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Optimal operation of multi-storage tank multi-source system based on storage policy*

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Abstract: A two-stage method is developed to solve a new class of multi-storage tank multi-source (MTMS) systems. In the first stage, the optimal storage policy of each tank is determined according to the electricity tariff, and the ground-level storage tank is modeled as a node. In the second stage, the genetic algorithm, combined with a repairing scheme, is applied to solve the pump scheduling problem. The objective of the pump scheduling problem is to ensure that the required volume is adequately provided by the pumps while minimizing the operation cost (energy cost and treatment cost). The decision variables are the settings of the pumps and speed ratio of variable-speed pumps at time steps of the total operational time horizon. A mixed coding methodology is developed according to the characteristics of the decision variables. Daily operation cost savings of approximately 11% are obtained by application of the proposed method to a pressure zone of S. Y. water distribution system (WDS), China.

Key words: Multi-storage tank system, Storage policy, Genetic algorithm, Repairing scheme, Pump scheduling
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1 Introduction

Due to the increased high-density residential areas and consumer demand, storage tanks and pressurization equipments are widely used in most water distribution systems (WDSs) in China. There are hundreds of storage tanks in some large cities where the total regulating capacity of the storage tanks is almost 20% of the whole daily water demand of the water supply system. Taking advantage of the capacity of storage tanks, together with the optimal control of the pump stations with time varying electricity tariffs, can lead to significant cost savings.

The optimal operation of multi-storage tank multi-source (MTMS) WDS is a large-scale non-linear optimization problem with discrete and continuous variables, which represents one of the most difficult problems to solve. Dynamic programming has been a very efficient algorithm in the optimal operation of WDSs (Ormsbee *et al*., 1989; Ormsbee and Lansey, 1994; Ertin *et al*., 2001; Ulanicki *et al*., 2007). Extension of the approach to large scale multi-tank systems is greatly limited due to the inherent problem that results from explosion in dimension. Spatial decomposition techniques (Coulbeck *et al*., 1988a; 1988b; Zessler and Shamir, 1989) are one of the methods to avoid this problem, but this technique is able for limited kinds of supply networks. Other techniques for optimal operation of multi-tank water supply systems include the linear programming (Pasha and Lansey, 2009), the nonlinear programming (Lansey and Zhong, 1990; Brion and Mays, 1991; Chase and Ormsbee, 1991), the mixed-integer linear programming (MILP) (Ulanicki and Orr, 1991), the genetic algorithm (Mackle *et al*., 1995; Beckwith and Wong, 1996; Yu *et al*., 2005; Vamvakeridou-Lyroudia *et al*., 2007), and the Ant colony algorithm (López-Ibáñez *et al*., 2008).

The scheduling problem considered in this paper, which corresponds to an MTMS system, is not con-

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tained in any of the above system classes. In this class of MTMS system, the storage level can be controlled by an automatic water-level control valve. These storage tanks act to equalize pumping requirements and to provide emergency water for specific residential areas. A two-stage method is developed to solve this class of scheduling problem. In the first stage, the optimal storage policy of each tank is determined; in the second stage, the optimal pump scheduling is found by application of the genetic algorithm.

The purpose of this study was to investigate the possible savings that could be obtained through the implementation of the proposed method for the S. Y. WDS with a large number of storage tanks and multiple pump stations in China. Although the application presented herein is restricted to a single pressure zone of the system, it nevertheless demonstrates the potential savings that may be obtained as a result of the application of this technology.

2 Multi-storage tank multi-source system model

The main components of the WDS system are illustrated in Fig. 1. In this class of MTMS system, treated water is pumped to the tank to be pressurized to a given residential area with tall buildings. Most residential areas are equipped with ground-level storage tank. These tanks are used to balance differences in supply and demand, and operate in a simultaneous inflow/outflow mode. The maximum and minimum levels of the tank can be controlled by an automatic water-level control valve. In most cases, these tanks maintain the maximum storage for the purpose of reliability. Obviously, this kind of operation policy wastes the storage capacity and increases the water age. Therefore, a standard policy is needed to provide the rules for operating the storage tank to efficiently use the storage capacity. In this study, the storage tank in the MTMS system is modeled as a node, and its demand equals the inflow of the storage tank.

3 Rule-based storage policy

The storage policy is a set of rules for determining the quantities of water to be stored. It depends largely on the electrical tariff, the storage capacity, and the residential demand pattern, and aims to gather water during the inexpensive tariff periods, and to release it during the expensive periods. The continuity equation for the storage tank can be given as

$$V_t' = V_t + I_t - O_t, \quad (1)$$

$$V_{\min,t} \le V_t \le V_{\max,t}, \quad (2)$$

where V_t is the beginning storage; V_t' is the ending storage; I_t is the inflow during time period t; O_t is the consumer demand during time period t; $V_{\min,t}$ is the minimum storage, the lower operational bound of storage tank, and $V_{\max,t}$ is the maximum storage, the upper operational bound of storage tank. The S. Y. electric system is structured to promote off-peak period energy usage in night hours with lower rates and penalize peak period energy usage in the diurnal hours with higher rates.

The optimal storage policy is represented by the following rules:

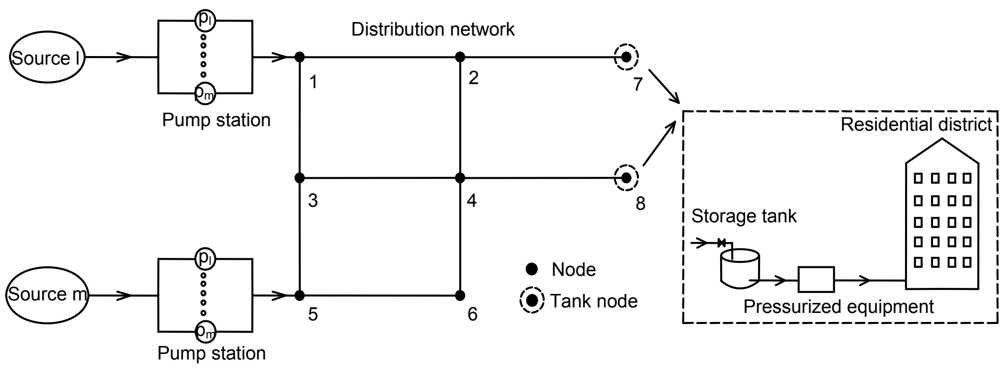

Fig. 1 Multi-storage tank multi-source water distribution system

Rule 1 During the lower electric tariff period, conservation storage is maintained as high as possible.

The inflow of tank can be calculated by the formulation:

$$I_t = \pi D^2 vt / 4, \tag{3}$$

where D is the diameter of pipe inflow, and v is the velocity of inflow water, which can be estimated by the history data. If the tank achieves the maximum storage, keep the maximum storage until the end of lower electricity tariff period.

Rule 2 During the higher electricity tariff period, keep I_t=0 until the tank achieves the minimum storage. If the tank achieves the minimum storage, keep the minimum storage until the end of higher electricity tariff period.

The initial tank storage is usually known from field conditions. The final tank storage is set depending on the desired storage policy. In addition to the initial and final tank storages, the residential consumer demand for each time interval must be known. These demands can be forecasted from operator experience or through the use of mathematical demand forecasting models (Alvisi *et al.*, 2007; Gato *et al.*, 2007; Ghiassi *et al.*, 2008; Magini *et al.*, 2008). In this stage, the inflow of each tank at each time interval is determined by the optimal storage policy, and then the optimal volumes of water produced at each time interval by the pumping stations are determined. In the next stage, the optimal pump scheduling can be found by the optimization method.

4 Pump scheduling problem

4.1 Objective function

The treatment cost and electric energy cost are considered as optimization objective. Electric energy cost is the cost of electric energy consumed by all pumps at the pumping station. It is not the same throughout the whole day, so the change in tariff structure is considered. The energy cost for pressurized equipments is not considered in this study.

$$f = \sum_{t=1}^{T}\sum_{i=1}^{I} S_i Q_{i,t} + \sum_{t=1}^{T}\sum_{k=1}^{K} \frac{C \cdot NP_{k,t} \cdot QP_{k,t} \cdot HP_{k,t}}{\eta_{k,t}} \cdot SP_{k,t}, \tag{4}$$

where S_i represents the treatment cost of pump station i per cubic meter, $Q_{i,t}$ is discharge of pump station i at time interval t, T is the total time steps of operational time horizon, and I is the total number of pump stations. $SP_{k,t}$ is the electricity tariff at time interval t, $NP_{k,t}$ is the on-off state of pump k at time interval t, K is the total number of pumps in all pump stations, C is the unit conversion factor, $HP_{k,t}$ is the hydraulic head of pump k at time interval t, $QP_{k,t}$ is the flow rate of pump k, and $\eta_{k,t}$ is the efficiency of pump k at time interval t.

4.2 Operational constraints

The optimization is subject to a set of constraints as follows.

1. Pressure control point constraint

Control point pressure is required to maintain greater than a minimum pressure level to ensure adequate water service, and less than a maximum pressure level to reduce water leakage or burst within a system.

$$P_{\min(i,t)} \le P_{\mathrm{c}(i,t)} \le P_{\max(i,t)}, \quad i = 1, 2, \cdots, NN, \tag{5}$$

where $P_{\min(i,t)}$ and $P_{\max(i,t)}$ are the minimum and maximum control pressures at point i, respectively, and NN is the number of control pressure points.

2. Limitation on pump station discharge

The discharge of pump station is limited by the capacity of pumps and the treatment capacity of treatment plant at time interval t.

$$Q_{p,t} \le Q_{\max p,t}, \tag{6}$$

where $Q_{p,t}$ is the actual discharge of pump station p at time interval t, $Q_{\max p,t}$ denotes the maximum discharge capacity of pump station p at time interval t.

3. Pump speed ratio constraint

Here, the pump speed ratio is defined as a ratio which equals the actual operating speed divided by its normal speed. Limited by current and voltage of power distribution, a pump installed with variable frequency drive is allowed to vary its speed within the range of minimum and maximum speeds.

$$n_{\min i} \le n_i \le n_{\max i}, \tag{7}$$

where $n_{\min i}$ is the minimum speed ratio of pump i, and $n_{\max i}$ is the maximum speed ration of pump.

4. Pump efficiency constraint

A pump should run in the designed high efficiency interval, which is often required to be not smaller than a minimum efficiency.

$$\eta_{\min i} \le \eta_i , \tag{8}$$

where η_i is the actual pump efficiency, and $\eta_{\min i}$ is the minimum efficiency required during the operation.

5 Genetic algorithm approach for pump scheduling

To solve the pump scheduling optimization problem as formulated above, an efficient hydraulic network simulator EPANET2.0 (Rossman, 2002) is directly embedded into the genetic algorithm (Goldber, 1989). The hydraulic simulation result (i.e., flow rate, junction pressure) is passed back to the optimization model to quantify the violations in the implicit bound constraints and objective function. Genetic algorithm is a general optimization technique based on the genetic processes of biological organisms. As a robust global search method, genetic algorithm has been widely used in many fields. In this study, a new genetic algorithm based on a mixed coding method and repair mechanism is presented to solve the pump scheduling problem.

Fig. 2 shows the flow chart of genetic algorithm combined with repairing mechanism. Invalid and infeasible solutions are handled following a methodology proposed by Deb (2000), where solutions are partially ordered depending on their feasibility.

5.1 Mixed coding scheme

The difficulty in solving the problem of optimizing a pump system arises from the mixture of constant speed pumps and variable speed pumps. In this study, a mixed coding scheme is designed to represent control variables (Table 1). Pump status variables and corresponding speed ratios are represented. A binary code refers to the status of a pump. The real code represents the operating speed ratio of the pump. A zero value represents a non-working pump, while one represents a working pump. For constant speed pumps, the speed ratios are set to 0. Here, the binary code and real code are combined as one gene, for example, (1,0.92) means the pump is running with speed ratio of 0.92; while (0,0.95) means the pump is switched off.

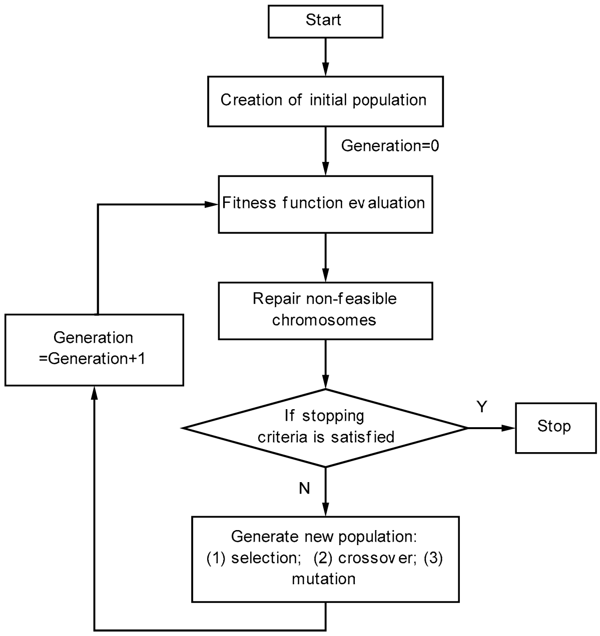

Fig. 2 Flow chart of genetic algorithm with repairing mechanism

Table 1 Mixed coding of pump scheduling problem

Time interval	1				…	n			
Pump	P_1	P_2	…	P_m	…	P_1	P_2	…	P_m
Status	0	1	…	1	…	1	1	…	1
Speed ratio	0	0	…	0.94	…	0	0	…	0.90

A new mixed variable crossover operator has been developed which takes the relation of binary and real variables into account. The main idea of this operator is to apply a uniform crossover operator to the binary variables. If they are equal for the two chosen parents, the simulated binary crossover (SBX) (Deb and Jainz, 2002) is applied to the corresponding real variables. For example, parent1 (1,0.87) and parent2 (1,0.82) generate child1 (1,0.84) and child2 (1,0.86) by the SBX. If they are not equal, the binary variables and corresponding real variables are switched together. For example, parent1 (0,0.72) and parent2 (1,0.79) generate child1 (1,0.79) and child2

(0,0.72). The SBX operator simulates the operation principle of the single point crossover operator on binary strings and respects the interval schemata processing. The mixed crossover operator is illustrated by the example for the pump scheduling problem (Fig. 3). By this method, both the status and the speed ratio of variable speed pumps can be considered. This helps to find the optimal pump scheduling and variable speed.

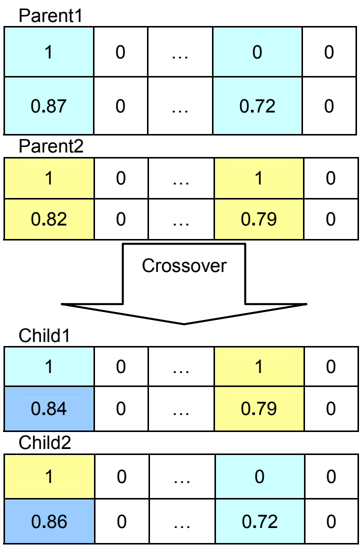

Fig. 3 New crossover operator applied to pump scheduling

5.2 Repairing chromosomes

Most solutions examined during optimization are grossly inadequate or non-feasible, where minimum pressure requirements are severely violated. If the pressures of control points are higher than the maximum pressure, some pumps should be turned off or the speed of variable speed pumps should be reduced. If the pressures of control points are lower than the minimum pressure, some pumps should be turned on or the speed of variable speed pumps should be raised. In this study, a sensitivity analysis method (Pezeshk and Helweg, 1996) is used to confirm which pump station has the highest influence on the pressure control point. Then one pump is randomly selected in this station to correct the pressure problem. In more details below, the following algorithm is executed for each new chromosome:

```
try=0;
while (invalid (chromosome) and try<=matrices)
  if (P_c(i,t)<P_c(min,t))
    confirm the pump station P_s that has the highest
      influence;
    if (variable speed pump running and n!=n_max)
      set n=n_max;
      // n is the speed ratio of variable speed pump
         in pump station P_s; n_max is the maximum
         speed ratio
    else if (n==n_max)
      switch on one pump randomly selected in
        pumps station P_s;
  end if
  else if (P_c(i,t)>P_c(max,t))
      if (n!=n_min)  // n_min is the minimum speed ratio
        set n=n_min;
      else if (n==n_min)
        switch off one pump randomly selected in
          pumps station P_s;
  end else
  if (Q_(p,t)>Q_max(p,t))
      if (n!=n_min)   set n=n_min;
      else if (n==n_min or no variable speed pump
        operating)
        switch off one pump randomly selected in
          pumps station;
  end if
  try=try+1;
end while
```

6 Case study

6.1 Description of water distribution system

The S. Y. city in the Northeast of China is divided into nine pressure zones. The eighth pressure zone is selected as a case study. The network is shown in Fig. 4. It has an average daily demand of 274 000 m^3/d and comprises two sources and two pump stations. The physical characteristics of the sources and pumps are shown in Table 2. There are 30 tank nodes in this network. Each tank node represents a storage tank connecting to a residential area (Fig. 1). The total regulating capacity of storage tanks is 22% of the whole daily water demand of the pressure zone, and the regulating capacity of each tank is more than 50% of daily consumption by the given residential area. In this network, all residential consumer demands are metered, and detailed historical data are available. As shown in Fig. 5, there are three electricity tariff periods in the day, 0.43 CNY/(kW·h) from 23:00 to

7:00, then 1.29 CNY/(kW·h) until 16:00, and 0.86 CNY/(kW·h) from 16:00 to 23:00.

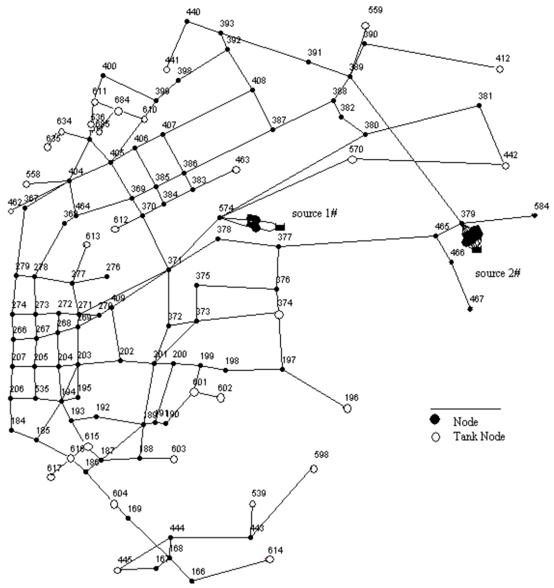

Fig. 4 Layout of the network

Table 2 Pump station data

Water source	Pump No.	Pump type	Treatment cost (CNY/m^3)	Constant or variable
1#	1#	28SA-10A	0.25	Constant
	2#	28SA-10B		Constant
2#	3#	28SA-10A	0.33	Constant
	4#	28SA-10A		Variable
	5#	28SA-10B		Variable
	6#	32SA-19E		Variable

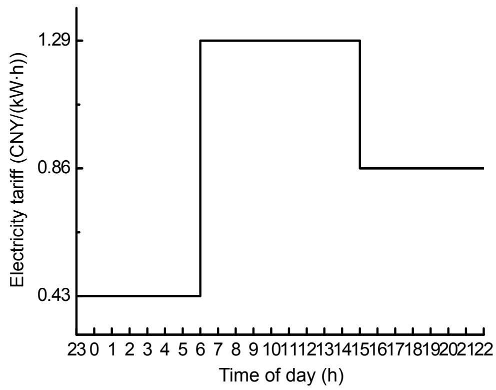

Fig. 5 Electricity tariff of S. Y. water distribution system

6.2 Results and discussion

Tank node 598 is illustrated in terms of how the optimal storage policy works. It represents a residential area with daily consumption of 1000 m^3. The equipped storage tank has the regulating capacity of 500 m^3, its minimum and maximum storage volume is 150 and 650 m^3, respectively. The diameter of inflow pipe is 200 mm.

During the time period of lower electricity tariff, the velocity of inflow pipe is estimated by 1.0 m/s and the beginning storage is 150 m^3. Then the inflow of tank can be calculated by Eq. (3). When the tank storage is at the maximum storage, it maintains the maximum storage till 7:00.

During the time period of higher electricity tariff (7:00–16:00), the control valve is turned off. When the storage achieves the minimum storage, the tank maintains the minimum storage till 23:00 and the inflow of tank can be obtained by Eq. (1).

Fig. 6 shows the consumer demand pattern of residential area and the optimal tank inflow resulting from a rule-based storage policy. Fig. 7 illustrates the optimal tank storage over the day. The tank is as full as possible during the inexpensive period and as empty as allowed during the expensive period. By this storage policy, the tank volume commonly achieves

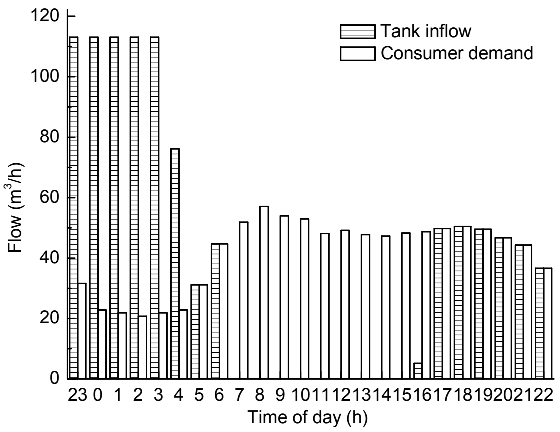

Fig. 6 Optimal tank inflow vs. residential consumer demand

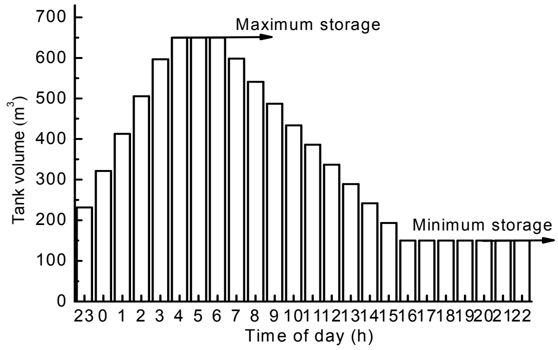

Fig. 7 Optimal tank storage during the day

the minimum storage at the end of expensive period, and easily equals to the storage at the beginning of the inexpensive period. Note that the storage policy largely depends on the electricity tariff and the regulating capacity of storage tank.

In the same way, inflows of all storage tanks at each time interval are determined, and furthermore, the pumped volumes in each time period are known. In this network, the beginning volumes of all tanks are assumed to be minimum storages. Fig. 8 shows the original water demand in the pressure zone and the pumped volume with storage policy. One can see that the pumped volumes during the inexpensive period are much larger than the original water demand. Make sure the pumped volume at each time interval do not exceed the maximum supply capacity of the network.

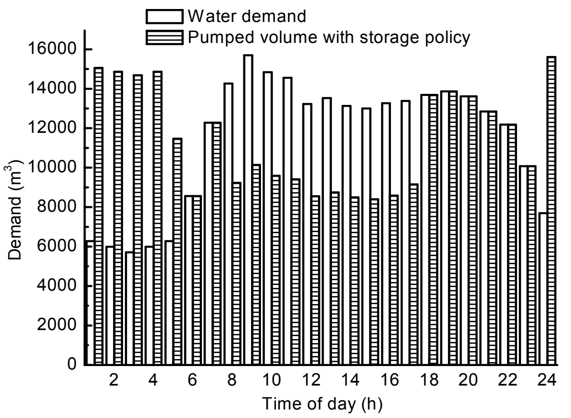

Fig. 8 Pumped volume with storage policy vs. water demand

In the next stage, the genetic algorithm combined with repairing scheme is applied to solve the pump scheduling problem. The system is optimized over a 24-h period with 1-h time steps. The genetic algorithm parameters are chosen at generation=100, P_{cross}=0.8 and $P_{mutation}$=0.01. Each population contains 200 chromosomes. For this example network, the time taken on a Pentium4 (1.86 GB) with 1 GB RAM computer was about 4.2 h for 100 generations. With the increase of generations, the computation time will be longer. Note that the computation time depends greatly on the simulation engine and on the complexity of the WDS. The program is run five times to find the best solution.

The resulting curves of the best and average objective solutions at each generation are illustrated in Fig. 9. The results of pump scheduling are shown in Table 3. The real number in Table 3 indicates the speed ratio of variable speed pump. The best solution with an operation cost of 95 645 CNY/d is produced to meet the hour's water demand requirement, approximately 11% less than the operation cost before optimization. The hourly average service pressure in this network is also compared in Fig. 10. The figure shows that average pressure of all nodes during the day has a smaller fluctuation than before. The average pressure is much lower during the night and higher from 7:00 to 13:00.

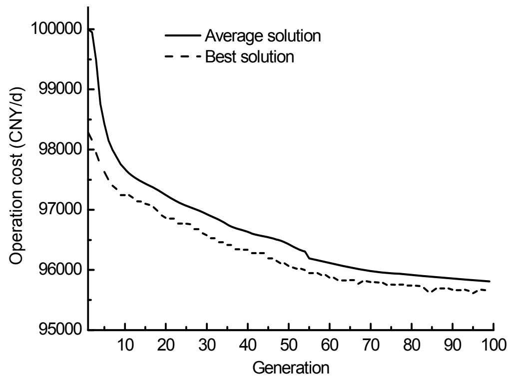

Fig. 9 Average and the best solutions of GA

Table 3 Result of pump scheduling

Time interval	Pump station 1			Pump station 2		
	1#	2#	3#	4#	5#	6#
1	On	On	On	0.91	0.94	0.92
2	On	On	On	0.82	0.85	0.94
3	On	On	On	0.77	0.92	0.96
4	On	On	Off	0.99	0.98	0.96
5	Off	On	Off	0.98	0.82	0.85
6	Off	On	Off	0.73	Off	0.71
7	Off	On	Off	0.89	Off	0.97
8	Off	On	Off	0.71	Off	0.70
9	Off	On	Off	0.70	Off	0.70
10	Off	On	Off	0.71	Off	0.71
11	Off	On	Off	0.70	Off	0.75
12	Off	On	Off	0.70	Off	0.94
13	Off	On	Off	0.70	Off	0.72
14	Off	On	Off	0.71	Off	0.71
15	Off	On	Off	0.75	Off	0.70
16	Off	On	Off	0.70	Off	0.71
17	Off	On	Off	0.70	0.73	0.72
18	Off	On	Off	0.81	0.85	0.96
19	Off	On	Off	0.96	0.74	0.97
20	Off	On	Off	0.99	0.74	0.93
21	Off	On	Off	0.89	0.76	0.87
22	Off	On	Off	0.90	0.79	0.82
23	Off	On	On	0.71	0.72	0.76
24	On	On	On	0.99	0.89	0.90

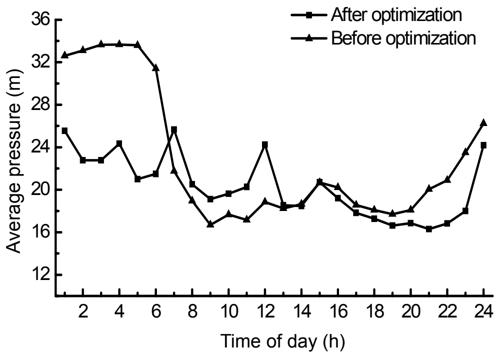

Fig. 10 Comparison of hourly average service pressure before and after optimization

7 Conclusions

We have described a two-stage method for the optimal scheduling of a new class of MTMS water supply systems containing fixed and variable speed pumps. This MTMS system is common in China's WDS. In the first stage, the ground-level storage tanks are modeled as demand nodes, and the optimal tank storage policy is determined according to the electricity tariff. It aims to store as full as possible during the inexpensive period and as empty as possible during the expensive period. This storage policy fully takes advantage of the storage capacity, but reduces the reliability in water supply to some extent.

Genetic algorithm combined with a repairing scheme is used to solve the pump scheduling problem. Application of the proposed method to a pressure zone of the S. Y. water system indicates the potential for the great cost savings and improvement in the network service pressure.

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