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Model for cascading failures in congested Internet*

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Abstract: Cascading failures often occur in congested networks such as the Internet. A cascading failure can be described as a three-phase process: generation, diffusion, and dissipation of the congestion. In this account, we present a function that represents the extent of congestion on a given node. This approach is different from existing functions based on betweenness centrality. By introducing the concept of 'delay time', we designate an intergradation between permanent removal and nonremoval. We also construct an evaluation function of network efficiency, based on congestion, which measures the damage caused by cascading failures. Finally, we investigate the effects of network structure and size, delay time, processing ability and packet generation speed on congestion propagation. Also, we uncover the relationship between the cascade dynamics and some properties of the network such as structure and size.

Key words: Complex network, Cascading failures, Congestion effects, Propagation model
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INTRODUCTION

The problem of attacks on the Internet has attracted much attention (Chang and Young, 2005; Ash and Newth, 2007; Galstyan and Cohen, 2007; Wu *et al.*, 2007a; Wang and Chen, 2008; Sharma and Srivastava, 2008) since the Internet topology with scale-free property was discovered (Albert *et al.*, 2000). In scale-free networks, attacks on nodes with more connections may lead to a drop in efficiency, and may even cause abnormalities due to overload. Consequently, successive packets on the nodes will be compelled to reroute to avoid the congested nodes. However, the bypassing may congest other downstream routers and lead to more traffic detours and node congestion. As the congestion diffuses, the cascade is quite likely to propagate through the entire network.

In most previous studies, the load on the node was estimated by its betweenness centrality (BC), and the overloaded nodes were always removed from the networks during the cascading process. However, in many real-world networks such as the Internet, the overloaded nodes result only in more traveling time for some packets without destroying any network connectivity. Also, a node becomes isolated when all of its neighbors are overloaded. It still works normally but it cannot forward any packets to other nodes, so such a node should also be regarded as disabled. Furthermore, in a transportation network such as the Internet, BC cannot accurately reflect the load processed by nodes.

In contrast to BC, we propose a congestion function to represent the extent of congestion and define a new evaluation function of network efficiency. Together they help us to obtain a more realistic modeling of Internet failures. By introducing the concept of 'delay time', we can establish an inter-

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gradation between permanent removal and nonremoval. In an investigation of the effects of network structure and size, delay time, processing ability, and packet generation speed on congestion propagation, we uncover some factors affecting cascade propagation. These results are expected to improve our understanding of the impact of disturbances in complex communication and transportation systems.

A MODEL OF CASCADING DYNAMICS

We represent the Internet as an undirected and unweighted graph G with N nodes and E edges. For a given network, suppose that at every step λ, units of packets are exchanged between each pair of nodes. The load $L_i(t)$ of node i at time t is defined as the total number of units processed by node i at that time. When simulation begins, $L_i(0)$ is allocated randomly. The definition of $L_i(t)$ distinguishes the method from BC. It is expressed as follows:

$$K_i = \lceil k_i / \langle k \rangle \rceil K, \tag{1}$$

$$K_i \geq 1, \tag{2}$$

$$L_i(t) = L_i(t-1) - K_i + \sum_{j=1}^{k_i} x_{ji}(t), \quad t \geq 1, \tag{3}$$

$$L(t) = \frac{1}{N} \sum_i L_i(t), \tag{4}$$

where i=1, 2, ..., N; K_i is the processing ability of node i and $K_i \propto k_i$, which is reasonable because the processing ability of a node with more connections is always stronger in transportation networks such as the Internet; k_i is the degree of node i; $\langle k \rangle$ is the average degree of the network; K is the processing baseline of the network, which is a constant, and whose value is greater than 1; $x_{ji}(t)$ is the total number of packets sent by neighbor j to neighbor i at time t; $L(t)$ is the global load of the network at time t.

Following (Motter and Lai, 2002), we assume the capacity C_i of node i to be proportional to its initial load $L_i(0)$ as follows:

$$C_i = (1+\alpha) L_i(0), \quad i=1, 2, ..., N, \tag{5}$$

where the constant $\alpha \geq 0$ is the tolerance parameter. This is a realistic assumption in the design for an infrastructure network such as the Internet. The capacity cannot be infinitely large because it is severely limited by the cost.

In contrast to previous models, we define a congestion function (CF) for each node, which enables us to assign a dynamic weight to each node and denotes the extent of congestion, as follows:

$$f_i(t) = \begin{cases} 1, & L_i(t) \leq C_i^*, \\ 1 + \dfrac{L_i(t) - C_i^*}{C_i^{**} - C_i^*}(N-1), & C_i^* < L_i(t) \leq C_i^{**}, \\ N, & L_i(t) > C_i^{**}, \end{cases} \tag{6}$$

where $f_i(t)$ denotes the extent of congestion of node i at time t, and C_i^* is the upper bound limit load for node i at normal status. The value of C_i^* is related to the resources occupied by node i as follows:

$$C_i^* = \begin{cases} C_i - \dfrac{1}{K_i} C_i, & K_i \neq 1, \\ L_i(0), & K_i = 1. \end{cases} \tag{7}$$

From Eqs.(2) and (7), we know $C_i^* \propto K_i \propto k_i$, i.e., the stronger the processing ability K_i of node i, the bigger the ratio of C_i^* to C_i. Such a definition can avoid the disagreement caused by allocating initial load randomly about the relation of K_i and C_i^* in the present model and in the actual situation. C_i^{**} is the upper bound limit load of node i at overloaded status and is equal to its capacity C_i:

$$C_i^{**} = C_i = (1+\alpha) L_i(0), \quad i=1, 2, ..., N. \tag{8}$$

The three cases on the right-hand side of Eq.(6) correspond to three statuses of nodes: *normal*, *congested* and *overloaded*. The former two statuses are called *non-overloaded*. Nodes with all neighbors overloaded are called *isolated* nodes. Both *overloaded* nodes and *isolated* nodes are called *disabled* nodes and the others are called *non-disabled* nodes. The symbol $\tau(s)$, $\forall s \in \{$*normal*, *congested*, *overloaded*, *isolated*, *non-overloaded*, *non-disabled*$\}$, refers to the ratio of the total number of nodes at status s to the network size N. Unlike most models which remove *overloaded* nodes, we set a delay time Δt for *over-*

loaded node i. The node i drops the load $L_i(t)$ as soon as it becomes *overloaded*. During Δt the node i cannot receive or forward any packets. The value of Δt is related to the ratio of the restarting time of nodes to the propagating time of cascading failures. The closer the ratio is to 1, the bigger the Δt value, and vice versa. When Δt nears infinity, it is similar to permanent removing, whereas when Δt is close to 0, it is similar to nonremoving. We propose a new routing rule that differs from the Shortest Path (SP) rule: the best paths between two nodes are those whose sum of CFs is minimal, expressed as

$$\bar{P}_{ij}(t) = \{i,\ i+1,\ \ldots,\ n,\ \ldots,\ j\},\quad 1 \le i,\ j \le N, \tag{9}$$

$$\psi_{P_{ij}}(t) = \sum\nolimits_{p} f_p(t),\quad p \in P_{ij}(t), \tag{10}$$

$$\text{s.t.}\qquad \psi_{\bar{P}_{ij}}(t) = \min\{\psi_{P_{ij}}(t)\}, \tag{11}$$

where $\bar{P}_{ij}(t)$ is the node set of the best path between nodes i and j at time t; $P_{ij}(t)$ is the feasible path between nodes i and j at time t; $\psi_{P_{ij}}(t)$ is the sum of CFs of nodes at the path P_{ij} at time t.

If all the nodes on the best path are *normal*, the sum of CFs is equal to that of SP. If more than one shortest route exists, we select one of them at random.

From Eq.(10) we know that $\psi^{-1}_{\bar{P}_{ij}}(t)$ indicates how difficult it is to communicate between nodes i and j at time t. So we define the average efficiency of G at time t as

$$U_G(t) = \frac{1}{N(N-1)} \sum_{i \ne j \in G} \frac{1}{\psi_{\bar{P}_{ij}}(t)}. \tag{12}$$

SIMULATIONS AND DISCUSSIONS

We mainly focus on two topologies: the Erdös-Rényi (ER) model with a probability that two nodes have a link (Bollobás, 1985) and the Barabási-Albert (BA) model (Barabási and Albert, 1999). For a network with N nodes and average degree $\langle k \rangle$, the average BC of the ER and BA models are (Shen and Gao, 2008)

$$\langle b \rangle_{\text{ER}} \sim \frac{(N-1)\ln N}{\langle k \rangle \ln \langle k \rangle},$$

and

$$\langle b \rangle_{\text{BA}} \sim \frac{(N-1)\ln N}{\langle k \rangle \ln\ln N}.$$

We can find that the BC is only related to N and $\langle k \rangle$, which just reflects the static properties of the network. Each simulation result based on CF corresponds to an average over 50 independent experiments. Like (Motter and Lai, 2002), we focus on global cascades [i.e., $1-U_G(t)=O(1)$] triggered by initial attacks on a small fraction q of most loaded nodes.

The effects of the structure and size of topology on cascading failures are taken into account as shown in Fig.1. In Fig.1a, there is a big difference between τ(*non-disabled*) and τ(*non-overloaded*). Ignoring the *isolated* nodes and considering only the *overloaded* nodes may result in serious inaccuracy.

Figs.1b and 1c show that the propagation process can be divided into three phases: slow start, fast propagation and saturation. This is consistent with Wu *et al*.(2007b). In addition, the stability of the network declines with the growth of N.

Similar to (Wu *et al*., 2007a), in the following simulations, we set the network size N=150, $\langle k \rangle$=4 and the topology is the BA model, based on three conditions: (1) The Internet topology is scale-free. (2) The main purpose of the following simulations is to uncover the effects of different parameters on cascading failures. This relatively small system can be seen as a simulation of the Internet backbone and provides a sufficiently sized network to obtain statistically significant attributes. (3) The whole cascading process costs too much in running time due to computational constraints and the time complexity of computing CF.

Now we present numerical simulations of the effects of delay time Δt on cascading failures. The results are shown in Fig.2.

In Fig.2 it is clear that τ(*non-disabled*)$\propto 1/\Delta t$, $U_G(t) \propto 1/\Delta t$. The time-series deviations of τ(*non-disabled*) as well as $U_G(t)$ increase with the delay time Δt. The bigger delay time can lead to the less synchronously active nodes, resulting in the increased fluctuations at these active nodes. This result shows that the stability of the system is related to the stability of the network topology.

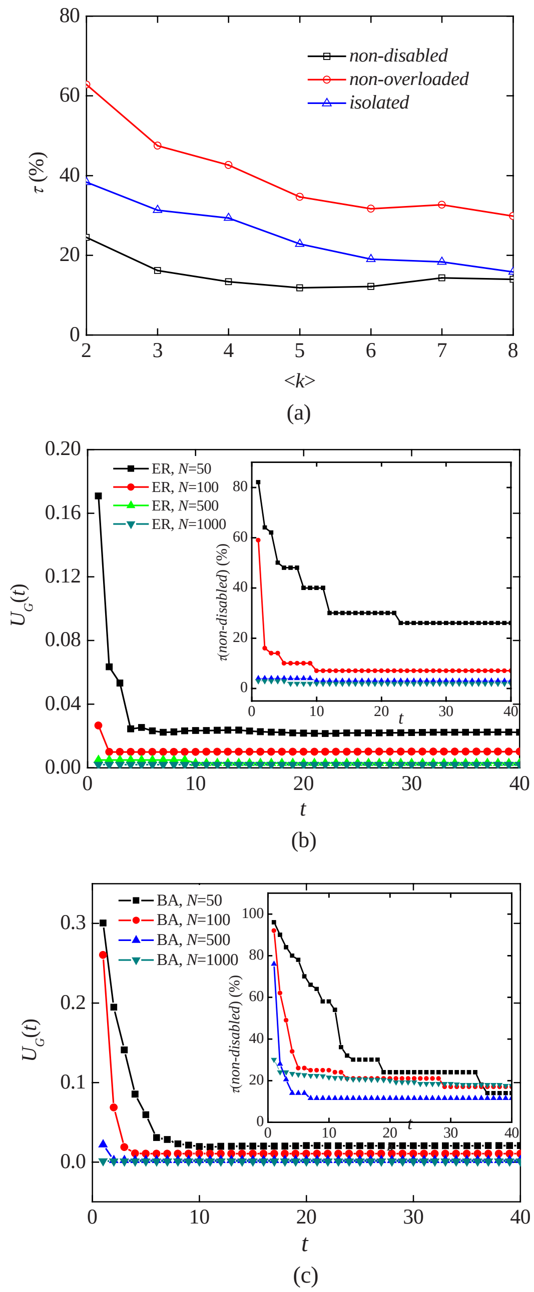

Fig.1 Effects of topology on cascading failures with α=0.2, λ=1, K=1, Δt=∞. (a) Effects of <k> on $\tau(s)$, $s\in$ {*non-disabled*, *non-overloaded*, *isolated*} based on the BA model after stabilizing with N=150; (b) Efficiency $U_G(t)$ of the network as a function of time t based on the ER model with <k>=4; (c) Efficiency $U_G(t)$ of the network as a function of time t based on the BA model with <k>=4. The insets of (b) and (c) show a plot of the relationship between τ(*non-disabled*) and t

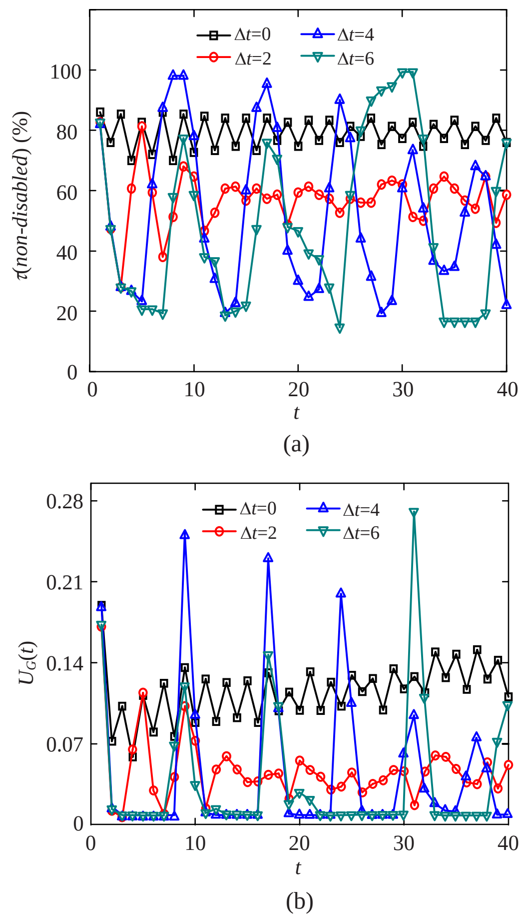

Fig.2 Effects of Δt on cascading failures with α=0.2, λ=1, K=1. (a) τ(*non-disabled*) as a function of time t; (b) Efficiency $U_G(t)$ of the network as a function of time t

We now turn to the effects of generating speed λ and processing ability K on cascading failures. Unlike previous studies which considered these two parameters separately (Cholvi, 2006), we set $\eta=\lambda/K$ as a pressure parameter. The results are shown in Fig.3.

From Fig.3, we find that τ(*non-disabled*)$\propto 1/\eta$, $U_G(t)\propto 1/\eta$ and $L(t)\propto 1/\eta$ when cascade propagation is steady ($t\geq 30$). When $\eta>1$, the traffic produced is higher than that transmitted and the load pressure is serious, leading to reduced performance; whereas when $\eta<1$, the load pressure is relieved, so both τ(non-disabled) and the network efficiency recover. In Fig.3c, the reaction of $L(t)$ to η is counter-intuitive. This is because only the *non-disabled* nodes are considered in computing the global load in this paper.

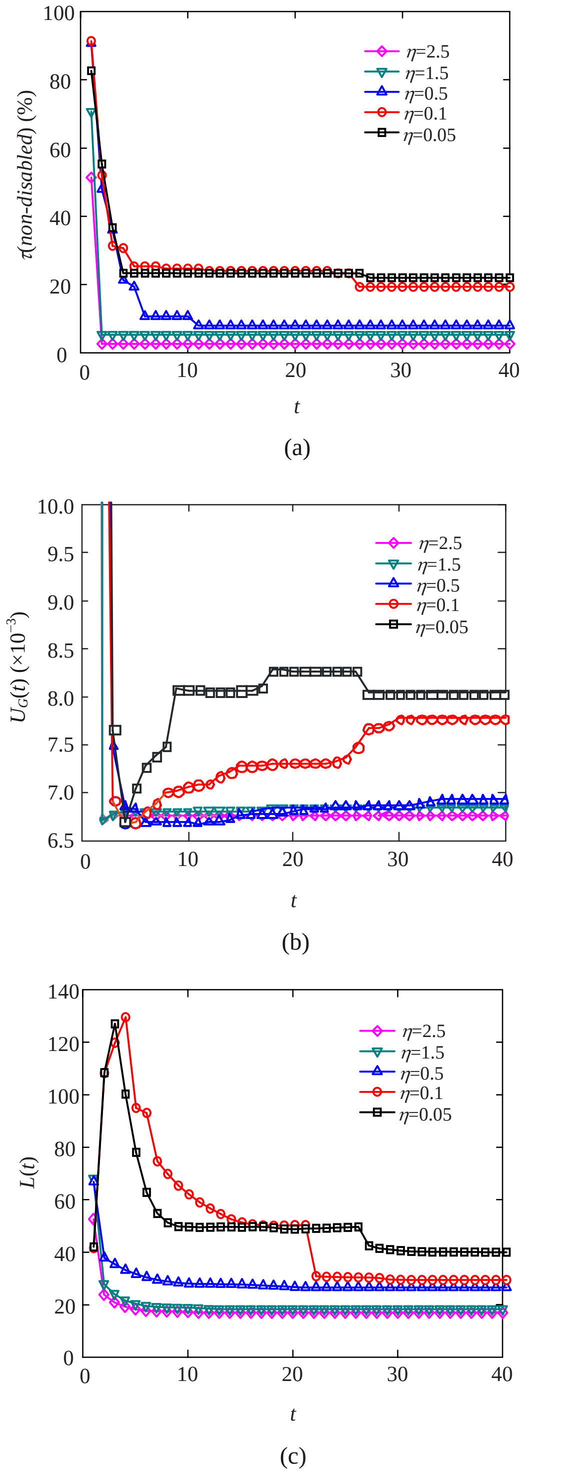

Fig. 3 Effects of η on cascading failures with $\Delta t=\infty$, α=0.2. (a) τ(*non-disabled*) as a function of time *t*; (b) Efficiency $U_G(t)$ of the network as a function of time *t*. (c) Global load $L(t)$ of the network as a function of time *t*

CONCLUSION

We propose a new cascading failure model based on congestion effects. Simulation results showed that the stability of the network declined with the growth of the number of nodes. The congestion propagation process is composed of three phases: slow start, fast propagation, and saturation. An increase in delay time can lead to the decline of Internet stability, and the ratio of generating speed to processing ability of packets has similar effects on congestion propagation.

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