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Optimal design of pressure vessel using an improved genetic algorithm*

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Abstract: As the idea of simulated annealing (SA) is introduced into the fitness function, an improved genetic algorithm (GA) is proposed to perform the optimal design of a pressure vessel which aims to attain the minimum weight under burst pressure constraint. The actual burst pressure is calculated using the arc-length and restart analysis in finite element analysis (FEA). A penalty function in the fitness function is proposed to deal with the constrained problem. The effects of the population size and the number of generations in the GA on the weight and burst pressure of the vessel are explored. The optimization results using the proposed GA are also compared with those using the simple GA and the conventional Monte Carlo method.

Key words: Pressure vessel, Optimal design, Genetic algorithm (GA), Simulated annealing (SA), Finite element analysis (FEA)
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INTRODUCTION

It is well known that the pressure vessel has been widely used in a variety of areas such as chemical engineering, medical treatment, aviation and astronautics as well as nuclear engineering. Currently the pressure vessel tends to be developing in large-scale and high-parameter directions, especially in chemical industry. However, the pressure vessel is generally subjected to a complex environment such as high pressure and high temperature. This means not only a strong challenge regarding the performance of the material and structure, but also concerning the design of the pressure vessel. How to achieve a perfect combination of excellent performance and low cost in the design of a pressure vessel under certain design conditions is an important topic.

Several studies have concentrated on the optimal design for the structural shape of a pressure vessel under strength and stiffness constraints. However, they are all based on conventional optimization methods such as simple mathematical calculations (Liu *et al.*, 2001; Magnucki *et al.*, 2004) and the gradient search method (Hyder and Asif, 2008). These methods may easily lead to local optimization results and largely depend on the initial values of the optimized variables. Besides, the search capacity is also limited for a big design space with a large number of discrete variables.

Currently, two intelligent optimization algorithms are suitable for a specific optimization problem with a large-scale discrete design space: genetic algorithm (GA) (Holland, 1975; Schmitt, 2001; Blachut and Eschenauer, 2001) and simulated annealing (SA) (Kirkpatrick *et al.*, 1983; de Vicente *et al.*, 2003). Using these two methods the optimization solutions are obtained from the stochastic sampling of the designed space. The GA was originally employed to simulate the evolution of biology and exhibits a good capacity for achieving global optimization results. However, the "hill-climbing" capacity, that is

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the local search capacity, is relatively weak. In contrast, the SA was originally used to simulate the annealing process of the physical multi-particle system in statistical thermodynamics and proved to have a strong ability in the local search of discrete space. Unfortunately, no report about the application of these two algorithms in the optimal design of a pressure vessel has been found so far.

In this paper the GA is used to optimize the weight of the pressure vessel under the burst pressure constraint. The concept of SA is introduced into the fitness function to improve the search efficiency of the GA. A penalty function is proposed to ensure the bidirectional search in both feasible and infeasible directions for the discrete design variables. Effects of various parameters in the genetic iteration of the optimization results are explored. Also, the results obtained using the proposed GA are compared with those using the simple GA and the conventional Monte Carlo method.

FINITE ELEMENT MODELING OF PRESSURE VESSEL

The following analysis concentrates on a vessel as shown in Fig.1, in which a Cartesian coordinate system is defined. It includes a cylinder, a spherical head and nozzles A and B. The finite element model and material properties are obtained by Liu *et al.*(2008).

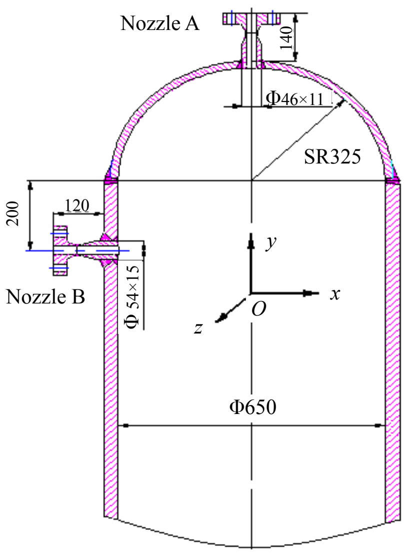

Fig.1 Schematic representation of vessel (unit: mm)

BASIC DESIGN CONDITIONS

The design of the pressure vessel aims to achieve the minimum weight under the burst pressure constraint. The design variables are the thicknesses t_1 and t_2 of the cylinder and the head, respectively. The status variable is the actual burst pressure P. Here the basic design conditions are summarized as

$$\text{minimize } W=f(t_1, t_2)\ (t_1 \in D_1, t_2 \in D_2), \\ P(t_1, t_2) \geq n \times P_d, \tag{1}$$

where the weight of vessel W and the actual burst pressure P are functions of t_1 and t_2. P_d represents the design pressure. D_1 and D_2 are the variable ranges for t_1 and t_2, respectively. The constant n is the safety coefficient of the vessel.

OPTIMIZATION MODEL

In the following, an improved GA is proposed and it includes seven steps:

(1) Binary encoding of the design variables. If the string length of bits for each variable t is L and the number of the design variables is Num, the length of the chromosome for each individual is $L \times Num$. If the range of the variable t is within $[t_{min}, t_{max}]$, the relationship between the decimal value t and binary value t_b is given by

$$t = t_{min} + \frac{t_b}{2^L - 1}(t_{max} - t_{min}). \tag{2}$$

(2) Generation of the initial population. The population with Pop individuals is generated using a completely stochastic method.

(3) Calculations of the objective function and burst pressure. In terms of the parametric model, the weight of the vessel is calculated using ANSYS-APDL and the burst pressure of the vessel is calculated by us using the proposed algorithm, in which the arc-length algorithm and the restart analysis in the finite element analysis (FEA) are employed (Liu *et al.*, 2008). Also, other research was performed to calculate the burst pressure of the pressure vessel, such as the methods proposed by Blachut and Vu (2007).

(4) Calculations of the fitness values for the individuals. In general, the calculated burst pressure may be smaller than the burst pressure constraint and the binary codes lead to infeasible results. Here a penalty function is proposed to deal with the constrained problem. The fitness function F for the minimum problem in the simple GA is written as

$$\begin{aligned} &f'(t_1, t_2)=f(t_1, t_2)+\lambda\times\max[n\times P_{\mathrm{d}}-P,\ 0],\\ &F=1/f'(t_1, t_2), \end{aligned} \tag{3}$$

where λ is the penalty factor and generally is a large positive number. Thus, the penalty function forces the genetic search to approach the optimization solution from both the feasible and infeasible directions. The larger the fitness value is, the more excellent the individual is.

However, two problems appear for the fitness function above: First, the difference between the individuals in the early genetic iteration is large and the excellent individuals can easily occupy the whole population, which leads to the phenomenon of "prematurity"; Second, the fitness values tend to be consistent in the later genetic iteration and the excellent individuals cannot exhibit their predominance, which results in the stagnancy of the whole population evolution.

In order to settle the two problems above, a novel fitness function F is proposed using the idea of SA to improve the search efficiency. The SA, which was proposed by Kirkpatrick *et al.*(1983), has been well applied to solve complex stochastic optimization problems with discrete space and demonstrated to have good local search capacity, which is also called the "hill-climbing" capacity.

The SA is described as follows: if a system is in a configuration A at time t and a new configuration B of the system at time $t+\Delta t$ is generated randomly, the configuration B is accepted according to the acceptance probability $P(B)$:

$$P(B)=\exp\left(-\frac{E(B)-E(A)}{K_{\mathrm{B}}T}\right), \tag{4}$$

where E represents the energy, K_{B} is the Boltzmann constant and T is the annealing temperature. In this case the acceptance probability $P(B)$ decreases with decreasing temperature T, which means that the material energy will gradually approach a low energy status and finally converge with the minimum energy status.

By introducing the idea of SA, the fitness function in the GA is modified as

$$\begin{cases} F=\exp\left[-f'(t_1, t_2)/(K_{\mathrm{B}}T)\right],\\ T=kT_0, \end{cases} \tag{5}$$

where T_0 denotes the initial annealing temperature and T is the real-time annealing temperature after each genetic iteration. k is the cooling schedule and generally is a number approximating unity.

(5) Selection. The roulette wheel selection proposed by Holland (1975) is the most famous selection method and its basic principle is to determine the birth probability of each individual according to the proportion of the fitness value of each individual in the whole population.

Corresponding to Eqs.(3) and (5), the probabilities of selection are respectively expressed as

$$P_{\mathrm{s}}=\sum_{i=1}^{Pop} f_i' \Big/ f_i',\quad P_{\mathrm{s}}=\frac{\exp(-f_i'/T)}{\sum_{i=1}^{Pop}\exp(-f_i'/T)}. \tag{6}$$

(6) Crossover. A one-point crossover is adopted in this work. The position of the crossover is randomly selected and the crossover may be carried out with a crossover probability. Taking a 20-bit chromosome for example, the operating representation of the crossover is shown in Fig.2.

(7) Mutation. The bits for each individual are randomly changed from zero to one and vice versa with a mutation probability. The operating representation of mutation is shown in Fig.2.

Parent 1	**10001001011**	**100011100**
Parent 2	00111100110	011011001
	⇓	
Child 1	**10001001011**	011011001
Child 1	00111100110	**100011100**

Fig.2 Operating representation of crossover

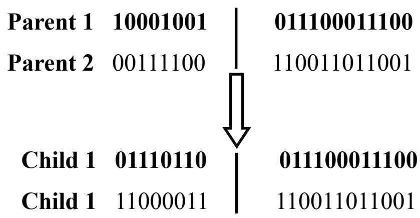

Fig.3 Operating representation of mutation

Based on the steps above, the optimal design of the pressure vessel is implemented by associating the software MATLAB with ANSYS. The flowchart is shown in Fig.4.

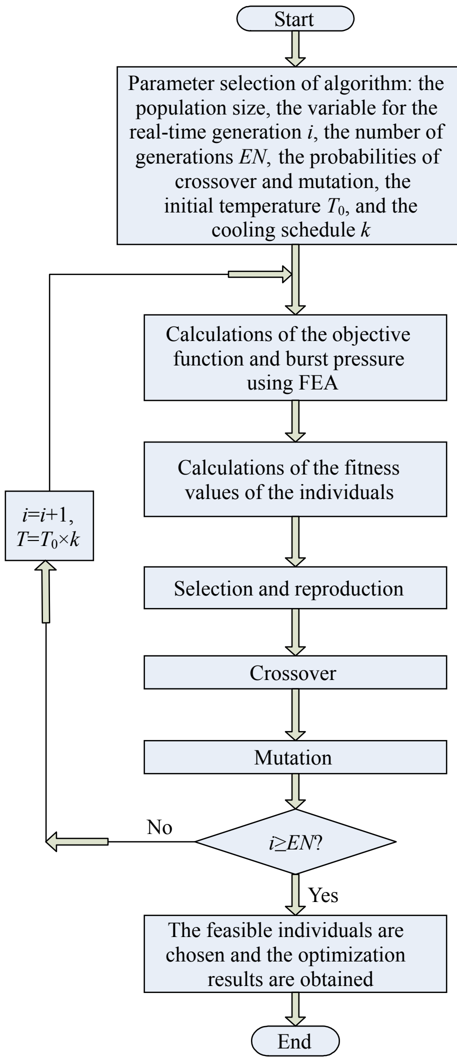

Fig.4 Flowchart of the improved genetic algorithm

NUMERICAL EXAMPLE

The constant parameters in the optimization analysis of the pressure vessel are chosen as follows: the thicknesses (mm) of the cylinder and head are $t_1 \in [36, 44]$, $t_2 \in [20, 28]$; the inner radius and length of the cylinder are 325 mm and 1200 mm, respectively; the inner radius of the hemispherical head is 325 mm; the size types for the nozzles A and B are Φ46 mm×11 mm and Φ54 mm×15 mm, respectively; the string length of bits for each variable is L=10; the design pressure is P_d=18 MPa; the safety coefficient of the pressure vessel is n=3; the penalty factor is λ=50; the probabilities of crossover and mutation are 0.85 and 0.01, respectively; the initial annealing temperature is T_0=100 K; the cooling schedule is k=0.99; the Boltzmann constant is K_B=1.

In terms of the simple GA, Table 1 shows the optimization results of the weight, burst pressure and fitness values of each individual for 10 populations and 100 generations. It can be seen that the optimization results are approximately obtained by gradually forcing both the infeasible and feasible results to approach the real solution. This is actually attributed to the effect of the proposed penalty function. By comparison, the results for Individual 2 are optimum among all individuals.

Figs.5a and 5b show the effects of the population size on the weight and burst pressure, respectively, and also compare the optimization results using the simple GA and those using the proposed GA. In the early genetic iteration, the results for the weight and burst pressure decrease remarkably with increasing population size. However, the results remain almost unchanged as the population size continues to grow. With the same population size, the improved GA leads to more optimal results for the weight and burst pressure than the simple GA does. For the burst pressure, only 100 populations produce the optimization results using the proposed GA, compared with 500 populations using the simple GA. Besides, the convergence velocity and precision of the simple GA is highly improved by introducing the concept of SA.

Figs.6a and 6b show the effects of the number of generations on the weight and burst pressure, respectively, and also compare the optimization results

Table 1 Optimization results for 10 populations after 100 generations in the simple GA

Individual number	Weight (kg)	Burst pressure (MPa)	Fitness	Feasible or infeasible
1	368.962	52.29	0.0022	Infeasible
2	**383.053**	**54.18**	**0.0026**	**Feasible**
3	385.116	54.46	0.0026	Feasible
4	356.084	50.57	0.0019	Infeasible
5	405.515	57.20	0.0025	Feasible
6	379.766	53.74	0.0025	Infeasible
7	408.940	57.66	0.0024	Feasible
8	398.701	56.28	0.0025	Feasible
9	350.544	49.82	0.0018	Infeasible
10	365.055	51.77	0.0021	Infeasible

Bold data: optimum result; Black data: feasible results; Grey data: infeasible results

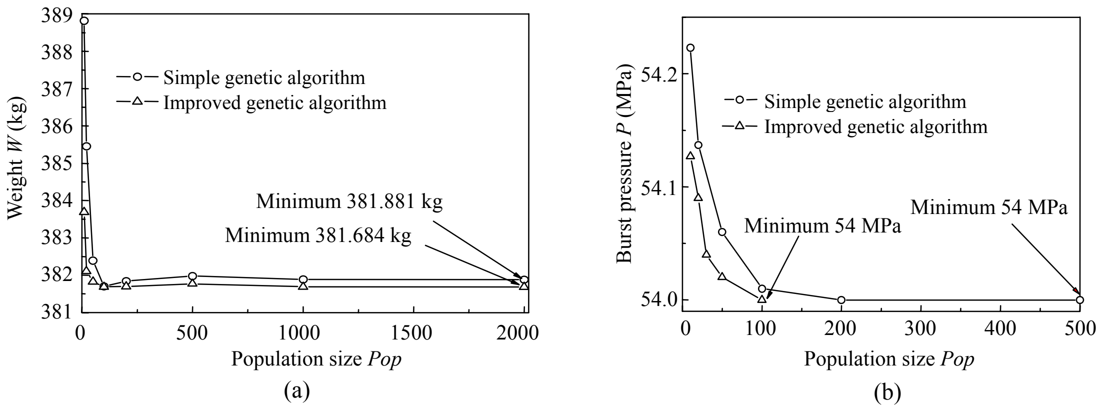

Fig.5 Effects of the population size on the (a) weight and (b) burst pressure after 100 generations

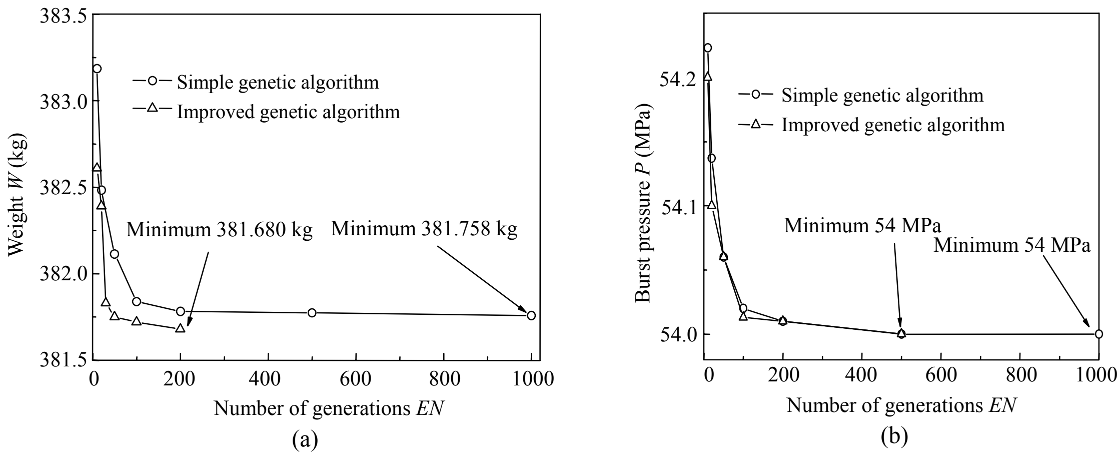

Fig.6 Effects of the number of generations on the (a) weight and (b) burst pressure for 100 populations

using the simple GA and those using the proposed GA. Similarly, the proposed GA distinctly improves the convergence efficiency and avoids the "prematurity". Therefore, the capacity of "hill- climbing" of the simple GA is well enhanced.

In order to confirm the efficiency of the GA, the optimization results using the GA are also compared with those using the conventional Monte Carlo method, as listed in Table 2. The Monte Carlo method assumes that the variables t_1 and t_2 are uniformly

Table 2 Comparisons of the optimization results using the simple GA, the improved GA and the Monte Carlo method

Optimization method	*Pop*	*EN*	Weight *W* (kg)	Burst pressure *P* (MPa)	t_1 (mm)	t_2 (mm)
Simple GA	1000	1000	381.764	54.01	39.77	20
Proposed GA	100	200	381.684	54.00	39.76	20
Monte Carlo method*			382.034	54.04	39.79	20
Monte Carlo method**			381.821	54.01	39.77	20

Pop: population size; *EN*: number of iterations; t_1: thickness of cylinder; t_2: thickness of head; *1000 iterations; **10000 iterations

distributed and the completely random values are chosen within their variable ranges. The Monte Carlo method is also carried out using MATLAB and the effect of the iteration times on the optimization results is calculated. Only when the iteration time becomes very large can the calculations reach relatively optimal results using the Monte Carlo method. This practically neglects the effect of the evolution in the GA and becomes time-consuming and inefficient. Based on the simple GA, the proposed GA further takes into account the different characteristics of the early and later iterations by adjusting the selecting probability of the individuals, and improves the search efficiency and convergence velocity. Therefore, the proposed GA exhibits more advantages than the simple GA and the Monte Carlo method. From Table 2 the optimization results using the proposed GA are 381.684 kg weight and 54 MPa burst pressure and the corresponding thickness configurations are t_1=39.76 mm and t_2=20 mm.

CONCLUSION

In this analysis, an improved GA which is associated with the concept of SA is proposed to optimize the weight of the pressure vessel under the burst pressure constraint. The actual burst pressure of the vessel is calculated using the arc-length and restart analysis in the FEA. A penalty function is proposed to deal with the constrained problem. Effects of the population size and the number of generations on the optimization results are explored. By comparison, the following conclusions are obtained:

(1) In terms of the convergence velocity and precision, the proposed GA exhibits more advantages than the simple GA and the conventional Monte Carlo optimization method.

(2) The conventional Monte Carlo method is more time-consuming and inefficient than the GA because the completely random search lacks the strong capacity to reach the real optimization solutions.

(3) The bidirectional search from the feasible and infeasible solutions are more rational than the complete exclusion of the infeasible solutions for dealing with the constraint conditions by using the GA.

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