lambda)$ can be found by setting its derivative with respect to $\boldsymbol{c}=\boldsymbol{0}$, that is,

$$\frac{\partial}{\partial\boldsymbol{c}'}P(\boldsymbol{c}',\lambda)=\tilde{\boldsymbol{R}}(\gamma)\boldsymbol{c}'-\lambda\boldsymbol{c}'=\boldsymbol{0}. \quad \text{(B2)}$$

Thus the optimization problem is equivalent to solving an eigenvalue equation

$$\tilde{\boldsymbol{R}}(\gamma)\boldsymbol{c}'=\lambda\boldsymbol{c}', \quad \text{(B3)}$$

where the optimum solution vector $\boldsymbol{c}'$ is an eigenvector of matrix $\tilde{\boldsymbol{R}}(\gamma)$ with corresponding eigenvalue λ. Multiplying $\tilde{\boldsymbol{R}}(\gamma)\boldsymbol{c}'$ by $(\boldsymbol{c}')^{\mathrm{H}}$, we obtain

$$(\boldsymbol{c}')^{\mathrm{H}}\tilde{\boldsymbol{R}}(\gamma)\boldsymbol{c}'=\lambda. \quad \text{(B4)}$$

Therefore, the eigenvector corresponding to the minimum eigenvalue $\lambda_{\min}$ of matrix $\tilde{\boldsymbol{R}}(\gamma)$ becomes the optimal coefficient vector and the minimum $(\boldsymbol{c}')^{\mathrm{H}}\tilde{\boldsymbol{R}}(\gamma)\boldsymbol{c}'$ is

$$E(\gamma)=\lambda_{\min}. \quad \text{(B5)}$$