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Peak-power reduction by the lattice-reduction-aided closest point search for MIMO broadcast channels*

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Abstract: In this letter, we present a modified vector-perturbation precoding scheme for the multiple-input multiple-output broadcast channel, where a perturbation vector is chosen to take into account both the instantaneous power and the instantaneous peak power of the transmitted signal. This perturbation vector is obtained by using the closest point search, with the aid of the lattice-reduction algorithm. Simulation results show that the proposed scheme yields a tradeoff among power efficiency, peak-to-average power ratio reduction, and complexity.

Key words: Multiple-input multiple-output (MIMO), Broadcast channel (BC), Precoding, Peak-to-average power ratio (PAPR), Lattice-reduction (LR)

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INTRODUCTION

The multiple-input multiple-output (MIMO) broadcast channel (BC) has been an area of extensive research in recent years. In future cellular systems, it may be used to model the downlink channel in which there are multiple transmit antennas at the base station and multiple receive antennas at each user. The capacity region of the Gaussian MIMO BC has been thoroughly investigated (Caire and Shamai, 2003; Vishwanath *et al*., 2003; Viswanath and Tse, 2003; Yu and Cioffi, 2004; Weingarten *et al.*, 2006). Weingarten *et al.*(2006) showed that the dirty-paper coding (DPC) (Costa, 1983) achievable rate region is actually the capacity region of the Gaussian MIMO BC. A well-known general approach for achieving the capacity of DPC is based on multidimensional lattice quantization and minimum mean-square error (MMSE) scaling (Erez and Brink, 2005; Erez *et al.*, 2005). Other developments based on the DPC scheme include trellis and convolutional precoding (Yu *et al.*, 2005) and superposition coding (Bennatan *et al.*, 2006).

However, the high nonlinearity of the DPC technique prevents its practical implementation. This has motivated several researchers to consider some other precoding schemes, including channel inversion (Peel *et al.*, 2005), regularized channel inversion (Peel *et al.*, 2005), vector-perturbation based on the sphere encoder (Hochwald *et al.*, 2005), lattice-reduction-aided (LRA) Tomlinson-Harashima precoding (Windpassinger *et al.*, 2004) and block diagonalization (Spencer *et al.*, 2004). Among these precoding schemes, the vector-perturbation scheme can achieve the near optimal performance of the DPC when multiuser diversity is not available. However, the vector-perturbation scheme does not take into account the peak-to-average power ratio (PAPR) of the resulting transmitted signal.

Recent work by Boccardi and Caire (2006) based on the *p*-sphere encoder (*p*-SE) promises to reduce the PAPR but suffers from high complexity. In

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this letter, we first apply the polynomial-time Lenstra-Lenstra-Lovasz (LLL) algorithm (Lenstra *et al.*, 1982) on the precoding matrix to obtain a lattice-reduced basis. Then, using the approximately orthogonal character of the columns of the LLL-reduced basis, the closest point search (CPS) can be used to find the optimal perturbation vector. The proposed LRA-CPS scheme can achieve a tradeoff between power efficiency and PAPR. Moreover, the LRA-CPS scheme has a lower complexity and is more intuitive than the *p*-SE.

SYSTEM MODEL AND PRELIMINARIES

Consider a MIMO BC precoding model with M transmit antennas at the base station and K users, each with one receive antenna. The complex baseband input-output model can be represented as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{w}, \tag{1}$$

where in each symbol interval, $\boldsymbol{y}=[y_1, y_2, \ldots, y_K]^{\mathrm{T}}$ contains the data received at each user, $\boldsymbol{x}=[x_1, x_2, \ldots, x_M]^{\mathrm{T}}$ contains the signal transmitted from M transmit antennas, $\boldsymbol{H}$ is a $K\times M$ complex-valued channel matrix with each entry h_{km} representing the channel gain between transmit antenna m and user k, and $\boldsymbol{w}=[w_1, w_2, \ldots, w_K]^{\mathrm{T}}\sim\mathcal{N}_C(0, \sigma^2\boldsymbol{I}_K)$ is an independently and identically distributed (i.i.d.) complex Gaussian noise vector. We assume that the MIMO channel matrix $\boldsymbol{H}$ is flat Rayleigh-fading ($h_{km}\sim\mathcal{N}_C(0, 1)$) and known to the transmitter.

The average power constraint $E(\|\boldsymbol{x}\|_2^2)\le P$ is imposed on the transmitted signal. In the vector perturbation scheme, it is convenient to also consider the unnormalized transmitted signal $\boldsymbol{s}$ such that

$$\boldsymbol{x} = \frac{\boldsymbol{s}}{\sqrt{\rho}} = \frac{\boldsymbol{G}(\boldsymbol{u}+\tau\boldsymbol{\lambda})}{\sqrt{\rho}}, \tag{2}$$

with $\rho = E(\|\boldsymbol{s}\|_2^2)/P$. Here, $\boldsymbol{u}=[u_1, u_2, \ldots, u_K]^{\mathrm{T}}\in\mathcal{X}^K$ is the information-bearing signal containing the modulation symbols to be transmitted to the users, $\mathcal{X}$ denotes some complex signal set such as quadrature amplitude modulation (QAM) constellation, $\boldsymbol{G}$ is an $M\times K$ precoding matrix which is often set to be the Moore-Penrose pseudoinverse of $\boldsymbol{H}$, i.e., $\boldsymbol{G}=\boldsymbol{H}^{\dagger}=\boldsymbol{H}^{\mathrm{H}}(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}})^{-1}$ with superscript H denoting the conjugate transpose of its argument (vector or matrix), and $\tau\boldsymbol{\lambda}$ is a data-dependent perturbation vector with $\boldsymbol{\lambda}=[\lambda_1, \lambda_2, \ldots, \lambda_K]^{\mathrm{T}}\in\mathcal{Z}_C^K$ and $\mathcal{Z}_C=\{a+\mathrm{j}b|a, b\in\mathbb{Z}\}$, $\mathrm{j}=\sqrt{-1}$.

For the case of square QAM constellation, $\tau=2(d_{\max}+\Delta/2)$, where $d_{\max}$ is the absolute value of the constellation symbols with the largest magnitude and Δ is the spacing between constellation points. For example, for the 16-QAM constellation with amplitudes {±1, ±3} in real dimension, $d_{\max}=3$, $\Delta=2$, and $\tau=8$.

PROBLEM FORMULATION

In the vector-perturbation scheme, the optimal perturbation vector, denoted by $\bar{\boldsymbol{\lambda}}$, is the one which minimizes the instantaneous transmit power $\|\boldsymbol{s}\|_2^2$, i.e.,

$$\bar{\boldsymbol{\lambda}} = \arg\min_{\boldsymbol{\lambda}\in\mathcal{Z}_C^K} \|\boldsymbol{G}(\boldsymbol{u}+\tau\boldsymbol{\lambda})\|_2^2 . \tag{3}$$

This can be efficiently implemented by the sphere decoder (SD) (Damen *et al.*, 2003) and this precoding scheme is also referred to as 'sphere encoder' (SE). However, this strategy minimizes only the average transmit power $E(\|\boldsymbol{s}\|_2^2)$ without any control on the peak power $\|\boldsymbol{s}\|_\infty^2$, where $\|\cdot\|_\infty^2$ denotes the vector ∞-norm. To take into account both the average transmit power and the instantaneous peak power, we introduce the instantaneous PAPR, defined as

$$\gamma(\boldsymbol{u},\boldsymbol{G}) = \frac{\|\boldsymbol{s}\|_\infty^2}{E(\|\boldsymbol{s}\|_2^2/M)}. \tag{4}$$

The goal is to find a perturbation vector that reduces the instantaneous PAPR without increasing the average transmit power too much. That means, for a given power penalty $\beta\ge1$, we would solve the problem

$$\min\|\boldsymbol{s}\|_\infty^2 \quad \text{s.t.}\ E(\|\boldsymbol{s}\|_2^2)\le\beta E(\|\bar{\boldsymbol{s}}\|_2^2). \tag{5}$$

Here, $\bar{\boldsymbol{s}}=\boldsymbol{G}(\boldsymbol{u}+\tau\bar{\boldsymbol{\lambda}})$, and $\bar{\boldsymbol{\lambda}}$ is the solution of Eq.(3), which means that $\bar{\boldsymbol{s}}$ has the lowest instantaneous transmit power.

In the p-SE (Boccardi and Caire, 2006), $\bar{\boldsymbol{s}}$ is first obtained through the SD, and then the p-SD is used to solve Eq.(5). The expected complexity of the p-SD is about $O(K^3)$ (Damen *et al.*, 2003). In the next section, we will present a low-complexity scheme, where the p-SD is replaced by the CPS. Moreover, the size of the search space of CPS is only linear or quadratic in K.

LRA-CPS SCHEME

In the proposed LRA-CPS scheme, we first apply the polynomial-time LLL algorithm (Lenstra *et al.*, 1982) on $\boldsymbol{G}$ to obtain

$$\boldsymbol{G}=\boldsymbol{TR}, \tag{6}$$

here $\boldsymbol{T}$ is the $M\times K$ LLL-reduced basis with approximately orthogonal columns, and $\boldsymbol{R}$ is an integer matrix with $|\det\boldsymbol{R}|=1$, which describes this transform. Therefore, it is obvious that the vector $\boldsymbol{v}=[v_1, v_2, \ldots, v_K]^{\mathrm{T}}=\boldsymbol{R}(\boldsymbol{u}+\tau\boldsymbol{\lambda})$ has integer entries.

Then the instantaneous transmit power can be approximated as

$$\|\boldsymbol{G}(\boldsymbol{u}+\tau\boldsymbol{\lambda})\|_2^2=\|\boldsymbol{Tv}\|_2^2\approx\sum_{k=1}^{K}\|\boldsymbol{T}^{[k]}v_k\|_2^2=\sum_{k=1}^{K}|v_k|^2\|\boldsymbol{T}^{[k]}\|_2^2, \tag{7}$$

where $\boldsymbol{T}^{[k]}$ denotes the kth column of $\boldsymbol{T}$ and the approximation follows from the approximately orthogonal character of $\boldsymbol{T}^{[k]}$ for k=1, 2, …, K. Furthermore, set $\bar{\boldsymbol{v}}=\boldsymbol{R}(\boldsymbol{u}+\tau\bar{\boldsymbol{\lambda}})$ and $\boldsymbol{v}=\bar{\boldsymbol{v}}+\Delta\boldsymbol{v}$, where the entry of $\Delta\boldsymbol{v}$ should be an integer multiple of τ to ensure the vector $\boldsymbol{\lambda}=(\boldsymbol{R}^{-1}\boldsymbol{v}-\boldsymbol{u})/\tau$ is an integer. Then the average transmit power constraint in Eq.(5) can be approximated as

$$\sum_{k=1}^{K}\left(|\bar{v}_k+\Delta v_k|^2-|\bar{v}_k|^2\right)\|\boldsymbol{T}^{[k]}\|_2^2\leq(\beta-1)E(\|\bar{\boldsymbol{s}}\|_2^2)=C. \tag{8}$$

Note that the differences between the lengths of all the columns of $\boldsymbol{T}$ are guaranteed small in the LLL basis reduction algorithm (Lenstra *et al.*, 1982). We assume that all $\|\boldsymbol{T}^{[k]}\|_2^2$ are equal for k=1, 2, …, K, which shows good performance in our results. Then, Eq.(8) can be further simplified as

$$\sum_{k=1}^{K}\left(|\bar{v}_k+\Delta v_k|^2-|\bar{v}_k|^2\right)\leq C/\|\boldsymbol{T}^{[1]}\|_2^2=C', \tag{9}$$

and it is obvious that most vectors gained by a suitable CPS strategy will satisfy the average power constraint. Here, we consider only the three simplest CPS strategies, defined as:

Strategy A Only one entry of $\bar{\boldsymbol{v}}$ changes and the four closest points of this entry are considered, i.e., the closest point set is defined as

$$S_{\mathrm{A}}=\left\{\begin{matrix}\boldsymbol{v}\,|\,\boldsymbol{v}=\bar{\boldsymbol{v}}+\Delta\boldsymbol{v},\ \Delta\boldsymbol{v}=\delta\boldsymbol{e}_k,\\ \delta=\{\pm\tau,\ \pm\mathrm{j}\tau\},\ k=1,2,\ldots,K\end{matrix}\right\},$$

where $\boldsymbol{e}_k$ is the all-zero vector except 1 at the kth element.

Strategy B Similar to Strategy A but with the eight closest points considered, i.e., the closest point set is defined as

$$S_{\mathrm{B}}=\left\{\begin{matrix}\boldsymbol{v}\,|\,\boldsymbol{v}=\bar{\boldsymbol{v}}+\Delta\boldsymbol{v},\ \Delta\boldsymbol{v}=\delta\boldsymbol{e}_k,\\ \delta=\{\pm\tau,\ \pm\mathrm{j}\tau,\ \pm\tau\pm\mathrm{j}\tau\},\ k=1,2,\ldots,K\end{matrix}\right\}.$$

Strategy C Two entries of $\bar{\boldsymbol{v}}$ change simultaneously and the four closest points of each of these two entries are considered, i.e., the closest point set is defined as

$$S_{\mathrm{C}}=\left\{\begin{matrix}\boldsymbol{v}\,|\,\boldsymbol{v}=\bar{\boldsymbol{v}}+\Delta\boldsymbol{v},\ \Delta\boldsymbol{v}=\delta_1\boldsymbol{e}_{k_1}+\delta_2\boldsymbol{e}_{k_2},\\ \delta_i=\{\pm\tau,\ \pm\mathrm{j}\tau\},\ k_i=1,2,\ldots,K;\ i=1,2;\ k_1\neq k_2\end{matrix}\right\}.$$

Note that in these three search strategies, for simplicity, we do not consider the transmit power constraint, which means that there are vectors obtained by the CPS that do not satisfy Eq.(9). Moreover, some of those vectors satisfying Eq.(9) do not satisfy the transmit power constraint in Eq.(5), since the columns of $\boldsymbol{T}$ are not perfectly orthogonal. To address these problems, the initial transmit power constraint in Eq.(5) is still necessary. Then, the optimal vector can be obtained by solving the following constrained problem:

$$\min\|\boldsymbol{Tv}\|_\infty^2$$
$$\text{s.t.}\ \ \|\boldsymbol{Tv}\|_2^2\leq\beta\|\boldsymbol{T}\bar{\boldsymbol{v}}\|_2^2\ \text{and}\ \boldsymbol{v}\in S_{\mathrm{A}},\ S_{\mathrm{B}},\ \text{or}\ S_{\mathrm{C}}. \tag{10}$$

After obtaining the optimal vector $\hat{\boldsymbol{v}}$, we can calculate the corresponding perturbation vector by

$\hat{\boldsymbol{\lambda}} = (\boldsymbol{R}^{-1}\hat{\boldsymbol{v}} - \boldsymbol{u}) / \tau.$

For clarity, the proposed LRA-CPS scheme can be summarized as follows:

Step 1: Perform the LLL algorithm Eq.(6);

Step 2: Obtain $\bar{\boldsymbol{\lambda}}$ using the SD and let $\bar{\boldsymbol{v}} = \boldsymbol{R}(\boldsymbol{u} + \tau\bar{\boldsymbol{\lambda}})$;

Step 3: Find the optimal vector $\hat{\boldsymbol{v}}$ according to Eq.(10);

Step 4: Calculate the perturbation vector $\hat{\boldsymbol{\lambda}} = (\boldsymbol{R}^{-1}\hat{\boldsymbol{v}} - \boldsymbol{u}) / \tau.$

To end this section, we briefly compare the complexity of the *p*-SE and the proposed LRA-CPS scheme. It is obvious that the complexity of the *p*-SE is dominated by the SD and the *p*-SD, while the complexity of the LRA-CPS scheme is dominated by the SD and the CPS. Since the complexity of the SD will be reduced with the LLL basis-reduction pre-processing (Damen *et al.*, 2003), the LLL algorithm is applied in the SD of both the *p*-SE and the LRA-CPS schemes. Therefore, it is sufficient to compare the complexity of only the *p*-SD and the CPS.

Recall that the expected size of the search space is $O(K^3)$ in the SD (Damen *et al.*, 2003). However, it is obvious that the size of the search spaces of Strategies A, B and C are $4K$, $8K$ and $8K(K-1)$, respectively. Therefore, the proposed LRA-CPS scheme has a lower complexity compared to the *p*-SE.

NUMERICAL RESULTS

Our experimental setup considers an $M=K=8$ MIMO BC with 16-QAM modulation constellation where the standard Gray binary mapping is used. To assess the efficiency of PAPR reduction, an effective measure is to consider the tail of the complementary cumulative distribution function (CCDF) of the PAPR, which is defined as $Pr(\gamma(\boldsymbol{u}, \boldsymbol{G})>x)$ (Boccardi and Caire, 2006).

In simulations, we considered the SE, the *p*-SE ($p=\infty$ and 4) and the proposed LRA-CPS scheme (power penalty $\beta=2$ and Strategies A, B and C; $\beta=3$ and Strategy A). Fig.1a shows the uncoded *BER* (bit error rate) vs *SNR* (signal-to-noise ratio, $SNR=P/\sigma^2$) performance, while Fig.1b shows the CCDF of the PAPR in logarithmic scale when SNR=25 dB.

The advantage of the proposed LRA-CPS scheme over the SE, in terms of PAPR, is evident from Figs.1a and 1b. Both the loss of the uncoded BER and the advantage of the PAPR increase with the size of search space in fixed β and with β in fixed search strategy. Furthermore, our scheme has similar performances in terms of uncoded BER and PAPR, compared to the *p*-SE. In particular, the LRA-CPS scheme with $\beta=2$ and Strategy A achieves nearly the same BER and PAPR performances as the *p*-SE with $p=4$.

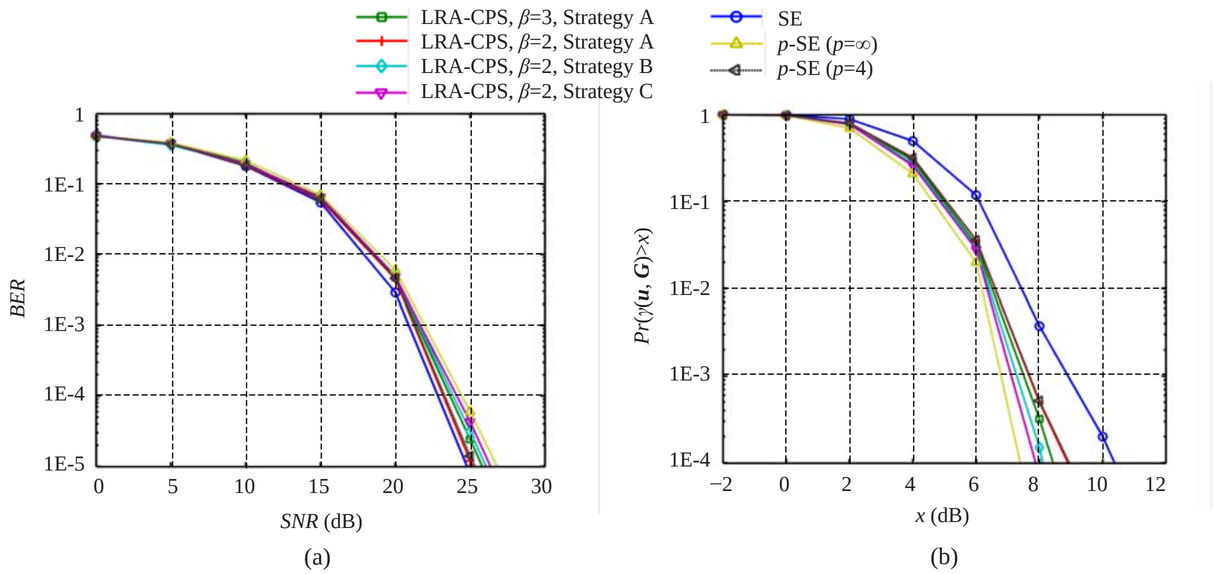

Fig.1 *BER* vs *SNR* (a) and comparison in terms of peak-to-average power ratio of the complementary cumulative distribution function (b) for the sphere encoder (SE), the *p*-SE and the lattice-reduction-aided closest point search (LRA-CPS), with $p=\infty$ and 4 in the *p*-SE, β=2, 3 and Strategies A, B and C in the LRA-CPS

CONCLUSION AND OPEN PROBLEM

The proposed LRA-CPS precoding scheme takes into account both the average transmit power and the instantaneous peak power. It has been shown that this scheme yields a tradeoff among power efficiency, PAPR reduction, and complexity.

In this letter, we assume that the orthogonal columns of the LLL-reduced basis have the same length. Without this assumption, we need to perform weighted CPS, which is a more complex weighted combination and search problem. Further, in both the *p*-SE and the LRA-CPS methods, the perturbation vector with minimum transmit power has to be calculated first by sphere decoder. How to find the optimal solution of Eq.(5) in an integrated process with lower complexity is still an unresolved problem.

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