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Direct and relaxation methods for soil-structure interaction due to tunneling*

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Abstract: This paper deals with the problem of tunneling effects on existing buildings. The direct solution, using the condensation method, is presented. This method allows the structural and geotechnical engineers to treat the problem separately and then assemble a relatively small matrix that can be solved directly, even within a spreadsheet. There are certain concerns that the resultant matrix may be ill-conditioned when the structure is very stiff. This paper suggests an alternative method that essentially relaxes the system from an infinitely rigid structure solution. As such, it does not encounter the problems associated with stiff systems. The two methods are evaluated for an example problem of tunneling below a framed structure. It is found that while the direct method may fail to predict reasonable values when the structure is extremely rigid, the alternative method is stable. The relaxation method can therefore be used in cases where there are concerns about the reliability of a direct solution.

Key words: Tunneling, Soil-structure interaction, Foundation settlements, Excavations, Soil mechanics
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1 Introduction

The evaluation of tunneling effects on existing structure is an important process that engineers need to confront when dealing with the construction of new tunnels. Nomograms for relatively simple structures, such as buried pipelines (Klar *et al*., 2005; 2007; 2008) or beam like structures (Potts and Addenboorke, 1997; Franzius *et al*., 2006) can be used for a first (crude) estimation of the effects of tunneling. However, more often than not, a more representative solution, which entails detailed characteristics of the structure, is required.

Obtaining a solution for a general structure may pose a problem, as the solution involves significant soil structure interaction, and falls exactly in between the two disciplines of geotechnical and structural engineering. It would be desirable to have a procedure that would allow a division of tasks between the two disciplines, such that the structural engineer would deal elaborately with the structure and the geotechnical engineer with the soil, and any coupling between the two would be conducted by a simple process that could even be performed on a spreadsheet (e.g., Microsoft Excel).

One possible method, suggested by Attewell *et al.* (1986), for such decoupling is to conduct matrix condensation for the structure and the soil for a direct solution of the foundation displacement:

$$(\boldsymbol{S}^{\mathrm{c}}+\boldsymbol{K}^{\mathrm{s}})\boldsymbol{u}=\boldsymbol{K}^{\mathrm{s}}\boldsymbol{u}^{\mathrm{gf}}, \tag{1}$$

where $\boldsymbol{S}^{\mathrm{c}}$ is the condensed stiffness matrix of the structure, $\boldsymbol{K}^{\mathrm{s}}$ is the condensed stiffness matrix of the soil, $\boldsymbol{u}$ is the resultant displacement of the foundation system and $\boldsymbol{u}^{\mathrm{gf}}$ is the green field displacement (i.e., the displacement due to the tunnel if the structure did not exist). Vector $\boldsymbol{u}$ includes only the foundations' degree

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of freedom, and hence is limited in size (compared to the overall degrees of freedom involved in the structure). Consequently, if $\boldsymbol{S}^{\mathrm{c}}$ and $\boldsymbol{K}^{\mathrm{s}}$ can be established individually by the structural and geotechnical engineers, respectively, Eq. (1) can be solved, even within a spreadsheet.

$\boldsymbol{S}^{\mathrm{c}}$ and $\boldsymbol{K}^{\mathrm{s}}$ can be obtained by separate analytical solutions for the soil and structure, or by two separate finite element analyses, one for the soil and the other for the structure. Fig. 1 illustrates the process of obtaining the condensed stiffness matrix from finite element models. Each column in the condensed stiffness matrix of the structure is the reaction forces that develop in the fixed foundations due to unit displacement of one of the foundations. That is, $\boldsymbol{S}_{ij}^{\mathrm{c}}$ is the reaction in foundation i due to unit displacement of foundation j. The condensed stiffness matrix can also be obtained directly from manipulation of the complete structure matrix (Weaver and Johnston, 1987). The condensed stiffness matrix of the soil can be obtained by a similar procedure to that of the structure, or by inverting the soil's flexibility matrix $\boldsymbol{G}$.

The construction of the condensed matrices can be completely decoupled; the structural engineer can easily build $\boldsymbol{S}^{\mathrm{c}}$ using relevant finite elements codes (Lusas, 2007; Oasys, 2007), while the geotechnical engineer can build $\boldsymbol{K}^{\mathrm{s}}$ with finite difference/element codes suitable for geotechnical engineering (Plaxis, 2006; Itasca, 2005). The two matrices can then be combined for the solution of the foundation displacement using a mathematical program or even a spreadsheet. Once the foundation displacements are obtained, the structural engineer can return to his finite element model and evaluate the stresses (or strains) in the structure by displacing the restrained foundations by $\boldsymbol{u}$.

Attewell *et al*. (1986) reviewed this approach and noted that solutions based on Eq. (1) might be unreliable with significant numerical errors since ($\boldsymbol{S}^{\mathrm{c}}$+$\boldsymbol{K}^{\mathrm{s}}$) may be ill-conditioned when the structure is stiff. This is because $\boldsymbol{S}^{\mathrm{c}}$ is singular, and may dominate when the structure is very stiff compared to the soil.

To overcome this problem Attewell *et al*. (1986) suggested applying $\boldsymbol{K}^{\mathrm{s}}\boldsymbol{u}^{\mathrm{gf}}$ as a force vector to the interface (or connection) nodes of a complete finite element model that includes both the structure and the soil. This, however, eradicates the advantage of the decoupled procedure that allows the separate treatments of structure and soil by different engineers.

This paper presents a relaxation scheme that allows the desired decoupling but does not result in an ill-conditioned system. The relaxation method can be used for solution of the system or as a tool to evaluate the correctness of solutions obtained from Eq. (1).

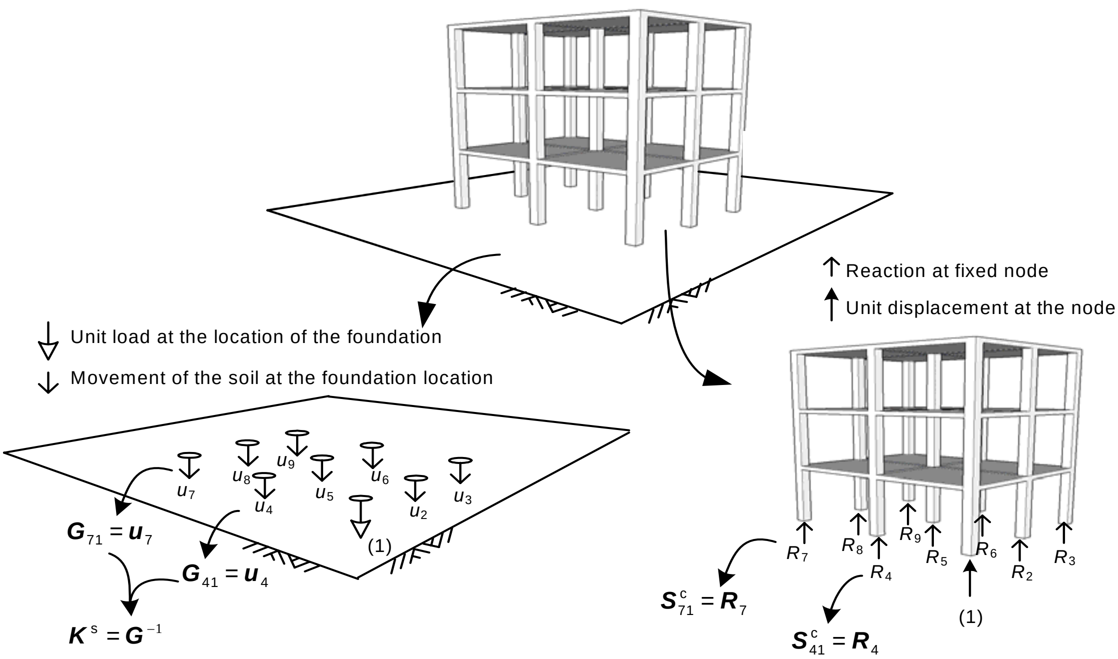

Fig. 1 Decoupling and condensation of the soil and structure behavior

The paper is composed of three main sections. The first section presents the assumptions and governing equations which lead to the set of equations of the direct method. The second section presents the suggested relaxation method, which avoids the problems associated with the direct method. The third section evaluates the suggested relaxation method through a comparison between the two methods.

2 Formulation

In the following formulations boldface capitals represent matrices and boldface letters represent vectors. When they appear with a subscript they represent individual values in the matrices or vectors.

The formulation is based on the following four assumptions: (1) the structure is linear elastic; (2) the structure is in contact with the soil at all time; (3) the soil is a linear elastic continuum; and (4) the tunnel itself is not affected by the structure.

Assumptions (1) and (2) are generally legitimate, particularly for heavy buildings, while assumptions (3) and (4) may be less legitimate. Soil behavior is neither linear nor elastic, and therefore assumption (3) may hinder the validity of the solution. Consequently, solutions based on such an assumption may only be used for crude evaluation of the true response. Nonetheless, since this assumption is involved both in the direct method and in the suggested relaxation method, it does not constitute a limitation on the comparison conducted in this study. Some consideration of soil nonlinearity may be achieved by employing a linear equivalent scheme. However, this requires repeated analysis of the soil finite element model for each iteration to update the condensed stiffness matrix of the soil. Consequently, such a scheme will be computationally expensive and inconvenient, and would probably not justify the additional time invested in the solution. Therefore, the suggested method is presented solely for the case of linear elastic soil.

The structure behavior may be represented as follows:

$$\boldsymbol{f}^{\mathrm{c}}=\boldsymbol{S}^{\mathrm{c}}\boldsymbol{u}, \tag{2}$$

where $\boldsymbol{f}^{\mathrm{c}}$ are the forces acting on the structure, and $\boldsymbol{u}$ is the displacement of the foundations. $\boldsymbol{S}^{\mathrm{c}}$ is the condensed stiffness matrix for the structure foundations nodes. Note that unlike the complete stiffness matrix of the structure $\boldsymbol{S}^{\mathrm{c}}$ is fully populated.

The displacement in the soil continuum, $\boldsymbol{u}^{\mathrm{C}}$, can be represented using flexibility coefficients as

$$\boldsymbol{u}^{\mathrm{C}}=\boldsymbol{G}\boldsymbol{f}^{\mathrm{s}}, \tag{3}$$

where $\boldsymbol{f}^{\mathrm{s}}$ are the forces acting on the soil, and arguments $\boldsymbol{G}_{ij}$ give the displacement at point i due to unit loading at point j. In the simple case of a homogenous half space, $\boldsymbol{G}_{ij}$ can be constructed analytically using the fundamental solutions of Boussinesq, Cerutti and Mindlin (Poulos and Davis, 1974). For more complicated cases $\boldsymbol{G}_{ij}$ can be constructed by loading simulations in a finite element model of the ground system (without any structure). Writing Eq. (3) for a given point results in

$$\boldsymbol{u}_i^{\mathrm{C}}=\sum_j \boldsymbol{G}_{ij}\boldsymbol{f}_j^{\mathrm{s}}. \tag{4}$$

This soil continuum displacement can be decomposed into two components: $\boldsymbol{u}_i^{\mathrm{CL}}$ the displacement at the point due to its own loading, and $\boldsymbol{u}_i^{\mathrm{CA}}$ the additional displacement of the point due to loading at different locations (i.e., at the other foundations or the tunnel lining):

$$\boldsymbol{u}_i^{\mathrm{C}}=\overbrace{\boldsymbol{G}_{ii}\boldsymbol{f}_i^{\mathrm{s}}}^{\boldsymbol{u}_i^{\mathrm{CL}}}+\overbrace{\sum_{j,\,j\neq i}\boldsymbol{G}_{ij}\boldsymbol{f}_j^{\mathrm{s}}}^{\boldsymbol{u}_i^{\mathrm{CA}}}. \tag{5}$$

The additional displacement $\boldsymbol{u}_i^{\mathrm{CA}}$, can be further decomposed into: $\boldsymbol{u}_i^{\mathrm{CAS}}$ the additional displacement caused by the foundations of the structure (at other locations than i), and $\boldsymbol{u}_i^{\mathrm{CAT}}$ the additional displacement due to the tunneling:

$$\boldsymbol{u}_i^{\mathrm{C}}=\overbrace{\boldsymbol{G}_{ii}\boldsymbol{f}_i^{\mathrm{s}}}^{\boldsymbol{u}_i^{\mathrm{CL}}}+\overbrace{\sum_{\substack{j,\,j\neq i\\ j\neq tunnel}}\boldsymbol{G}_{ij}\boldsymbol{f}_j^{\mathrm{s}}}^{\boldsymbol{u}_i^{\mathrm{CAS}}}+\boldsymbol{u}_i^{\mathrm{CAT}}. \tag{6}$$

By utilizing assumption (4), that the tunnel is not affected by the structure (i.e., the forces on the tunnel are not affected by the structure), $\boldsymbol{u}^{\mathrm{CAT}}$ becomes the green field displacement, $\boldsymbol{u}^{\mathrm{gf}}$. Note that the forces acting on the foundations of the structure are equal but opposite to the forces acting on the soil (reactions):

$$\boldsymbol{f}_i^{\mathrm{c}} = -\boldsymbol{f}_i^{\mathrm{s}} = -\frac{\boldsymbol{u}_i^{\mathrm{CL}}}{\boldsymbol{G}_{ii}}. \tag{7}$$

Due to compatibility, the foundation displacements, $\boldsymbol{u}$, are equal to those of the soil continuum; that is, $\boldsymbol{u}=\boldsymbol{u}^{\mathrm{c}}=\boldsymbol{u}^{\mathrm{CL}}+\boldsymbol{u}^{\mathrm{CAS}}+\boldsymbol{u}^{\mathrm{CAT}}$. Introducing this with Eq. (7) to Eq. (2) results in

$$\boldsymbol{S}^{\mathrm{c}}\boldsymbol{u} = \boldsymbol{K}^{*}\boldsymbol{u}^{\mathrm{CAS}} + \boldsymbol{K}^{*}\boldsymbol{u}^{\mathrm{CAT}} - \boldsymbol{K}^{*}\boldsymbol{u}, \tag{8}$$

where $\boldsymbol{K}^{*}$ is a diagonal matrix, $\boldsymbol{K}_{ij}^{*}=1/\boldsymbol{G}_{ij}$ for $i=j$ and 0 for $i\neq j$. By multiplying the above equation by the inverse of $\boldsymbol{K}^{*}$ and rearranging the arguments we obtain

$$(\boldsymbol{G}^{*}\boldsymbol{S}^{\mathrm{c}} + \boldsymbol{I})\boldsymbol{u} = \boldsymbol{u}^{\mathrm{CAS}} + \boldsymbol{u}^{\mathrm{CAT}}, \tag{9}$$

where $\boldsymbol{G}^{*}$ is also diagonal, $\boldsymbol{G}_{ij}^{*}=\boldsymbol{G}_{ij}$ for $i=j$ and 0 for $i\neq j$. Note that $\boldsymbol{u}^{\mathrm{CAS}}=(\boldsymbol{G}-\boldsymbol{G}^{*})\boldsymbol{f}^{\mathrm{s}}=-(\boldsymbol{G}-\boldsymbol{G}^{*})\boldsymbol{S}^{\mathrm{c}}\boldsymbol{u}$. Introducing this to Eq. (9) and rearranging the arguments results in

$$(\boldsymbol{G}\boldsymbol{S}^{\mathrm{c}}+\boldsymbol{I})\boldsymbol{u}=\boldsymbol{u}^{\mathrm{CAT}}. \tag{10}$$

Eqs. (10) and (1) represent the same system; that is, Eq. (1) can be obtained by multiplying Eq. (10) by the inverse of $\boldsymbol{G}$, which is $\boldsymbol{K}^{\mathrm{s}}$, to get $(\boldsymbol{S}^{\mathrm{c}}+\boldsymbol{K}^{\mathrm{s}})\boldsymbol{u}=\boldsymbol{K}^{\mathrm{s}}\boldsymbol{u}^{\mathrm{CAT}}$. The above derivation reveals the meaning of the formulation (Eq. (1)), expressed mainly in the superposition principle for the elastic soil and the assumption that the soil-structure interaction does not affect the tunnel behavior, which results in $\boldsymbol{u}^{\mathrm{CAT}}$ being equal to $\boldsymbol{u}^{\mathrm{gf}}$.

3 Suggested relaxation method

The problem with Eq. (1) is that it fails to solve stiff or nearly rigid structures. When the structure is stiff, $\boldsymbol{S}^{\mathrm{c}}$ is much larger than $\boldsymbol{K}^{\mathrm{s}}$, and the matrix on the left term of Eq. (1) becomes ill-conditioned. Solutions from such ill-conditioned system may be unreliable. Essentially, Eq. (1) is not suitable for solution of infinitely stiff structures, and hence may fail to solve cases involving highly stiff structures. On the other hand, the following relaxation method begins with the exact solution of an infinitely stiff structure, and relaxes it, using an iterative procedure, until convergence is achieved.

For the sake of clarity and simplicity, we shall consider interaction only due to vertical displacement at the foundations of the structure (i.e., the structure's foundations are free to translate horizontally with zero reaction from the soil). However, the basis of the formulation is also correct for the case where horizontal soil resistance is considered.

Let us decompose the structure displacement, $\boldsymbol{u}$, into a rigid body displacement, $\boldsymbol{u}^{\mathrm{R}}$, and relative displacement, $\boldsymbol{u}^{\mathrm{rel}}$:

$$\boldsymbol{u}=\boldsymbol{u}^{\mathrm{R}}+\boldsymbol{u}^{\mathrm{rel}}. \tag{11}$$

The rigid body displacement can be presented by vertical displacement and rotations, conveniently about the origin of the coordinate system:

$$\boldsymbol{u}_i^{\mathrm{R}} = u_z^{\mathrm{R}} + \theta_y \boldsymbol{x}_i + \theta_x \boldsymbol{y}_i, \tag{12}$$

where u_z^{R} is the vertical displacement at the coordinates origin, θ_y is the rotation about the y-axis, θ_x is the rotation about the x-axis, $\boldsymbol{x}$ and $\boldsymbol{y}$ are vectors representing the x-location and y-location of the structure's foundations. In the more complete case, where horizontal movement of the foundations is also considered, 3 additional rigid-displacement parameters are required (i.e., 2 for horizontal movements and 1 for rotation about the z-axis).

Rearranging Eq. (6) to extract the interaction forces between the soil and the structure results in the following relation, expressed in matrix form:

$$\begin{aligned}\boldsymbol{f}^{\mathrm{s}} &= \boldsymbol{G}^{-1}\boldsymbol{u} - \boldsymbol{G}^{-1}\boldsymbol{u}^{\mathrm{CAT}} = \boldsymbol{K}^{\mathrm{s}}\boldsymbol{u} - \boldsymbol{K}^{\mathrm{s}}\boldsymbol{u}^{\mathrm{CAT}} \\ &= \boldsymbol{K}^{\mathrm{s}}(\boldsymbol{u}^{\mathrm{R}} + \boldsymbol{u}^{\mathrm{rel}}) - \boldsymbol{K}^{\mathrm{s}}\boldsymbol{u}^{\mathrm{CAT}} \\ &= \boldsymbol{K}^{\mathrm{s}}\boldsymbol{u}^{\mathrm{R}} - \boldsymbol{K}^{\mathrm{s}}(\boldsymbol{u}^{\mathrm{CAT}} - \boldsymbol{u}^{\mathrm{rel}}).\end{aligned} \tag{13}$$

The force system (which is the soil reaction to the structure), $\boldsymbol{f}^{\mathrm{s}}$, must be in equilibrium with the external loads on the structure. However, since the solution deals only with the additional effect of tunneling (beyond the static gravitational load solution) there are no external loads on the structure, and the following equilibrium equations (i.e., vertical, moment about y-axis, and moment about x-axis) can be written as

$$\sum_{i=1}^{N} \boldsymbol{f}_i^{\mathrm{s}} = 0,\ \sum_{i=1}^{N} \boldsymbol{f}_i^{\mathrm{s}} \boldsymbol{x}_i = 0,\ \sum_{i=1}^{N} \boldsymbol{f}_i^{\mathrm{s}} y_i = 0, \tag{14}$$

where N is the number of foundations. In the more complete case where the soil resistance to horizontal movement is also considered, 3 additional equilibrium equations are required (2 for horizontal equilibrium and 1 for moment balance about the z-axis). Introducing Eq. (13) into Eq. (14) and explicitly expressing $\boldsymbol{u}^{\mathrm{R}}$ by Eq. (12) results in the following set of equations for the rigid displacement parameters, u_z^{R}, θ_y and θ_x:

$$\begin{bmatrix} \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}} & \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}}\boldsymbol{x}_j & \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}}\boldsymbol{y}_j \\ \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}}\boldsymbol{x}_i & \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}}\boldsymbol{x}_j\boldsymbol{x}_i & \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}}\boldsymbol{y}_j\boldsymbol{x}_i \\ \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}}\boldsymbol{y}_i & \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}}\boldsymbol{x}_j\boldsymbol{y}_i & \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}}\boldsymbol{y}_j\boldsymbol{y}_i \end{bmatrix} \begin{Bmatrix} u_z^{\mathrm{R}} \\ \theta_y \\ \theta_x \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}}(\boldsymbol{u}_j^{\mathrm{CAT}}-\boldsymbol{u}_j^{\mathrm{rel}}) \\ \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}}\boldsymbol{x}_i(\boldsymbol{u}_j^{\mathrm{CAT}}-\boldsymbol{u}_j^{\mathrm{rel}}) \\ \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{K}_{ij}^{\mathrm{s}}\boldsymbol{y}_i(\boldsymbol{u}_j^{\mathrm{CAT}}-\boldsymbol{u}_j^{\mathrm{rel}}) \end{Bmatrix}. \tag{15}$$

Once the above system is solved, the force vector $\boldsymbol{f}^{\mathrm{s}}$ can be established by Eq. (13). To evaluate the relative displacement, $\boldsymbol{u}^{\mathrm{rel}}$, a reduced stiffness matrix for the structure needs to be constructed from $\boldsymbol{S}^{\mathrm{c}}$. This reduced matrix, $\boldsymbol{S}_{\mathrm{r}}^{\mathrm{c}}$, needs to be restrained at 3 nodes which are not positioned on a single line, to prevent formation of a mechanism. $\boldsymbol{S}_{\mathrm{r}}^{\mathrm{c}}$ is built from $\boldsymbol{S}^{\mathrm{c}}$ by dropping the lines and columns associated with the 3 chosen foundations, or making them null with a unit value on the main diagonal. Consequently, $\boldsymbol{S}_{\mathrm{r}}^{\mathrm{c}}$ is no longer singular as $\boldsymbol{S}^{\mathrm{c}}$, and can be inverted to obtain $\boldsymbol{u}^{\mathrm{rel}}$:

$$\boldsymbol{u}^{\mathrm{rel}} = \left(\boldsymbol{S}_{\mathrm{r}}^{\mathrm{c}}\right)^{-1} \boldsymbol{f}^{\mathrm{c}} = -\left(\boldsymbol{S}_{\mathrm{r}}^{\mathrm{c}}\right)^{-1} \boldsymbol{f}^{\mathrm{s}}. \tag{16}$$

Note that the relative displacement is with respect to the 'rigid plane' and the $\boldsymbol{u}^{\mathrm{rel}}$ value of restrained nodes is zero by definition. In the general case, the displacement of the 'rigid plane', $\boldsymbol{u}^{\mathrm{R}}$, will depend on the choice of the 3 restrained foundations, but the total displacement $\boldsymbol{u}^{\mathrm{R}}+\boldsymbol{u}^{\mathrm{rel}}$ will be independent of them, when the solution has converged. Only in the case of an infinitely stiff structure will the value of $\boldsymbol{u}^{\mathrm{R}}$ be independent of the choice of the 3 restrained foundations.

The solution of Eqs. (15) and (16) is conducted iteratively as follows. First, Eq. (15) is solved to obtain $\boldsymbol{u}^{\mathrm{R}}$, assuming $\boldsymbol{u}^{\mathrm{rel}}$ is zero at the first iteration. Then forces at the foundation points are evaluated using Eq. (13) with $\boldsymbol{u}=\boldsymbol{u}^{\mathrm{R}}+\boldsymbol{u}^{\mathrm{rel}}$ (Note, $\boldsymbol{u}^{\mathrm{rel}}$ is zero here as well, at the first iteration). Using the derived $\boldsymbol{f}^{\mathrm{s}}$, the relative displacement, $\boldsymbol{u}^{\mathrm{rel}}$, is evaluated from Eq. (16). $\boldsymbol{u}^{\mathrm{rel}}$ is then introduced into Eq. (15) at the beginning of the next iteration. The process continues until convergence of $\boldsymbol{u}$, $\boldsymbol{u}^{\mathrm{R}}$ and $\boldsymbol{u}^{\mathrm{rel}}$ with an error smaller than the defined tolerance (ε). Note that $\boldsymbol{u}^{\mathrm{R}}$ in the first iteration constitutes the solution for an infinitely rigid structure.

While the above procedure was found stable for stiff structure, oscillations and instabilities were observed for flexible structure. This is because any small change in $\boldsymbol{f}^{\mathrm{s}}$ may lead to a large change in $\boldsymbol{u}^{\mathrm{rel}}$ in a flexible structure. To overcome this problem, $\boldsymbol{u}^{\mathrm{rel}}$ can be artificially damped using the following expression, which is executed after Eq. (16), just before the start of the next iteration:

$$\boldsymbol{u}_{k+1}^{\mathrm{rel}} = \beta \boldsymbol{u}_{k+1/2}^{\mathrm{rel}} + (1-\beta)\boldsymbol{u}_{k}^{\mathrm{rel}}, \tag{17}$$

where $\boldsymbol{u}_{k+1}^{\mathrm{rel}}$ is the $\boldsymbol{u}^{\mathrm{rel}}$ used in the subsequent iteration in Eq. (15), $\boldsymbol{u}_{k}^{\mathrm{rel}}$ is the $\boldsymbol{u}^{\mathrm{rel}}$ used in the solution of Eq. (15) at the beginning of the current iteration, and $\boldsymbol{u}_{k+1/2}^{\mathrm{rel}}$ is the solution of Eq. (16) at the current iteration. The required value of β for stability and convergence decreases with reduction of structure stiffness. One should remember that for a flexible structure there is no real need to use the current formulation, since Eq. (1) is suitable for flexible structures. Fig. 2 shows the flowchart of the iterative procedure.

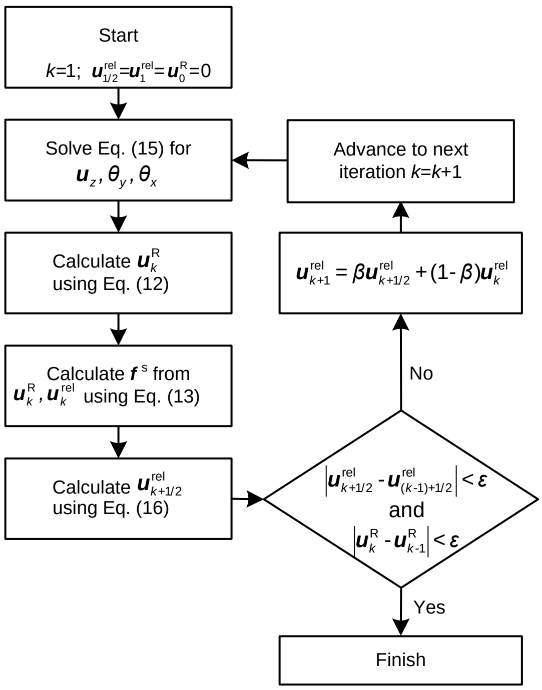

Fig. 2 Flowchart for solution by the suggested relaxation method

4 Examples

To compare between the methods and evaluate the concerns about ill-conditioned systems, let us consider the following example problem. A tunnel is excavated 15 m beneath a 6-storey building with a 30 m×30 m base. The building is a framed structure composed of 0.5 m×0.5 m columns and 0.4 m×0.8 m beams. The ceiling height is 3.5 m and the distance between the columns is 5 m. The building has 49 foundations. Two cases of tunnel excavations are considered (Fig. 3): (1) where the tunnel passes beneath the symmetry line of the building, and (2) where the tunnel passes at an offset to the symmetry line. In case 1 the deformation is associated with the final stage, in which the tunnel has passed the building, while in case 2 the tunnel front is positioned below the building. Case 2 is associated with rotation of the structure, unlike case 1.

Essentially, a structure response to tunneling should be computed for the complete sequence of the advancing tunnel, for evaluation of the worst-case scenario. In this example, however, only a single 'snapshot' is considered for each of the tunneling cases. One of the advantages of the decoupling procedure is that many solutions for different positions of the tunnel can be quickly generated, simply by modifying the input green field displacement.

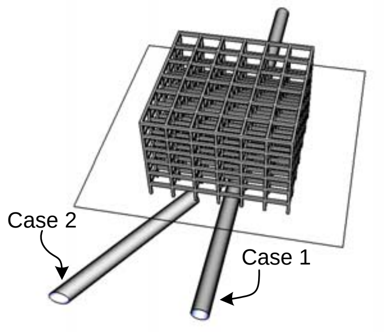

Fig. 3 Illustration of the example problem

A reasonable representation of the green field settlement trough may be obtained by the error curve (Attewell *et al.*, 1986):

$$s_{\text{v}}(\Delta x, \Delta y) = s_{\max} \frac{1}{2} \exp\left(-\frac{1}{2}\frac{\Delta x^2}{i^2}\right)\left[1 - Erf\left(\frac{1}{\sqrt{2}}\frac{\Delta y}{i}\right)\right], \tag{18}$$

where s_{v} is the green field vertical settlement at a transverse distance Δx from the tunnel centerline and longitudinal distance Δy ahead of the advancing tunnel face. $s_{\max}$ is the maximum settlement and i the transverse distance to the inflection point. Erf is the Gauss error function. The value of i taken for the 15 m depth tunnel is 7.5 m and is based on the empirical relation of Mair *et al*. (1993). Fig. 4 illustrates the shape of this 3D settlement trough.

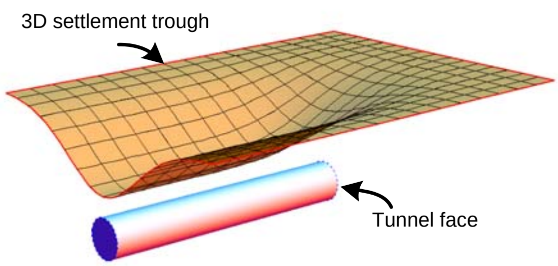

Fig. 4 3D green field settlement trough

Fig. 5 shows the green field displacements with respect to the considered structure for each of the tunneling cases. Note that the ground displacement will not be equal to the green field displacements, unless the structure is infinitely flexible, and that the green field is merely an input for the calculation. The resultant ground displacements differ from the green field displacements due to the deformation resistance of the structure. The current interaction analysis treats that issue rigorously, under the assumption of linear soil-structure interaction. Also note that nonlinearity may be associated with the excavation process, but as long as it is limited to the vicinity of the tunnel and does not propagate to the surface the solution may be considered valid.

Generally, the method is not limited to the green field displacement given in Eq. (18), or to a certain tunneling technique. Different tunneling techniques will be associated with different green field displacements, all of which can be introduced into the analysis as different $\boldsymbol{u}^{\mathrm{gf}}$.

In the current example, the condensed stiffness matrix of the structure, $\boldsymbol{S}^{\mathrm{c}}$, was obtained from analyses using the finite element code LUSAS (2007). The condensed stiffness matrix of the soil, $\boldsymbol{K}^{\mathrm{s}}$, was obtained analytically by inverting the flexibility matrix based on the solution of a circular foundation on elastic soil (Davis and Selvadurai, 1996):

$$\boldsymbol{G}_{ij}=\begin{cases}\dfrac{1-\nu^2}{E_{\mathrm{g}}d}, & i=j;\\[2ex] \dfrac{1-\nu^2}{E_{\mathrm{g}}d}\left(\dfrac{2}{\pi}\arcsin\left(\dfrac{d/2}{\sqrt{(\boldsymbol{x}_i-\boldsymbol{x}_j)^2+(\boldsymbol{y}_i-\boldsymbol{y}_j)^2}}\right)\right), & i\neq j,\end{cases} \tag{19}$$

where E_{g} and ν are the Young's modulus and Poisson's ratio of the ground, respectively, d is the diameter of the foundation, taken as 2 m in the current example. For more complicated conditions (inhomogeneous soil, uneven ground level, etc.) $\boldsymbol{G}$ can be built from finite element analyses of the soil. In the current example, Poisson's ratios were 0.25 for the soil and 0.2 for the structure.

The absolute and differential settlements in a linear elastic system are only a function of the ratio between $\boldsymbol{S}^{\mathrm{c}}$ to $\boldsymbol{K}^{\mathrm{s}}$. Therefore, let us define a nominal (reference) structure, with a Young's modulus 1000 times greater than that of the soil (if the structure presented in Fig. 3 is made of concrete, then the Young's modulus of the soil will be approximately 30 MPa, in the reference case). A parametric study, evaluating the difference between the methods, can be conducted by increasing and decreasing the relative rigidity of this nominal structure. For example, if the soil is ten times stiffer (i.e., $\boldsymbol{K}^{\mathrm{s}}$ is ten times larger), then the rigidity factor is 0.1; if the soil is ten times softer the rigidity factor is 10. If in the last case the structure is stiffer by 100, say due to additional walls (or floors), then the rigidity factor is 1000. Strictly speaking, if additional walls are added to the structure, matrix $\boldsymbol{S}^{\mathrm{c}}$ should be revised using a suitable finite element analysis. In the current analysis, such stiffening is approximated by increasing the value of the nominal $\boldsymbol{S}^{\mathrm{c}}$.

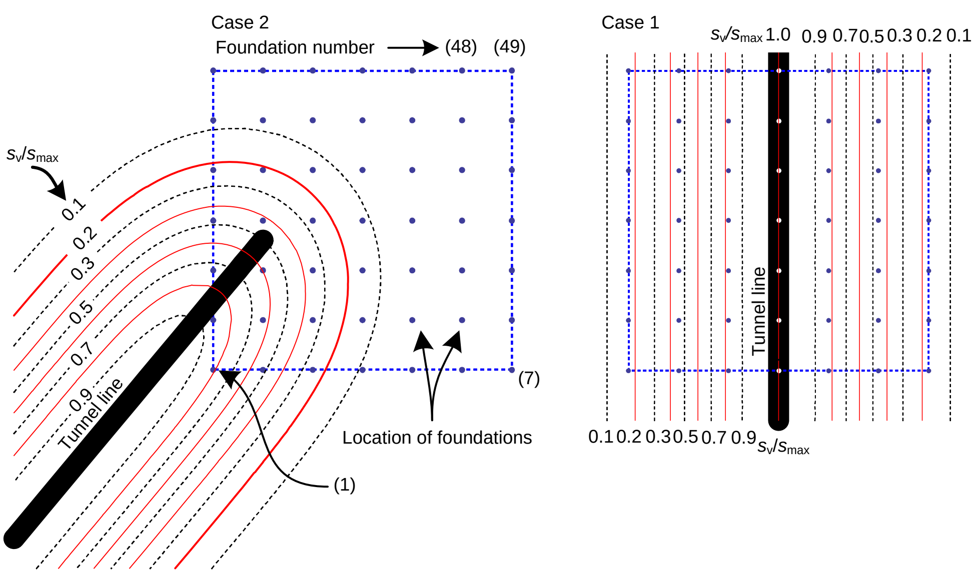

Fig. 5 Contours of green field displacement with respect to the structure layout

Tables 1 and 2 show results of the parametric study for the two tunneling cases respectively. The relaxation method values were converged to an accuracy of 10 digits. Column 1 represents the increase and decrease of the nominal structure rigidity. A value of zero refers to an infinitely flexible building, while a value of infinity to an infinitely rigid structure. Column 2 represents the relaxation method results of the differential vertical settlement between the corner and the central foundations in case 1 and between foundations (1) and (49) (Fig. 5) in case 2. Column 3 shows the same differential settlement, but with the direct method (Eq. (1)) using Gauss elimination with single and double precision. Column 4 shows the difference between the direct method and the relaxation method. Values in Column 2 and 3 are with respect to the maximum settlement in the green field (i.e., s_{max} in Eq. (18)).

While the relaxation method is not suitable for an infinitely flexible building, as Eq. (16) cannot be formed, Eq. (1) is suitable, resulting exactly in the green field differential settlement between the two locations (i.e., $0.8276s_{max}$ in case 1 and $0.8123s_{max}$ in case 2). On the other hand, Eq. (1) is not suitable for an infinitely rigid structure, while the relaxation method is, resulting in a differential settlement of 0 in case 1 and $0.7692s_{max}$ in case 2, due to rigid body rotation.

For rigidity factors between 0.01~1 the two methods result in the same values. The single precision calculation starts to deviate significantly at stiffness factor above 10 for case 1 and 100 for case 2, while the double precision remains accurate up to extremely rigid structures. It should be noted that the rigidity factors for which the direct method with double precision fails to predict reasonable results are

Table 1 Comparison between the methods for case 1

1	2	3		4	
Rigidity factor	Relaxation method	Direct method		Difference from relaxation method (%)	
		DP	SP	DP	SP
0	NR	0.8276	0.8276	–	–
0.01	0.8250	0.8250	0.8250	0.00	0.00
0.1	0.8025	0.8025	0.8023	0.00	−0.02
1	0.6338	0.6338	0.6340	0.00	0.02
10	0.2072	0.2072	0.2089	0.00	0.80
100	2.685×10^{-2}	2.685×10^{-2}	0.0419	0.00	56.14
1000	2.767×10^{-3}	2.766×10^{-3}	0.4092	−0.05	14 685
10000	2.776×10^{-4}	2.761×10^{-4}	−1.5381	−0.54	−554 194
100000	2.777×10^{-5}	2.631×10^{-5}	−0.3472	−5.24	−1 250 559
∞	0	NR	NR	–	–

NR: not relevant, cannot be calculated with the method; DP: double precision; SP: single precision

Table 2 Comparison between the methods for case 2

1	2	3		4	
Rigidity factor	Relaxation method	Direct method		Difference from relaxation method (%)	
		DP	SP	DP	SP
0	NR	0.8123	0.8123	–	–
0.01	0.8126	0.8126	0.8126	0.00	0.00
0.1	0.8145	0.8145	0.8145	0.00	0.00
1	0.8183	0.8183	0.8183	0.00	0.00
10	0.7910	0.7910	0.7915	0.00	0.06
100	0.7722	0.7722	0.7779	0.00	0.74
1000	0.7695	0.7694	0.8020	−0.01	4.23
10000	0.7692	0.7682	2.4304	−0.13	215.97
100000	0.7692	0.7589	−0.0850	−1.33	−111.05
∞	0.7692	NR	NR	–	–

NR: not relevant, cannot be calculated with the method; DP: double precision; SP: single precision extremely high. That is, although the direct method may fail, it is not likely that this would occur for conventional structures when double precision is used.

5 Conclusion

Evaluation of tunneling effects on existing buildings is a problem that requires the attention of both structural and geotechnical engineers. The structural engineer requires the differential settlement of the foundation to evaluate the strains and stresses in the building. However, the geotechnical engineer, lacking detailed information about the structure, can only evaluate the green field displacements. These, however, are not the differential settlements of the structure as the structure itself modifies the ground displacement (unless it is infinitely flexible). It is desirable to have a numerical method that allows simple communication between the engineers and still includes the soil structure interaction, without the need for each of the engineers to explicitly consider the other system in his solution (i.e., without having the structural engineer model the soil and the geotechnical engineer model the structure in each of their finite element models).

This paper discusses the direct condensation method that allows structural and geotechnical engineers to communicate and solve the coupled system with a very small matrix, even within a spreadsheet. Since the direct condensation method may result in an ill-conditioned system when the structure is extremely rigid, additional methods are required, that can be used either for solution of the problem or for evaluation of the correctness of the direct condensation solution. This paper suggests a relaxation method that does not yield erroneous values when the structure is rigid.

The two methods were used in the analysis of an example problem—a parametric study, evaluating the differential settlements for varying stiffness of a simple framed structure. It was shown that when the structure stiffness is about 10 times greater than a nominal value, the direct method indeed yields erroneous values, but only when a single precision calculation is conducted. The double precision calculation yields erroneous values only for extremely rigid structures that may not exist in reality. The results indicate that previously raised concerns about the direct method are not necessarily justified. It should be noted that previous observation of ill-conditioned systems were made when most computers used single precision calculations, which contributed to the instability. Today, most calculations are made with double precision. Nonetheless, in cases of concern the suggested relaxation method can be used for analysis.

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