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# Subspace identification for continuous-time errors-in-variables model from sampled data*

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**Abstract:** We study the subspace identification for the continuous-time errors-in-variables model from sampled data. First, the filtering approach is applied to handle the time-derivative problem inherent in continuous-time identification. The generalized Poisson moment functional is focused. A total least squares equation based on this filtering approach is derived. Inspired by the idea of discrete-time subspace identification based on principal component analysis, we develop two algorithms to deliver consistent estimates for the continuous-time errors-in-variables model by introducing two different instrumental variables. Order determination and other instrumental variables are discussed. The usefulness of the proposed algorithms is illustrated through numerical simulation.

**Key words:** System identification, Errors-in-variables, Continuous-time system, Subspace method
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## INTRODUCTION

A continuous-time (CT) system model is required in many applications such as modeling, diagnosis, and control. Interest in CT system identification from sampled data has been increasing in recent years (Garnier *et al.*, 2003; Wang *et al.*, 2004; Young and Garnier, 2006; Garnier and Wang, 2008). There are two different ways to identify the CT model: indirect and direct methods. The indirect approach uses the sampled data to estimate a discrete-time (DT) model first and then converts it into an equivalent CT model, while the direct approach identifies a CT model from the sampled data directly. There are many advantages in identifying the CT models directly from sampled data over the indirect approach, such as the sampling rate problem (Garnier and Wang, 2008). Some practical problems about this topic had been analyzed in (Sung *et al.*, 2001). Identifying a CT model directly implies measurement or generation of the time derivatives of the input-output data. Various methods have been proposed to overcome these time-derivative problems from the view point of signal pre-processing. A broad overview of available techniques can be referred to (Unbehauen and Rao, 1987; 1990; Sinha and Rao, 1991; Garnier *et al.*, 2003). In (Garnier *et al.*, 2003), the performances of these methods were evaluated on simulated examples by Monte Carlo simulations.

A class of multivariable system identification techniques based on the subspace method has recently received considerable attention (Larimore, 1990; Verhaegen and Dewilde, 1992; van Overschee and de Moor, 1994; Li and Qin, 2001; Wang and Qin, 2002; Bauer, 2005). These techniques are based on robust numerical tools such as QR factorization and singular value decomposition (SVD). A state space form for the identified system can be directly derived from input and output data by using subspace methods without nonlinear optimizations. Most of these methods have been developed for DT models. Several

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attempts have been proposed to extend the DT subspace identification to CT identification (Johansson *et al.*, 1999; Bastogne *et al.*, 2001; Haverkamp, 2001; Ohsumi *et al.*, 2002; Li *et al.*, 2003). However, little consideration has been proposed for identifying the CT errors-in-variables (EIV) model from the subspace method.

EIV models have been widely used in many fields such as array signal processing for direction-of-arrival estimation, blind channel equalization, image processing, and environmental modeling (van Huffel and Lemmerling, 2002). The main difficulty of EIV model identification is due to the violation of both input and output signals by the uncertainties or measurement noises. Most of subspace identification methods involve a projection of the future output on the orthogonal complement of the future input. Many methods have been proposed to solve the EIV problem for DT models. Söderström (2007) provided an overview of these methods.

Some attention has been paid to CT EIV model identification (Mahata and Garnier, 2006; Thil *et al.*, 2008); howerer, the subspace identification for the CT EIV model has not yet received enough attention so far. This paper aims to identify a CT EIV model in state-space form directly from sampled input/output data using the recently developed DT subspace identification method. The idea is derived from modifying the discrete subspace identification based on principal component analysis (PCA) proposed in (Li and Qin, 2001; Wang and Qin, 2002) and is associated with the filtering method which is used to cope with the time-derivative problem. A classic filtering approach —generalized Poisson moment functionals (GPMF) (Garnier *et al.*, 2003) is applied. This method has some advantages over other filtering methods (Mercère *et al.*, 2007). In order to cope with the EIV problem, two instrumental variables (IVs) are introduced to eliminate the noise terms: one is the orthogonal projection operator of the GPMF transformation of the high order time derivatives of past inputs; the other is the orthogonal projection operator of the GPMF transformation of the high order time derivatives of past outputs. A discussion of the IVs is made. Order determination and consistency properties of the proposed algorithms are also discussed. The numerical simulation results are in accordance with the conclusions of the discussion and illustrate the effectiveness of the proposed algorithms.

## PROBLEM STATEMENT AND NOTATIONS

Consider the following $n$th order CT linear time-invariant system represented in the state-space form (Mercère *et al.*, 2007):

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\tilde{\boldsymbol{u}}(t) + \boldsymbol{w}(t), \tag{1a}$$

$$\tilde{\boldsymbol{y}}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\tilde{\boldsymbol{u}}(t), \tag{1b}$$

where $\tilde{\boldsymbol{u}}(t) \in \mathbb{R}^m$ and $\tilde{\boldsymbol{y}}(t) \in \mathbb{R}^l$ are the noise-free input and output signals, respectively, $\boldsymbol{x}(t)\in\mathbb{R}^n$ the state vector, and $\boldsymbol{w}(t)\in\mathbb{R}^n$ the noise. We assume that the noise-free $\tilde{\boldsymbol{u}}(t)$ and $\tilde{\boldsymbol{y}}(t)$ are sampled at time instants $\{t_k\}_{k=1}^N$ with $t_k=kT_s$ ($T_s$ is the sampling period). $\boldsymbol{w}(t)$ keeps constant between two consecutive sampling instances: $\boldsymbol{w}(t)=\boldsymbol{w}(kT_s)$, for $kT_s \le t<(k+1)T_s$.

The noise-free sampled input and output signals are contaminated by discrete-time noise sequences $\boldsymbol{l}(t_k)\in\mathbb{R}^m$ and $\boldsymbol{v}(t_k)\in\mathbb{R}^l$, respectively:

$$\boldsymbol{u}(t_k) = \tilde{\boldsymbol{u}}(t_k) + \boldsymbol{l}(t_k), \tag{2a}$$

$$\boldsymbol{y}(t_k) = \tilde{\boldsymbol{y}}(t_k) + \boldsymbol{v}(t_k). \tag{2b}$$

Like in the DT subspace identification problem, we make the following assumptions:

**Assumption 1** ($\boldsymbol{A}$, $\boldsymbol{C}$) is observable and ($\boldsymbol{A}$, $\boldsymbol{B}$) is controllable.

**Assumption 2** The signals $\boldsymbol{u}$ and $\boldsymbol{y}$ are differentiable at least to the ($i$−1)th time derivative with $i>n$.

**Assumption 3** The system is stable.

**Assumption 4** The noises $\boldsymbol{w}(t_k)$, $\boldsymbol{l}(t_k)$ and $\boldsymbol{v}(t_k)$ are stationary white noise sequences with zero mean and independent of past noise-free $\tilde{\boldsymbol{u}}(t)$ and $\tilde{\boldsymbol{y}}(t)$. These noises satisfy the following correlation:

$$E\left(\begin{bmatrix}\boldsymbol{w}(t_q)\\ \boldsymbol{l}(t_q)\\ \boldsymbol{v}(t_q)\end{bmatrix}\begin{bmatrix}\boldsymbol{w}(t_j)\\ \boldsymbol{l}(t_j)\\ \boldsymbol{v}(t_j)\end{bmatrix}^{\mathrm{T}}\right) = \begin{bmatrix}\boldsymbol{R}_{ww} & \boldsymbol{R}_{wl} & \boldsymbol{R}_{wv}\\ \boldsymbol{R}_{lw} & \boldsymbol{R}_{ll} & \boldsymbol{R}_{lv}\\ \boldsymbol{R}_{vw} & \boldsymbol{R}_{vl} & \boldsymbol{R}_{vv}\end{bmatrix}\delta_{qj},$$

where $E\{\cdot\}$ is the mathematical expectation, $\boldsymbol{R}$ represents the covariance matrix, and $\delta_{qj}$ is the Kronecker delta function.

**Assumptions 5** The system order is a priori known.

The goal of CT subspace identification for the EIV model is to estimate the state space matrices [$\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$] up to a similarity transformation from noisy sampled I/O data $\{\boldsymbol{u}(t_k)\}_{k=1}^{N}$, $\{\boldsymbol{y}(t_k)\}_{k=1}^{N}$ with $t_k=kT_s$.

A stacked vector equation can be obtained by computing the $i$th derivative of the state-space equation (Mercère *et al.*, 2007):

$$\boldsymbol{y}_i(t)=\boldsymbol{\Gamma}_i\boldsymbol{x}(t)+\boldsymbol{H}_i\boldsymbol{u}_i(t)-\boldsymbol{H}_i\boldsymbol{l}_i(t)+\boldsymbol{G}_i\boldsymbol{w}_i(t)+\boldsymbol{v}_i(t), \quad (3)$$

where $i$ is a user-defined parameter, $i\geq n-1$. The stacked vector $\boldsymbol{y}_i(t)=[(\boldsymbol{y}(t))^{\mathrm{T}},(\dot{\boldsymbol{y}}(t))^{\mathrm{T}},...,(\boldsymbol{y}^{(i)}(t))^{\mathrm{T}}]^{\mathrm{T}}$ $\boldsymbol{y}^{(i)}(t)$ denotes the $i$th time derivative of $\boldsymbol{y}(t)$. $\boldsymbol{u}_i(t)$, $\boldsymbol{l}_i(t)$, $\boldsymbol{w}_i(t)$ and $\boldsymbol{v}_i(t)$ are defined similarly to $\boldsymbol{y}_i(t)$. The extended observability matrix $\boldsymbol{\Gamma}_i$ and the Toeplitz matrices $\boldsymbol{H}_i$, $\boldsymbol{G}_i$ are defined as

$$\boldsymbol{\Gamma}_i=[\boldsymbol{C}^{\mathrm{T}},(\boldsymbol{CA})^{\mathrm{T}},...,(\boldsymbol{CA}^i)^{\mathrm{T}}]^{\mathrm{T}},$$

$$\boldsymbol{H}_i=\begin{bmatrix}\boldsymbol{D} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{CB} & \boldsymbol{D} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{CAB} & \boldsymbol{CB} & \boldsymbol{D} & \cdots & \boldsymbol{0}\\ \vdots & \vdots & \vdots & & \vdots\\ \boldsymbol{CA}^{i-1}\boldsymbol{B} & \boldsymbol{CA}^{i-2}\boldsymbol{B} & \boldsymbol{CA}^{i-3}\boldsymbol{B} & \cdots & \boldsymbol{D}\end{bmatrix},$$

$$\boldsymbol{G}_i=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{C} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{CA} & \boldsymbol{C} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \vdots & \vdots & \vdots & & \vdots\\ \boldsymbol{CA}^{i-1} & \boldsymbol{CA}^{i-2} & \boldsymbol{CA}^{i-3} & \cdots & \boldsymbol{0}\end{bmatrix}.$$

For $N$ sampled input and output data, Eq.(3) can be extended to

$$\boldsymbol{Y}_{i,1,N}=\boldsymbol{\Gamma}_i\boldsymbol{X}_{1,N}+\boldsymbol{H}_i\boldsymbol{U}_{i,1,N}-\boldsymbol{H}_i\boldsymbol{L}_{i,1,N}+\boldsymbol{G}_i\boldsymbol{W}_{i,1,N}+\boldsymbol{V}_{i,1,N}, \quad (4)$$

with $\boldsymbol{Y}_{i,1,N}=[\boldsymbol{y}_i(t_1), \boldsymbol{y}_i(t_2), ..., \boldsymbol{y}_i(t_N)]$, $\boldsymbol{X}_{1,N}=[\boldsymbol{x}(t_1), \boldsymbol{x}(t_2), ..., \boldsymbol{x}(t_N)]$. The matrices $\boldsymbol{U}_{i,1,N}$, $\boldsymbol{L}_{i,1,N}$, $\boldsymbol{W}_{i,1,N}$ and $\boldsymbol{V}_{i,1,N}$ are constructed in an identical way to $\boldsymbol{Y}_{i,1,N}$.

From Eqs.(3) and (4), the first step to develop the subspace identification algorithm is to measure the successive time derivatives or integrals of I/O data, furthermore $\boldsymbol{Y}_{i,1,N}$ and $\boldsymbol{U}_{i,1,N}$. It seems impossible to directly compute these time derivatives from sampled I/O data. Fortunately, several techniques are specifically developed to deal with these time-derivative problems from the sampled data (Garnier *et al.*, 2003). In this study, we consider a typical approach: generalized Poisson moment functionals.

## GENERALIZED POISSON MOMENT FUNCTIONALS

Three main types of methods are considered to evaluate the time derivatives or integrals of I/O signals. These techniques are linear filters, integral methods and modulating functions. Sinha and Rao (1991) gave an overview of these techniques. In this paper, we introduce the generalized Poisson moment functionals (GPMF) method, which is an approach of the linear filters. Some of its good properties are discussed in Remark 1.

The GPMF approach was developed by Unbehauen and Rao (1987). The $i$th order GPMF transformation of a signal $r(t)$ at time instant $t$ is given by the following convolution product:

$$\mathscr{F}_i\{r(t)\}=\int_0^t r(\tau)f_i(t-\tau)\mathrm{d}\tau, \quad (5)$$

where $f_i(t)=\beta^{i+1}t^i\mathrm{e}^{-\lambda t}/(i!)$. $\lambda\in(0, \infty)$ is a user-defined parameter accounting for the filter cut-off frequency. $\beta\in(0, \infty)$ is the Poisson filter constant. In most of the time, $\lambda=\beta$.

The most important property of the GPMF approach is that the GPMF transformation of the high order time-derivative signals can be expressed by the combination of the lower order GPMF transformation of the signal from the above transformation after a transient period as

$$\mathscr{F}_i^{\,i}\{r(t)\}=\begin{bmatrix}\mathscr{F}_i\{r(t)\}\\ \mathscr{F}_i\{r^1(t)\}\\ \vdots\\ \mathscr{F}_i\{r^i(t)\}\end{bmatrix}=\boldsymbol{\phi}_i\begin{bmatrix}\mathscr{F}_i\{r(t)\}\\ \mathscr{F}_{i-1}\{r(t)\}\\ \vdots\\ \mathscr{F}_0\{r(t)\}\end{bmatrix}=\boldsymbol{\phi}_i\bar{\mathscr{F}}_i\{r(t)\}, \quad (6a)$$

where $\boldsymbol{\phi}_i$ is a constant low triangular weighting matrix related with $\lambda$ and $\beta$. Based on Eq.(6a),

$$\mathscr{L}(\mathscr{F}_i^{\,i}\{r(t)\})=\Delta_i(s)r(s), \quad (6b)$$

where $\mathscr{L}(\cdot)$ is the Laplace transformation operator, and $\Delta_i(s)$ is the filter derived from $\boldsymbol{\phi}_i$.

The $i$th order GPMF transformation of $r(t)$ can be considered as the output of a cascaded low-pass filter chain of $i$+1 identical stages, each with a transfer function given by

$$F(s)=\frac{\beta}{s+\lambda}.$$

Then we can deliver the GPMF transformation $\mathscr{F}_i^i\{r(t)\}$ from the Laplace inverse transformation. The multivariable signal can be extended in a similar way. The detailed transformation description can be referred to (Bastogne *et al.*, 2001).

The digital implementation of the GPMF filter can be attainted by the Runge-Kutta method, Tustin discretization or zero order hold (ZOH) discretization (Bastogne *et al.*, 2001; Garnier *et al.*, 2003). The Tustin and ZOH discretizations of the GPMF transformation have been implemented in CONTSID. Some discussions about the sampling period, filtering order, high time derivative order, and filter cut-off frequency have been presented in (Bastogne *et al.*, 2001). In our study, the ZOH discretization of the GPMF transformation is applied in the simulation.

**Remark 1** Two aspects have to be considered in coping with the time-derivative problem: the initial conditions and the design parameter choice. For a large observation time with the GPMF algorithm, the terms related to the initial conditions can be neglected. The design parameter to be specified by the user is the cut-off frequency of the filter. It is suggested to choose the cut-off frequency a little larger than the frequency bandwidth of the system to be identified (Garnier *et al.*, 2003).

Denote $\mathscr{F}_i\{\cdot\}$ as the GPMF transformation of the $i$th order time derivatives of signals. Applying the GPMF transformation to the CT extended input-output matrix Eq.(4) yields

$$\begin{aligned}\mathscr{F}_i\{\boldsymbol{Y}_{i,1,N}\}=&\boldsymbol{\Gamma}_i\mathscr{F}_i\{\boldsymbol{X}_{1,N}\}+\boldsymbol{H}_i\mathscr{F}_i\{\boldsymbol{U}_{i,1,N}\}-\boldsymbol{H}_i\mathscr{F}_i\{\boldsymbol{L}_{i,1,N}\}\\&+\boldsymbol{G}_i\mathscr{F}_i\{\boldsymbol{W}_{i,1,N}\}+\mathscr{F}_i\{\boldsymbol{V}_{i,1,N}\},\end{aligned}\quad(7)$$

where

$$\mathscr{F}_i\{\boldsymbol{X}_{1,N}\}=\left[\mathscr{F}_i\{\boldsymbol{x}(t_1)\},\mathscr{F}_i\{\boldsymbol{x}(t_2)\},\ldots,\mathscr{F}_i\{\boldsymbol{x}(t_N)\}\right],$$

$$\mathscr{F}_i\{\boldsymbol{Y}_{i,1,N}\}=\begin{bmatrix}\mathscr{F}_i^0\{\boldsymbol{y}(t_1)\} & \mathscr{F}_i^0\{\boldsymbol{y}(t_2)\} & \cdots & \mathscr{F}_i^0\{\boldsymbol{y}(t_N)\}\\ \mathscr{F}_i^1\{\boldsymbol{y}(t_1)\} & \mathscr{F}_i^1\{\boldsymbol{y}(t_2)\} & \cdots & \mathscr{F}_i^1\{\boldsymbol{y}(t_N)\}\\ \vdots & \vdots & & \vdots\\ \mathscr{F}_i^i\{\boldsymbol{y}(t_1)\} & \mathscr{F}_i^i\{\boldsymbol{y}(t_2)\} & \cdots & \mathscr{F}_i^i\{\boldsymbol{y}(t_N)\}\end{bmatrix},$$

$\mathscr{F}_i^b\{\boldsymbol{y}(t_k)\}$ ($b$=0, 1, ..., $i$; $k$=1, 2, ..., $N$) denotes the GPMF transformation with the $i$th order filter of the $b$th time derivative of $\boldsymbol{y}(t_k)$. Other expressions $\mathscr{F}_i\{\boldsymbol{U}_{i,1,N}\}$, $\mathscr{F}_i\{\boldsymbol{L}_{i,1,N}\}$, $\mathscr{F}_i\{\boldsymbol{W}_{i,1,N}\}$ and $\mathscr{F}_i\{\boldsymbol{V}_{i,1,N}\}$ are defined similarly to $\mathscr{F}_i\{\boldsymbol{Y}_{i,1,N}\}$.

## SUBSPACE IDENTIFICATION FOR THE CT EIV MODEL

For the DT EIV identification problem, some researchers have proposed a type of subspace identification algorithm based on PCA (Wang and Qin, 2002; 2006; Huang *et al.*, 2005). The main idea is to obtain the total least squares equation for eliminating the noise terms. Inspired by this idea, we extend the DT subspace identification based on PCA to the CT case. We consider the total least squares equation based on the filtering method. Assuming that the input is sufficiently exciting and that the transformation does not remove any state variable, it is possible to apply this DT identification algorithm to the CT problem by introducing suitable IVs.

First, the filtering-based total least squares equation is introduced. Here we truncate Eq.(7) as follows:

$$\begin{aligned}\mathscr{F}_i\{\boldsymbol{Y}_{i,f+1,N}\}=&\boldsymbol{\Gamma}_i\mathscr{F}_i\{\boldsymbol{X}_{f+1,N}\}+\boldsymbol{H}_i\mathscr{F}_i\{\boldsymbol{U}_{i,f+1,N}\}\\&-\boldsymbol{H}_i\mathscr{F}_i\{\boldsymbol{L}_{i,f+1,N}\}+\boldsymbol{G}_i\mathscr{F}_i\{\boldsymbol{W}_{i,f+1,N}\}+\mathscr{F}_i\{\boldsymbol{V}_{i,f+1,N}\},\end{aligned}\quad(8)$$

where $f$ is a user-defined parameter which satisfies $f$>$n$, and

$$\mathscr{F}_i\{\boldsymbol{Y}_{i,f+1,N}\}=\begin{bmatrix}\mathscr{F}_i^0\{\boldsymbol{y}(t_{f+1})\} & \mathscr{F}_i^0\{\boldsymbol{y}(t_{f+2})\} & \cdots & \mathscr{F}_i^0\{\boldsymbol{y}(t_N)\}\\ \mathscr{F}_i^1\{\boldsymbol{y}(t_{f+1})\} & \mathscr{F}_i^1\{\boldsymbol{y}(t_{f+2})\} & \cdots & \mathscr{F}_i^1\{\boldsymbol{y}(t_N)\}\\ \vdots & \vdots & & \vdots\\ \mathscr{F}_i^i\{\boldsymbol{y}(t_{f+1})\} & \mathscr{F}_i^i\{\boldsymbol{y}(t_{f+2})\} & \cdots & \mathscr{F}_i^i\{\boldsymbol{y}(t_N)\}\end{bmatrix}.$$

We can consider $\mathscr{F}_i\{\boldsymbol{Y}_{i,f+1,N}\}$, $\mathscr{F}_i\{\boldsymbol{U}_{i,f+1,N}\}$, $\mathscr{F}_i\{\boldsymbol{L}_{i,f+1,N}\}$, $\mathscr{F}_i\{\boldsymbol{W}_{i,f+1,N}\}$ and $\mathscr{F}_i\{\boldsymbol{V}_{i,f+1,N}\}$ as the GPMF transformations of the future I/O data and noises. In Eq.(8), by moving the second term of the right-hand side to the left-hand side, we obtain the following total least squares equation form:

$$(\boldsymbol{I}-\boldsymbol{H}_i)\begin{bmatrix}\mathscr{F}_i\{\boldsymbol{Y}_{i,f+1,N}\}\\ \mathscr{F}_i\{\boldsymbol{U}_{i,f+1,N}\}\end{bmatrix}=\boldsymbol{\Gamma}_i\mathscr{F}_i\{\boldsymbol{X}_{f+1,N}\}-\boldsymbol{H}_i\mathscr{F}_i\{\boldsymbol{L}_{i,f+1,N}\}+\boldsymbol{G}_i\mathscr{F}_i\{\boldsymbol{W}_{i,f+1,N}\}+\mathscr{F}_i\{\boldsymbol{V}_{i,f+1,N}\}. \quad (9)$$

Pre-multiplying both sides of Eq.(9) with $\boldsymbol{\Gamma}_i^{\perp}$ (i.e., the orthogonal complement of the extended observability matrix $\boldsymbol{\Gamma}_i$) leads to

$$\begin{bmatrix}\boldsymbol{\Gamma}_i^{\perp}\\ -\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{\perp}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathscr{F}_i\{\boldsymbol{Y}_{i,f+1,N}\}\\ \mathscr{F}_i\{\boldsymbol{U}_{i,f+1,N}\}\end{bmatrix}=(\boldsymbol{\Gamma}_i^{\perp})^{\mathrm{T}}(-\boldsymbol{H}_i\mathscr{F}_i\{\boldsymbol{L}_{i,f+1,N}\}+\boldsymbol{G}_i\mathscr{F}_i\{\boldsymbol{W}_{i,f+1,N}\}+\mathscr{F}_i\{\boldsymbol{V}_{i,f+1,N}\}). \quad (10)$$

To remove the noise term $-\boldsymbol{H}_i\mathscr{F}_i\{\boldsymbol{L}_{i,f+1,N}\}+\boldsymbol{G}_i\mathscr{F}_i\{\boldsymbol{W}_{i,f+1,N}\}+\mathscr{F}_i\{\boldsymbol{V}_{i,f+1,N}\}$, the IV can be used. According to the mathematical rigour of statistical properties, by multiplying Eq.(10) with $\boldsymbol{\xi}/(N-f)$,

$$\frac{1}{N-f}\begin{bmatrix}\boldsymbol{\Gamma}_i^{\perp}\\ -\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{\perp}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathscr{F}_i\{\boldsymbol{Y}_{i,f+1,N}\}\\ \mathscr{F}_i\{\boldsymbol{U}_{i,f+1,N}\}\end{bmatrix}\cdot\boldsymbol{\xi}=\frac{(\boldsymbol{\Gamma}_i^{\perp})^{\mathrm{T}}}{N-f}\cdot\left(-\boldsymbol{H}_i\mathscr{F}_i\{\boldsymbol{L}_{i,f+1,N}\}+\boldsymbol{G}_i\mathscr{F}_i\{\boldsymbol{W}_{i,f+1,N}\}+\mathscr{F}_i\{\boldsymbol{V}_{i,f+1,N}\}\right)\boldsymbol{\xi}, \quad (11)$$

where $\boldsymbol{\xi}$ is the IV, which should satisfy the following conditions (Huang *et al.*, 2005):

$$\lim_{N\to\infty}\frac{1}{N-f}(-\boldsymbol{H}_i\mathscr{F}_i\{\boldsymbol{L}_{i,f+1,N}\}+\boldsymbol{G}_i\mathscr{F}_i\{\boldsymbol{W}_{i,f+1,N}\}+\mathscr{F}_i\{\boldsymbol{V}_{i,f+1,N}\})\cdot\boldsymbol{\xi}=\boldsymbol{0}, \quad (12)$$

$$\lim_{N\to\infty}\frac{1}{N-f}\begin{bmatrix}\mathscr{F}_i\{\boldsymbol{Y}_{i,f+1,N}\}\\ \mathscr{F}_i\{\boldsymbol{U}_{i,f+1,N}\}\end{bmatrix}\cdot\boldsymbol{\xi}\neq\boldsymbol{0}. \quad (13)$$

This means that the IV should be uncorrelated with the GPMF transformation of future noises and sufficiently correlated with the informative GPMF transformations of future input-output data.

Several suggestions about the IVs are proposed in the literature (Johansson *et al.*, 1999; Mercère *et al.*, 2007). The details will be discussed in the following section. In this study, we adopt the orthogonal projection operator of the GPMF transformation of the high order time derivatives of past inputs or outputs, i.e.,

$$\boldsymbol{\xi}_{\Pi_{\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}}}=\Pi_{\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}}=(\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\})^{\mathrm{T}}\cdot(\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}(\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\})^{\mathrm{T}})^{\dagger}\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}, \quad (14)$$

$$\boldsymbol{\xi}_{\Pi_{\mathscr{F}_j\{\boldsymbol{Y}_{j,1,N-f}\}}}=\Pi_{\mathscr{F}_j\{\boldsymbol{Y}_{j,1,N-f}\}}=(\mathscr{F}_j\{\boldsymbol{Y}_{j,1,N-f}\})^{\mathrm{T}}\cdot(\mathscr{F}_j\{\boldsymbol{Y}_{j,1,N-f}\}(\mathscr{F}_j\{\boldsymbol{Y}_{j,1,N-f}\})^{\mathrm{T}})^{\dagger}\mathscr{F}_j\{\boldsymbol{Y}_{j,1,N-f}\}, \quad (15)$$

where $j$ is a user-defined parameter which satisfies $j>i+n+1$, $\Pi$ denotes the orthogonal projection operator, the superscript † denotes the Moore-Penrose pseudo inverse, and

$$\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}=\begin{bmatrix}\mathscr{F}_j^0\{\boldsymbol{u}(t_1)\} & \mathscr{F}_j^0\{\boldsymbol{u}(t_2)\} & \cdots & \mathscr{F}_j^0\{\boldsymbol{u}(t_{N-f})\}\\ \mathscr{F}_j^1\{\boldsymbol{u}(t_1)\} & \mathscr{F}_j^1\{\boldsymbol{u}(t_2)\} & \cdots & \mathscr{F}_j^1\{\boldsymbol{u}(t_{N-f})\}\\ \vdots & \vdots & & \vdots\\ \mathscr{F}_j^j\{\boldsymbol{u}(t_1)\} & \mathscr{F}_j^j\{\boldsymbol{u}(t_2)\} & \cdots & \mathscr{F}_j^j\{\boldsymbol{u}(t_{N-f})\}\end{bmatrix}.$$

$\mathscr{F}_j\{\boldsymbol{Y}_{j,1,N-f}\}$ has a similar form to $\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}$.

According to Assumption 4 and the properties of GPMF transformation, the GPMF transformation of future white noises will be uncorrelated with the GPMF transformation of the high order time derivatives of past inputs or outputs,

$$\lim_{N\to\infty}\frac{1}{N-f}(-\boldsymbol{H}_i\mathscr{F}_i\{\boldsymbol{L}_{i,f+1,N}\}+\boldsymbol{G}_i\mathscr{F}_i\{\boldsymbol{W}_{i,f+1,N}\}+\mathscr{F}_i\{\boldsymbol{V}_{i,f+1,N}\})(\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\})^{\mathrm{T}}=\boldsymbol{0}, \quad (16)$$

$$\lim_{N\to\infty}\frac{1}{N-f}(-\boldsymbol{H}_i\mathscr{F}_i\{\boldsymbol{L}_{i,f+1,N}\}+\boldsymbol{G}_i\mathscr{F}_i\{\boldsymbol{W}_{i,f+1,N}\}+\mathscr{F}_i\{\boldsymbol{V}_{i,f+1,N}\})(\mathscr{F}_j\{\boldsymbol{Y}_{j,1,N-f}\})^{\mathrm{T}}=\boldsymbol{0}. \quad (17)$$

Then Eq.(13) is satisfied with the IVs $\boldsymbol{\xi}_{\Pi_{\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}}}$ and $\boldsymbol{\xi}_{\Pi_{\mathscr{F}_j\{\boldsymbol{Y}_{j,1,N-f}\}}}$. Now that $\boldsymbol{\xi}_{\Pi_{\mathscr{F}_j\{\boldsymbol{Y}_{j,1,N-f}\}}}$ is similar to $\boldsymbol{\xi}_{\Pi_{\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}}}$, the following detailed procedure will be described using $\boldsymbol{\xi}_{\Pi_{\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}}}$ only. Similar steps will be followed for $\xi_{\Pi_{\mathscr{F}_j\{\mathbf{Y}_{j,1,N-f}\}}}$.

Substituting $\xi_{\Pi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}}}$ for $\xi$ in Eq.(11) if $N$ is large enough yields

$$\begin{bmatrix} \boldsymbol{\Gamma}_i^{\perp} \\ -\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{\perp} \end{bmatrix}^{\mathrm{T}} \frac{1}{N-f}\begin{bmatrix} \mathscr{F}_i\{\mathbf{Y}_{i,f+1,N}\} \\ \mathscr{F}_i\{\mathbf{U}_{i,f+1,N}\} \end{bmatrix}\Pi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}} = \mathbf{0}. \quad (18)$$

If the following Assumption 6

$$\operatorname{rank}\left(\begin{bmatrix} \mathscr{F}_i\{\mathbf{Y}_{i,f+1,N}\} \\ \mathscr{F}_i\{\mathbf{U}_{i,f+1,N}\} \end{bmatrix}\Pi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}}\right) = l(i+1)+n \quad (19)$$

is satisfied, then from Eq.(18), it is known that the orthogonal column space of $\frac{1}{N-f}\begin{bmatrix} \mathscr{F}_i\{\mathbf{Y}_{i,f+1,N}\} \\ \mathscr{F}_i\{\mathbf{U}_{i,f+1,N}\} \end{bmatrix}$ $\cdot\Pi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}}$ should be equal to the column space of $\begin{bmatrix} \boldsymbol{\Gamma}_i^{\perp} \\ -\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{\perp} \end{bmatrix}$. This is the backbone of the subspace identification based on PCA. In order to deliver the orthogonal column space of $\frac{1}{N-f}\begin{bmatrix} \mathscr{F}_i\{\mathbf{Y}_{i,f+1,N}\} \\ \mathscr{F}_i\{\mathbf{U}_{i,f+1,N}\} \end{bmatrix}$ $\cdot\Pi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}}$ we perform SVD:

$$\frac{1}{N-f}\begin{bmatrix} \mathscr{F}_i\{\mathbf{Y}_{i,f+1,N}\} \\ \mathscr{F}_i\{\mathbf{U}_{i,f+1,N}\} \end{bmatrix}\Pi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}} = [\boldsymbol{K}_1 \;\; \boldsymbol{K}_2]\begin{bmatrix} \boldsymbol{\Lambda}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{O}_1 \\ \boldsymbol{O}_2 \end{bmatrix}. \quad (20)$$

The following relation can be obtained by combing Eqs.(18) and (20) when Assumption 6 is satisfied:

$$\begin{bmatrix} \boldsymbol{\Gamma}_i^{\perp} \\ -\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{\perp} \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}_{21} \\ \boldsymbol{K}_{22} \end{bmatrix}\boldsymbol{Q}, \quad (21)$$

where $\boldsymbol{Q}$ is any constant nonsingular matrix with an appropriate dimension. $\boldsymbol{K}_{21}$ and $\boldsymbol{K}_{22}$ are parts of $\boldsymbol{K}_2$ with appropriate dimensions. Choosing $\boldsymbol{Q}$ as an identity matrix, the extended observability matrix $\boldsymbol{\Gamma}_i$ and Toeplitz matrix $\boldsymbol{H}_i$ can be estimated as

$$\boldsymbol{\Gamma}_i = \boldsymbol{K}_{21}^{\perp}, \quad (22)$$

$$-\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{\perp} = \boldsymbol{K}_{22}. \quad (23)$$

Then the system matrices ($\boldsymbol{A}$, $\boldsymbol{C}$) can be extracted:

$$\boldsymbol{A} = (\boldsymbol{\Gamma}_i(1:li,:))^{\perp}(\boldsymbol{\Gamma}_i(l+1:l(i+1),:)), \quad (24)$$

$$\boldsymbol{C} = \boldsymbol{\Gamma}_i(1:l,:). \quad (25)$$

Here we adopt the notation used in the Matlab package. For example, $\boldsymbol{U}(1{:}l,{:})$ denotes the first $l$ rows of a matrix $\boldsymbol{U}$. From Eq.(23) and the estimates of ($\boldsymbol{A}$, $\boldsymbol{C}$), the ($\boldsymbol{B}$, $\boldsymbol{D}$) can be estimated as in the DT identification case from the least squares algorithm (Wang and Qin, 2002).

From the above steps, we can estimate the system matrices from the IV $\xi_{\Pi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}}}$. Similar procedures can be performed by replacing $\xi_{\Pi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}}}$ with $\xi_{\Pi_{\mathscr{F}_j\{\mathbf{Y}_{j,1,N-f}\}}}$. We call these two algorithms with different IVs CORTPCAGPMF (Continuous-time system identification based on ORThogonal projection and PCA by Generalized Poisson Moment Functionals, the IV is $\xi_{\Pi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}}}$) and CORTPCAGPMFY (the IV is $\xi_{\Pi_{\mathscr{F}_j\{\mathbf{Y}_{j,1,N-f}\}}}$), respectively.

**Remark 2** The order of plant is assumed a priori known, though it may be determined from the SVD of Eq.(20) by Assumption 6. The details are referred to the following discussion. The future horizon $f$ should satisfy $f>n$. Some guidelines for choosing the user parameters such as $i$, $j$, $\lambda$ have been discussed in (Bastogne *et al.*, 2001). However, it will lead to numerical performance corruption using many approximations of time derivative. So the user-defined parameters ($i$, $j$) should not be set large. The most popular way to choose the user parameters is an iterative computation procedure.

**Remark 3** Similar to DT identification (Li and Qin, 2001; Wang and Qin, 2002), the IV may be considered as directing the GPMF transformation of the high order time derivatives of past inputs $(\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\})^{\mathrm{T}}$. The IV $\Pi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}}$ can be considered as adding an optimal column weighting to the IV $(\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\})^{\mathrm{T}}$. For the DT identification case, refer to (Huang *et al.*, 2005; Wang and Qin, 2006) for more details. The following simulation results also show that the IVs with an optimal column weighting can give better performance than those without column weighting. The same situation appears for other IVs.

## DISCUSSIONS

### Order determination

From Assumption 6, to decide the system order, we can choose the number of the non-zero singular values (the order of $\boldsymbol{\Lambda}_1$) from the SVD step in Eq.(20). We then obtain the system order by subtracting $l(i+1)$ from the order of $\boldsymbol{\Lambda}_1$. However, in most of the time, the form of Eq.(20) cannot be obtained from the SVD step. There is no zero singular value. In this situation, we can select the number of the relatively large values as the order of $\boldsymbol{\Lambda}_1$. However, this choice of order is very sensitive to the noise level.

Two important aspects should be considered for this determination. First, the differences between these singular values may become so great that fewer relatively large singular values will be selected. This leads to the 'small number' problem. On the other hand, the differences between these singular values may become small. In this case, more singular values may be selected as the relatively large values, which will lead to the 'large number' problem. Hence, the criterion of selecting the number of relatively large singular values is not favorable in a noisy framework. In some cases, other methods may be needed to determine the system order. There are some discussions for DT subspace identification (Li and Qin, 2001; Wang and Qin, 2002). However, few efforts have been made for the order determination of CT subspace identification. Future work will focus on this topic.

### Consistency and input conditions

The consistency properties of DT subspace identification have been researched in (Wang and Qin, 2002; Bauer, 2005). The consistency properties are related to the input conditions. In our algorithms, in order to obtain the consistent estimates, for all IVs $\boldsymbol{\xi}$'s, Assumption 6 must be satisfied:

$$\operatorname{rank}\left(\begin{bmatrix}\mathscr{F}_j\{\mathbf{Y}_{i,f+1,N}\}\\ \mathscr{F}_j\{\boldsymbol{U}_{i,f+1,N}\}\end{bmatrix}\cdot\boldsymbol{\xi}\right)=l(i+1)+n.$$

This condition guarantees the consistency of estimates by these algorithms. The detailed input condition is not easy to express explicitly, especially for CT identification. However, simulation results indicate that most classic input signals such as pseudo random binary sequences could give a satisfactory performance.

### Instrumental variables

Some IVs (i.e., the past input) satisfy the conditions of Eqs.(12) and (13) (Johansson *et al.*, 1999; Mercère *et al.*, 2007). However, simulation results show that using the past input as the IV is not suitable for this type of subspace identification based on PCA. In this subsection, another two IVs that may be appropriate for this type of subspace identification are introduced:

(1) The GPMF transformation of the high order time derivatives of past inputs. This IV is used for comparison with $\boldsymbol{\xi}_{\Pi_{\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}}}$. It can be constructed as

$$\boldsymbol{\xi}_{\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}}=(\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\})^{\mathrm{T}}. \tag{26}$$

(2) The orthogonal projection operator of the GPMF transformation of the high order time derivatives of combined past input-outputs. Similar to the DT subspace identification based on PCA (Wang and Qin, 2002), the following IV may be derived from the GPMF transformation of the high order time derivatives of combined past input-outputs:

$$\boldsymbol{Z}_{j,1,N-f}=[(\mathscr{F}_j\{\mathbf{Y}_{j,1,N-f}\})^{\mathrm{T}}\ \ (\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\})^{\mathrm{T}}]^{\mathrm{T}}.$$

However, we consider the orthogonal projection operator form

$$\boldsymbol{\xi}_{\Pi_{\mathscr{F}_j\{\boldsymbol{Z}_{j,1,N-f}\}}}=\Pi_{\mathscr{F}_j\{\boldsymbol{Z}_{j,1,N-f}\}}. \tag{27}$$

The algorithms with different IVs based on previous discussions are abbreviated as CPCAGPMF (Continuous-time system identification based on PCA by Generalized Poisson Moment Functionals, the IV is $\boldsymbol{\xi}_{\mathscr{F}_j\{\boldsymbol{U}_{j,1,N-f}\}}$) and CORTPCAGPMFZ (the IV is $\boldsymbol{\xi}_{\Pi_{\mathscr{F}_j\{\boldsymbol{Z}_{j,1,N-f}\}}}$). It can be seen from the following simulation that the performances of CORTPCAGPMFZ and CPCAGPMF are not so good as those of CORTPCAGPMF and CORTPCAGPMFY. The possible reason for a quite different performance among different algorithms with different IVs may be due to the numerical problem in PCA computation.

SIMULATIONS

A numerical example was simulated to illustrate the usefulness of the proposed algorithms and to verify the discussions in the previous section. The example is a third-order 2×2 system shown in CONTSID (Garnier *et al.*, 2006). The input measure noise was added. The example is an EIV problem.

The following third-order state space system is considered:

$$\dot{\boldsymbol{x}}(t)=\begin{bmatrix}0 & 1 & 0\\ -3 & -2 & -1\\ -1 & -2 & -1\end{bmatrix}\boldsymbol{x}(t)+\begin{bmatrix}1 & 1\\ 2 & 1\\ 1 & 2\end{bmatrix}\boldsymbol{u}(t),$$

$$\boldsymbol{y}(t)=\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\boldsymbol{x}(t)+\boldsymbol{v}(t).$$

The noise-free input was a pseudo random binary sequence of size $N$=2000. The noise contaminating the input signals was a white Gaussian noise with a variance of 0.01. A white Gaussian noise was added on output such that $SNR$=20 dB. The initial state vector was equal to zero. The sampling period was chosen as 0.1 s. In order to illustrate the discussion in the previous section, the four algorithms CORTPCAGPMF, CPCAGPMF, CORTPCAGPMFY and CORTPCAGPMFZ were applied for this simulation. The filter order $i$=3, the IV filter order $j$=9, and the past input and output were built up with $f$=5. The filter cut-off frequency was set as $\lambda$=3 in the CONTSID. The parameters were the same for all these four algorithms. Fifty Monte Carlo simulations were used to evaluate the performances of these algorithms.

The singular values of the numerical example are depicted in Fig.1. First, we consider these three algorithms: CORTPCAGPMF, CORTPCAGPMFY and CORTPCAGPMFZ. As Fig.1 shows, the number of relatively large singular values was 11. The term $l(i+1)$ was equal to 8, and thus the order of the system should be considered as 3, equal to the true order value. From Fig.1, a false order will be obtained by CPCAGPMF. However, the order determination of CORTPCAGPMF, CORTPCAGPMFY and CORTPCAGPMFZ algorithms are still not clear, as the plots show.

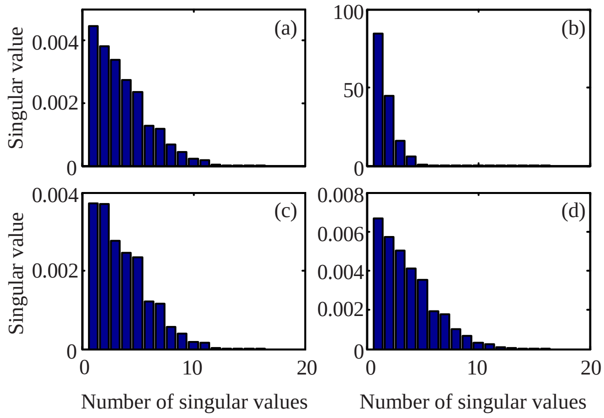


**Fig.1 Singular value plots**

(a) CORTPCAGPMF; (b) CPCAGPMF; (c) CORTPCAGPMFY; (d) CORTPCAGPMFZ

Fig.2 plots the poles of the estimated plant by using CORTPCAGPMF, CPCAGPMF, CORTPCAGPMFY and CORTPCAGPMFZ. CORTPCAGPMF, CORTPCAGPMFY and CORTPCAGPMFZ gave satisfactory estimates, while CPCAGPMF led to poor results. Fig.3 gives the Bode diagrams of the estimates of these four algorithms. This figure also shows that CORTPCAGPMF, CORTPCAGPMFY and CORTPCAGPMFZ gave better estimates than CPCAGPMF. CPCAGPMF still attained poor estimates for the frequency response, as expected in Remark 3. It can be seen that CORTPCAGPMF had a similar performance to CORTPCAGPMFY. CORTPCAGPMF and CORTPCAGPMFY obtained more accurate estimates than CORTPCAGPMFZ.

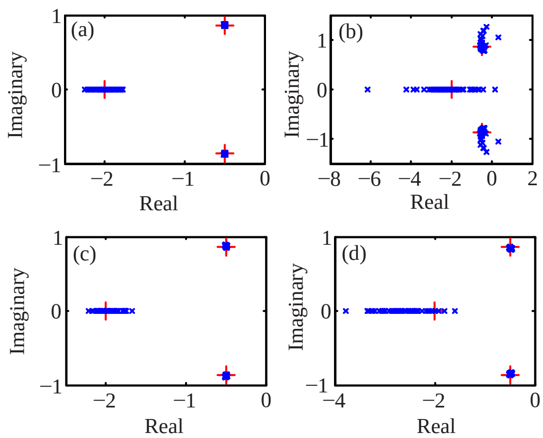


**Fig.2 Pole plots of the plant estimated by using CORTPCAGPMF (a), CPCAGPMF (b), CORTPCAGPMFY (c) and CORTPCAGPMFZ (d)**

The true poles are denoted by '+'

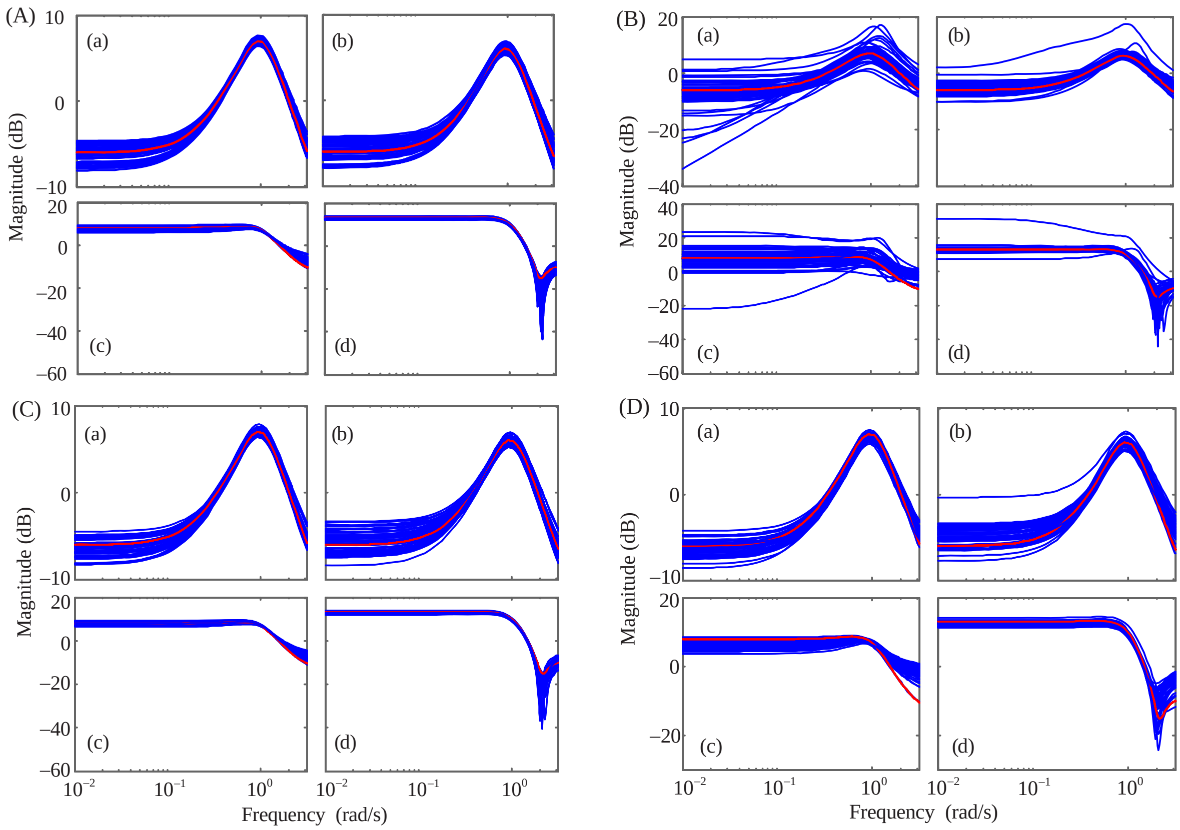


**Fig.3 Bode plots of the plant estimated by using CORTPCAGPMF (A), CPCAGPMF (B), CORTPCAGPMFY (C) and CORTPCAGPMFZ (D)**

The true Bode plot is covered by 50 curves. In each subfigure, (a) from input 1 to output 1, (b) from input 2 to output 1, (c) from input 1 to output 2, (d) from input 2 to output 2

From the simulation results, it can be seen that the performances by using the IVs $\xi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}}$ and $\xi_{\Pi_{\mathscr{F}_j\{\mathbf{Z}_{j,1,N-f}\}}}$ are not so good as those by using $\xi_{\Pi_{\mathscr{F}_j\{\mathbf{U}_{j,1,N-f}\}}}$ and $\xi_{\Pi_{\mathscr{F}_j\{\mathbf{Y}_{j,1,N-f}\}}}$.

## CONCLUSION

Subspace identification algorithms for the continuous-time errors-in-variables model from sampled data are proposed in this paper. In order to overcome the time-derivative problem inherent in continuous-time identification, we propose the filtering method GPMF. The backbone of these algorithms is to deliver the total least squares equation based on the GPMF transformation. Two IVs, the orthogonal projection operator of the GPMF transformation of the high order time derivatives of past inputs and the orthogonal projection operator of the GPMF transformation of the high order time derivatives of past outputs, are used to eliminate the noise terms. A numerical simulation study and an industrial example illustrate the capability of the proposed algorithms. Some discussions about another two IVs, order determination and consistency properties, are presented. As the simulation results show, the instrumental variables of CORTPCAGPMF and CORTPCAGPMFY algorithms are more suitable for this type of subspace identification method based on PCA.

It is also shown that the order determination method proposed in this paper is not good enough. Future work will focus on order determination and the color noise problem.

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