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Simulation of filling construction of permeable geosynthetic tubes*

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Abstract: The filling construction of permeable geosynthetic tubes is considered. First, an analytical approach is developed to determine the internal pressure, tension and shape of the cross section of a geosynthetic tube based on its volume. An analytical solution for the drainage rate of the tube is then derived. The course of the filling construction is divided into several time intervals and the volume of the tube after each interval is obtained from the equilibrium of flow calculated from the drainage rate and filling rate. The validity of our analytical approach is tested by comparing our results with previously published experimental result. The results of this comparison indicate that our method is applicable for simulating the filling construction of permeable geosynthetic tubes.

Key words: Geosynthetic tube, Filling construction, Simulation, Volume of the tube, Drainage rate
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1 Introduction

Tubes and containers made of geosynthetic sheets are often pump-filled with dredged material or mortar, and are widely used for emergency flood protection in dams and dykes, and as construction elements for erosion control, bottom scour protection, scour filled artificial reefs, groynes, seawalls, breakwaters and dune reinforcement (Saathoff *et al.*, 2007). The range of applications for geosynthetic tubes and containers continues to grow.

A number of studies of different aspects of geosynthetic tubes have been published. As the length of a tube is usually much bigger than its width, Wang and Watson (1981) developed a 2D analytical analysis of the cross section of a tube. The friction between the tube and the filling material and between the tube and the foundation were ignored. The geosynthetic was assumed to be an inextensible and impermeable membrane with negligible density, laid on a rigid, horizontal foundation, to be subjected to internal (and possibly external) hydrostatic pressure. Based on these assumptions, they established the governing equations according to the equilibrium of force and the geometric condition, and obtained an approximate solution and a numerical solution for a single water-filled geomembrane tube using a non-dimensional analysis method. Based on Wang and Watson (1981)'s results, Namias (1985) obtained an accurate integral equation for the shape and tension of a single tube enduring uniform load, which can be numerically solved. Leshchinsky *et al.* (1996) considered the changes in tube shape caused by the consolidation of the filling material. Plaut and Suherman (1998) studied tubes that are partially or fully submerged in an external fluid and that rest on a deformable foundation. They also studied the problem of asymmetrical tubes. Seay and Plaut (1998) simulated the filling process of a tube using the finite element method, and analyzed the 3D properties of a tube. Plaut and Klusman (1999) studied stacked geosynthetic tubes resting on a deformable foundation. Cantré (2002)

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investigated the properties of a tube during consolidation and stacking using a finite element program, ABAQUS. Cheng and Li (2000) reported the engineering characteristics, construction process and the monitoring results of geosynthetic tubes. Zhang and Tan (2004) summarized some studies of woven geotextile tube dykes. Song *et al.* (2004), Liu and Hajime (2007), and Liu and Wang (2007) made a comprehensive study of the use of tubes as a reinforcement mechanism. Plaut and Cotton (2005) considered the static and dynamic behavior of air-filled geomembrane tubes without external water. The material of the tube was assumed to be inextensible and to have no bending resistance. Equilibrium configurations and small vibrations about equilibrium were investigated for tubes on rigid, Winkler, and Pasternak foundations. Zhang and Tan (2006) made a comprehensive summary of research on the characteristics of tubes after filling and stacking, and on the effects of consolidation of the material in a tube. Song *et al.* (2006) summarized current knowledge about the design, construction and quality control of blow-fill cofferdams made from geotextile tubes, and presented some engineering problems needing urgent solution. Qiu *et al.* (2008) took the anisotropy of silty slurry into account and established equations to analyze the shape and mechanical behavior of geotextile tubes, assuming that the density of the slurry increased linearly. Ghavanloo and Daneshmand (2009) developed a new analytical method to determine the equilibrium shape of a geomembrane tube filled with air and resting on rigid foundations of an arbitrary shape, such as straight line, trapezoidal, parabolic or circular rigid.

Most of these studies considered the properties of a tube after being filled, but few have described the filling procedure of a permeable geosynthetic tube. Nowadays, the filling of tubes is controlled and depends mostly upon on-site practical experience. In this paper, the process of the tube filling is divided into several periods of time, and the volume of the tube at the end of each time period is calculated based on an analytical solution for the drainage speed of the tube. Then, the internal pressure, tension, and shape of the cross section of the tube are determined in terms of volume by using a new theoretical method. Finally, we compare our calculation results with experimental results reported by Shin and Oh (2003).

2 Solution in terms of the volume of a tube

The shape of the cross section of a tube is shown in Fig. 1. The following assumptions are adopted in this study: (1) The tube is long enough so that a 2D analysis of a cross section of the tube is appropriate. (2) The tube is considered to be inextensible with negligible weight and no bending resistance. (3) Friction between the tube and the filling material, and between the tube and the foundation are ignored. (4) The filling material in the tube remains liquid. (5) The tube rests on a rigid horizontal foundation. (6) The permeability coefficient of the tube is unchanged.

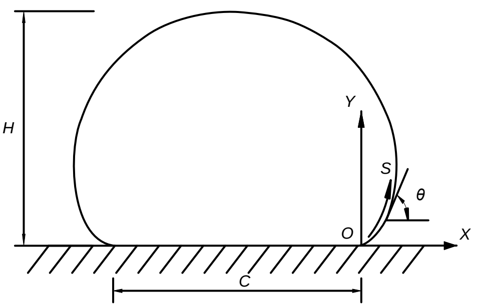

Fig. 1 Shape of the cross section of the soil tube
C is the contact part of the tube with the foundation, *S* is the length of the arc, *H* is the height of the tube, and *θ* is the tangential angle

The closed-form solution for the cross-sectional shape and the circumferential tension was presented by Namias (1985) and Plaut and Suherman (1998) in terms of the bottom (or the top) pressure. For a given pressure at the bottom, the parameter *k* can be calculated from

$$2[K(k)-E(k)]p_0=1, \tag{1}$$

where $K(k)$ and $E(k)$ are complete elliptic integrals of the first and second kinds, respectively, and p_0 is the nondimensional quantity of the bottom pressure P_0. The nondimensional tension τ can be determined:

$$\tau=\frac{k^2p_0^2}{4}. \tag{2}$$

After obtaining the parameter *k* and the tension τ from Eq. (1) and Eq. (2), the nondimensional contact length, height, and shape of the cross section can be calculated from the following equations:

$$c=2p_0\left[(1-k^2/2)K(k)-E(k)\right], \tag{3}$$

$$h = (1 - \sqrt{1-k^2})p_0, \tag{4}$$

$$x = p_0[E(k,\theta/2) - (1-k^2/2)F(k,\theta/2)], \tag{5}$$

$$y = p_0 - \sqrt{2\tau}\sqrt{p_0^2/(2\tau) - 1 + \cos\theta}, \tag{6}$$

$$s = \sqrt{\tau}kF(k,\theta/2), \tag{7}$$

where θ is the tangential angle, c is the nondimensional quantity of contact length C, h is the nondimensional quantity of the height H, x is the nondimensional quantity of the abscissa X, y is the nondimensional quantity of the ordinate Y, and s is the nondimensional quantity of the arc length S of the tube. $F(k,\theta/2) = \int_0^{\theta/2} 1\big/\sqrt{1-k^2\sin^2\psi}\,\mathrm{d}\psi$ is the elliptic integral of the first kind, and $E(k,\theta/2) = \int_0^{\theta/2}\sqrt{1-k^2\sin^2\psi}\,\mathrm{d}\psi$ is the elliptic integral of the second kind. When θ equals π, $F(k,\pi/2)=K(k)$ and $E(k,\pi/2)=E(k)$.

The pressure during filling cannot be obtained easily. However, the capacity of the tube at any time can be calculated from the drainage speed and the filling speed. Thus, we developed a method to compute the tube shape, internal pressure and tension in terms of the volume as follows.

According to the equilibrium of vertical force at the bottom of the tube,

$$\gamma V = P_0 C \quad \text{or} \quad v = p_0 c, \tag{8}$$

where V is the volume of the tube, γ is the specific weight of the material in the tube, $v=V/(l^2L)$ is the nondimensional quantity of the volume, L is the length of the tube (we take it as 1 m in the calculation), and l is the perimeter of the cross section.

At the top of the tube, $\theta=\pi$, and

$$x = -c/2. \tag{9}$$

Substituting Eq. (9) into Eq. (5), yields

$$p_0\left[E(k) - (1-k^2/2)K(k)\right] = -c/2. \tag{10}$$

Combining Eqs. (8) and (10) gives

$$2p_0^2\left[(1-k^2/2)K(k) - E(k)\right] - v = 0. \tag{11}$$

Substituting Eq. (1) into Eq. (11), leads to

$$\frac{[(1-k^2/2)K(k) - E(k)]}{2[K(k)-E(k)]^2} - v = 0. \tag{12}$$

If the volume v is given, the nondimensional pressure, tension, contact length, height and shape of the cross section can be computed from Eqs. (1)–(7) after obtaining the parameter k from Eq. (12). The relationship between k and v is shown in Fig. 2.

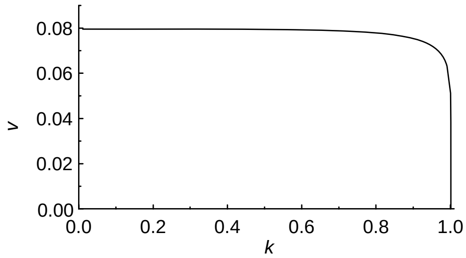

Fig. 2 Relationship between *k* and *v*

3 Solution for the drainage rate and the volume of the tube

The drainage rate perpendicular to the geosynthetic can be written as (Bao, 2008)

$$q_\mathrm{d} = k_\mathrm{g}\frac{\Delta h}{b}A, \tag{13}$$

where k_g is the perpendicular permeability coefficient of the geosynthetic, Δh is the water head difference between the inside and outside of the geosynthetic, b is the thickness of the geosynthetic, and A is its permeable area.

The pressure head difference is

$$\Delta h = (p_0 - y)\gamma l/\gamma_\mathrm{w}, \tag{14}$$

where γ_w is the specific weight of water.

The pore water pressure degrades steadily at the free tube surface. At the contact area between the tube and the foundation, there is little flow (Cantré, 2002). Thus, we assume that the water drains through the geotextile but not in areas where it is in contact with the foundation. The permeable area per unit length is (L=1 m)

$$A = l\int_s \mathrm{d}s = l\int_s \frac{1}{\sin\theta}\mathrm{d}y. \tag{15}$$

Substituting Eqs. (14) and (15) into Eq. (13), we obtain

$$q_{\mathrm{d}}=k_{\mathrm{g}}\frac{\Delta h}{b}A=\frac{k_{\mathrm{g}}l^{2}\gamma}{b\gamma_{\mathrm{w}}}\int_{s}\frac{(p_{0}-y)}{\sin\theta}\mathrm{d}y. \tag{16}$$

Substituting Eq. (6) into Eq. (16), we get

$$q_{\mathrm{d}}=\frac{k_{\mathrm{g}}l^{2}\gamma}{b\gamma_{\mathrm{w}}}\int_{0}^{2\pi}\tau\mathrm{d}\theta=\frac{2\pi k_{\mathrm{g}}l^{2}}{b\gamma_{\mathrm{w}}}\tau\gamma. \tag{17}$$

The course of the filling construction of the tube can be divided into several intervals, denoted as (t_n, t_{n+1}). Assuming that τ, γ remain unchanged and equal the initial value of each time interval, then the drainage discharge in the time interval (t_n, t_{n+1}) can be expressed as

$$\Delta V_{n}=q_{\mathrm{f}}(t_{n+1}-t_{n})-\frac{2\pi k_{\mathrm{g}}l^{2}}{b\gamma_{\mathrm{w}}}\tau_{n}\gamma_{n}(t_{n+1}-t_{n}), \tag{18}$$

where q_{f} is the filling speed, τ_n and γ_n are the tension and the specific weight of the material in the tube at time t_n. Then, the volume V_{n+1} at time t_{n+1} is obtained as

$$V_{n+1}=V_{n}+\Delta V_{n}. \tag{19}$$

The specific weight of the material in the tube at time t_{n+1} can be expressed as

$$\gamma_{n+1}=\frac{\gamma_{n}V_{n}+\gamma_{\mathrm{f}}q_{\mathrm{f}}(t_{n+1}-t_{n})-\gamma_{\mathrm{w}}\Delta V_{n}}{V_{n}+q_{\mathrm{f}}(t_{n+1}-t_{n})-\Delta V_{n}}, \tag{20}$$

where γ_{f} is the specific weight of the filling material.

4 Termination condition and procedure of computation

During the construction, it is necessary to not only ensure the security of the tube, but also to control its final height. So we can take the computation termination condition as follows:

1. Security control: the tension should not be greater than the tensile strength.

2. Height control: the final height of the tube should achieve the required height. The height and water content of the filling material can be calculated at any time, and then the final height after consolidation can be expressed as (Leshchinsky *et al.*, 1996)

$$H_{\mathrm{f}}=H_{0}\left(1-\frac{G_{\mathrm{s}}(w_{0}-w_{\mathrm{f}})}{1+w_{0}G_{\mathrm{s}}}\right), \tag{21}$$

where H_{f} is the final height of the tube after consolidation, H_0 is the initial height of the tube after filling, G_{s} is the specific gravity of solids in the filling material, and w_0, w_{f} are the water content after filling and consolidation, respectively.

A flowchart of the calculation is shown in Fig. 3, and the detailed explanation can be found in the next section.

5 Comparison with previous experimental results

To test the analysis method in this study, we compared our results with experimental results reported by Shin and Oh (2003). The permeability coefficient of the tube was 1×10^{-4}–1×10^{-2} cm/s, taken as an average value of 1×10^{-3} cm/s in this study. The tensile strength was 196 kN/m, the specific gravity of the sand was 2.65, the perimeter of the cross section was 8 m, and the water content of the dredged mortar was 190%. The final height was 0.6 m and as the height after filling was 1.22 m, then the water content after filling was 39% as calculated from Eq. (21), and the average filling speed was q_{f}=0.145 m^3/min.

Numerical integration was used in the calculation. k varies little as v changes from 0 to 0.05 (Fig. 2). The obtained outflow is small and shows very little variation. So the first time interval was set as 10 min and subsequent intervals as 2 min because the variation in k caused by v increases with time. The specific calculation was carried out according to the flow chart in Fig. 3. It is convenient to introduce the following nondimensional quantities:

$$\begin{aligned}&x=X/l,\ y=Y/l,\ s=S/l,\ c=C/l,\ h=H/l,\\&p_{0}=P_{0}/(\gamma l)=h,\ \tau=T/(\gamma l^{2}),\ v=V/(Ll^{2}).\end{aligned} \tag{22}$$

Step 1: input and non-dimensionalize the initial known quantities (volume of the tube) through Eq. (22), at the current moment: n=1, t_1=0, V_1=0, v_1=0.

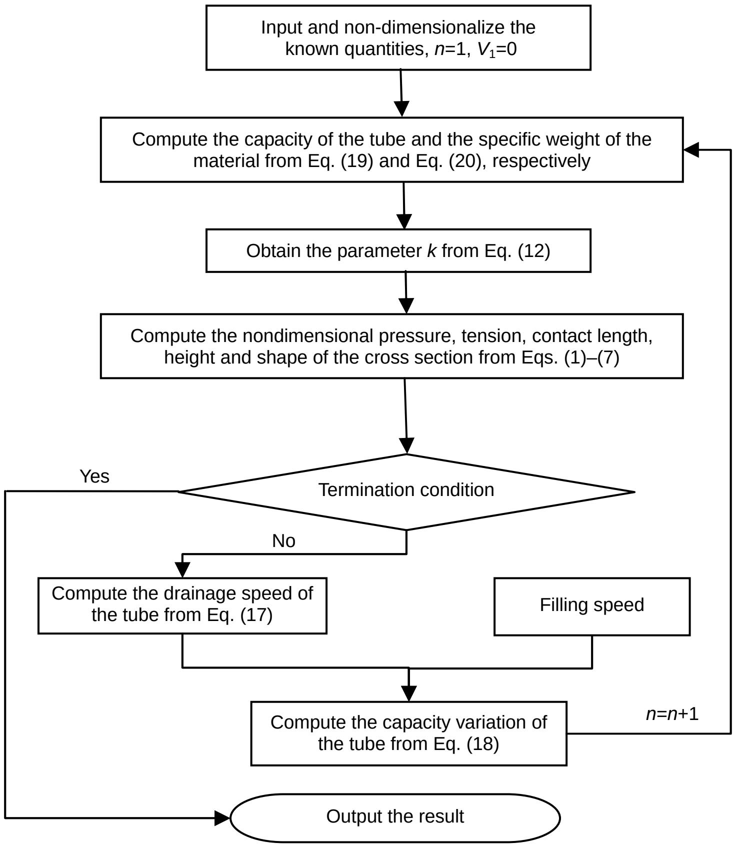

Fig. 3 Flowchart of the computation procedure

Step 2: obtain the volume of the tube and the weight of the material in the tube, γ_1=(1+1.9)/(1/26.5+1.9/10)=12.7341, v_1=0.

Step 3: obtain the parameter k_1=1 from Eq. (12) by v_1=0.

Step 4: compute the non-dimensional pressure p_{01}=0, tension τ_1=0, contact length c_1=0.5, height h_1=0 and shape of the cross section from Eqs. (1)–(7).

Step 5: termination condition: safety control: $T_1=\tau_1\gamma_1 l^2 = 0\times 12.7341\times 64 = 0 < 196$; height control:

$$H_{\mathrm{f1}} = H_{01}\left(1-\frac{G_{\mathrm{s}}(w_0 - w_{\mathrm{f}})}{1+w_0 G_{\mathrm{s}}}\right) = 0\times 8\left(1-\frac{2.65(1.9-0.39)}{1+1.9\times 2.65}\right) = 0 < 0.6.$$

Step 6: compute the drainage speed q_{d1}=0 of the tube from Eq. (17).

Step 7: compute the capacity variation of the tube from Eq. (18):

$$\Delta V_1 = q_{\mathrm{f}}(t_2 - t_1) - \frac{2\pi k_{\mathrm{g}} l^2}{b\gamma_{\mathrm{w}}}\tau_1\gamma_1(t_2 - t_1) = 0.145\times 10 - 0 = 1.45.$$

Go back to Step 2 and start the second loop calculation. We obtain in the second loop: n=2, t_2=10, v_2=0.0227, γ_2=12.7341, k_2=0.9999, p_{02}=0.0508, τ_2=0.00006457, c_2=0.4492, h_2=0.0508, h_{f2}=0.0171, q_{d2}=0.00007901 m^3/s. Continue to loop until the termination condition is met.

Fig. 4 shows the comparison between the calculated height of the tube and the height from experimental results. The deviation in the first 50 min may be because the first time step was too long, and because the filling speed used in the calculation was an average value which may not be identical to that of the experiment. Fig. 5 shows the estimated cross section of the tube after filling compared with that observed. The calculated results mostly coincide with the experimental results. There is a difference between the calculated and experimentally detected shapes of the tube shown in Fig. 5. This may be because the filling material was considered homogeneous while some consolidation may have taken place during the filling process leading to a decreased draining area. Thus, the calculated outflow would be greater than that found by experiment. However, the assumption is necessary to facilitate implementation of the calculation. The error caused by this assumption is within acceptable limits. It can be seen from the comparison that the proposed method can meet the needs of simulation of filling construction in engineering. The tension in the filling process increases more and more quickly as time progresses (Fig. 6).

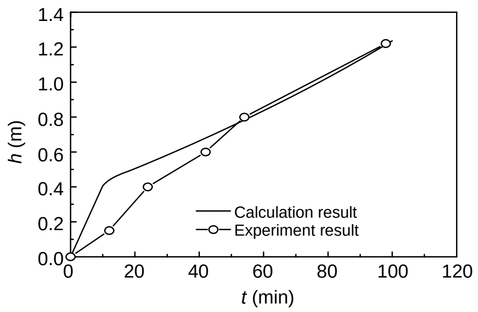

Fig. 4 Height of the tube during filling between the calculation result and experiment result. The experiment result comes from Shin and Oh (2003)

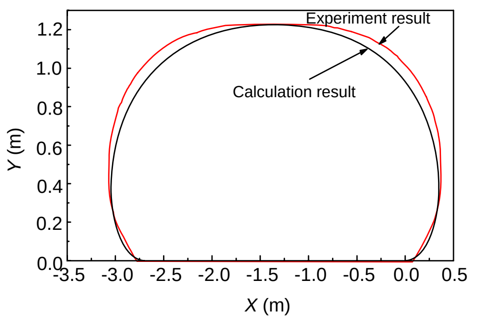

Fig. 5 Shape of the tube after filling between the calculation result and experiment result. The experiment result comes from Shin and Oh (2003)

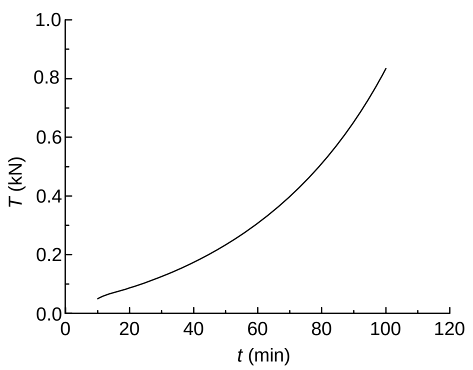

Fig. 6 Growth of tension of tube with time during filling

6 Conclusions

1. In this study, an analytical method is presented to simulate the filling construction of a permeable geosynthetic tube. The nondimensional internal pressure, tension, contact length, height and shape of the cross section are determined from the volume of the tube, which can be obtained easily during the filling procedure from the filling speed and drainage speed.

2. An analytical solution of drainage speed is also obtained for a permeable tube during the filling process. Given the parameters of the tube, the drainage speed depends only on the tension of the tube and the specific gravity of the material in the tube, as illustrated in Eq. (17).

3. A comparison of the results obtained from our method with those of a previous experiment was carried out, which showed that our method is applicable for simulating the filling construction of permeable geosynthetic tubes.

4. It is possible to control the construction theoretically using our approach rather than being dependent solely on actual construction experience. The computation method outlined in the flowchart (Fig. 3) could be applied when designing geotextile geotubes. Some important properties such as the height, tension, specific weight and water content of the material in the tube could be estimated at any time from an analytical analysis before starting filling construction.

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