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A simplified MMSE-based iterative receiver for MIMO systems

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Abstract: A simplified minimum mean square error (MMSE) detector is proposed for joint detection and decoding of multiple-input multiple-output (MIMO) systems. The matrix inversion lemma and the singular value decomposition (SVD) of the channel matrix are used to simplify the computation of the coefficient of the MMSE filter. Compared to the original MMSE detector, the proposed detector has a much lower computational complexity with only a marginal performance loss. The proposed detector can also be applied to MIMO systems with high order modulations.

Key words: Multiple-input multiple-output (MIMO), Minimum mean square error (MMSE), Matrix inversion lemma, Singular value decomposition (SVD)
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INTRODUCTION

The design of an efficient and effective multiple-input multiple-output (MIMO) detector is a challenging task in coded MIMO systems (Foschini and Gans, 1998; Hassibi, 2000; Zhu *et al*., 2004; Rontogiannis *et al*., 2006). The maximum a posterior probability (MAP) algorithm achieves optimal performance. However, its high computational complexity is a major obstacle to its application in real MIMO systems. Other complexity-reduced suboptimal detectors have been proposed, such as the sphere decoder (SD) (Hochwald and Brink, 2003), Markov chain Monte Carlo method (Farhang-Boroujeny *et al*., 2006) and tree search algorithm (TS) (de Jong and Willink, 2005). These can achieve near-optimal performance but their complexities are still too high for real applications.

Among the soft-input soft-output detectors, the simplest are those that use the minimum mean square error (MMSE) criteria (El Gamal and Geraniotis, 2000; Sellathurai and Haykin, 2002; Choi, 2006). The original soft-input soft-output MMSE requires the computation of N_t (number of transmit antennas) matrix inversions for each iteration, which is computationally costly. In this letter, a simplified soft-input soft-output MMSE algorithm is proposed. The matrix inversion lemma and the singular value decomposition (SVD) of the channel matrix are used to avoid the matrix inverse calculation per iteration and time period. The simplified method retains almost the same performance as the original method while greatly reducing the computational complexity.

SYSTEM MODEL

Consider a joint detection-decoding MIMO (Tonello, 2001; Sellathurai and Haykin, 2002) wireless system with N_t transmit antennas and N_r receive antennas. The information sequence $\boldsymbol{x}_2$ is encoded to a sequence $\boldsymbol{x}_2'$ with an error correction code. Then, $\boldsymbol{x}_2'$ is interleaved to produce $\boldsymbol{x}_1$, which is then divided into several frames, each consisting of N_tM_c bits. Each frame is then mapped onto a MIMO symbol, denoted as $\boldsymbol{s}=[s_1\ s_2\ \dots s_{N_t}]^{\mathrm{T}}$, where s_i (i=1, 2, ···, N_t) is obtained by mapping M_c coded bits out of the frame onto a modulation constellation of size 2^{M_c}. M_c is the number of bits per constellation symbol. Assuming the total transmit power is E_s, the power constraint on each antenna is $E\{||s_i||^2\}=E_s/N_t$. Let $\boldsymbol{y}=[y_1\ y_2\ ...\ y_{N_t}]^T$ be the received signal vector. The MIMO channel is assumed to be flat fading, modeled as

$$\boldsymbol{y}=\boldsymbol{H}\boldsymbol{s}+\boldsymbol{n}, \quad (1)$$

where $\boldsymbol{H}$ is the channel matrix, and each element $h_{i,j}$ of the matrix denotes the channel fading coefficient between the jth transmit antenna and the ith receive antenna. The elements of the channel matrix are independent, complex-valued Gaussian random variables with zero mean and unit variance. $\boldsymbol{n}$ is an $N_r\times1$ vector whose elements are independent zero-mean complex Gaussian random variables with variance σ_n^2 per dimension.

SOFT-INPUT SOFT-OUTPUT MMSE DETECTOR

Soft-input soft-output MMSE can provide good performance in the high signal-to-noise ratios (SNR) region and its computational complexity is much lower than that of MAP detectors (Benesty *et al*., 2003; Lee *et al*., 2006; Tomasoni *et al*., 2006). An MMSE detector does not perform joint detection among all transmit antennas; it detects signals on each transmit antenna individually. The principle of such a detector is to view the $N_r\times N_t$ channel as N_t interfering $N_r\times1$ sub-channels. It uses the information provided by the decoder in each iteration to cancel the interference of the sub-channels.

First, we compute the a priori based symbol mean $\bar{s}_i$ and variance $var\{s_i\}$ of s_i.

An MMSE detector for the symbol s_i on the ith transmit antenna is a linear filter $\boldsymbol{w}_i$ that provides the symbol estimation (Sellathurai and Haykin, 2002; Choi, 2006)

$$\tilde{s}_i=\boldsymbol{w}_i^H\left(\boldsymbol{y}-\sum_{j=1,\,j\neq i}^{N_t}\bar{s}_j\boldsymbol{h}_j\right) \quad (2)$$

by minimizing the mean square error $E\{\|s_i-\tilde{s}_i\|^2\}$. We have

$$\boldsymbol{w}_i=\left(E\{\boldsymbol{y}\boldsymbol{y}^H\}\right)^{-1}E\{s_i\boldsymbol{y}\}, \quad (3)$$

where

$$E\{\boldsymbol{y}\boldsymbol{y}^H\}=\boldsymbol{H}\boldsymbol{R}_{ss,i}\boldsymbol{H}^H+\sigma_n^2\boldsymbol{I}_{N_r}, \quad (4)$$

$$\boldsymbol{R}_{ss,i}=\text{diag}\{var\{s_0\},\ \cdots,\ var\{s_{i-1}\},\ E_s,\ var\{s_{i+1}\},\ \cdots,\ var\{s_{N_t-1}\}\}. \quad (5)$$

We can evaluate Eq.(2) as

$$\tilde{s}_i=E_s\boldsymbol{h}_i^H(\boldsymbol{H}\boldsymbol{R}_{ss,i}\boldsymbol{H}^H+\sigma_n^2\boldsymbol{I}_{N_r})^{-1}\left(\boldsymbol{y}-\sum_{j=1,\,j\neq i}^{N_t}\bar{s}_j\boldsymbol{h}_j\right). \quad (6)$$

Thus, the matrix inversion is required for the estimation of each symbol during each iteration and each time period. Finally, the soft information on each coded bit is calculated. We assume that the estimated signal is transmitted on an equivalent AWGN channel with the equivalent Gaussian noise $\eta_i\sim N(0,\sigma_{\eta_i}^2)$, where

$$\sigma_{\eta_i}^2=\mu_i(1-\mu_i)E_s, \quad (7)$$

$$\mu_i=\boldsymbol{w}_i^H\boldsymbol{h}_i. \quad (8)$$

According to the AWGN channel assumption and the Bayes rule, the a posterior probability of the symbol s_i can be obtained:

$$p(s_i\,|\,\boldsymbol{y})\approx p(s_i\,|\,\tilde{s}_i)=\frac{p(\tilde{s}_i\,|\,s_i)P(s_i)}{P(\tilde{s}_i)}, \quad (9)$$

where

$$p(\tilde{s}_i\,|\,s_i=a_m)=\frac{1}{\pi\sigma_{\eta_i}^2}\exp\left(-\frac{\|\tilde{s}_i-\mu_i a_m\|^2}{\sigma_{\eta_i}^2}\right). \quad (10)$$

Then the a posterior information about the bit x_k can be written as

$$L_D(x_k\,|\,\tilde{s}_i)=\ln\frac{p(x_k=+1\,|\,\tilde{s}_i)}{p(x_k=-1\,|\,\tilde{s}_i)}=\ln\frac{\sum_{s_i\in A,x_k=+1}p(\tilde{s}_i\,|\,s_i)\exp(\boldsymbol{x}_i^T\boldsymbol{L}_{A,i}/2)}{\sum_{s_i\in A,x_k=-1}p(\tilde{s}_i\,|\,s_i)\exp(\boldsymbol{x}_i^T\boldsymbol{L}_{A,i}/2)}. \quad (11)$$

Here the bit x_k belongs to the bit vector $\boldsymbol{x}_i$ from which the symbol s_i is mapped, and $(i-1)M_c\le k\le iM_c-1$. $\boldsymbol{L}_{A,i}$ denotes the vector of a priori information corresponding to the bit vector $\boldsymbol{x}_i$.

SIMPLIFIED MMSE DETECTOR

The soft-input soft-output MMSE requires the computation of N_t matrix inversions for each iteration and time period. Some simplified algorithms have been proposed (Choi, 2006; Lee *et al*., 2006; Wang *et al*., 2007). These algorithms use a special form of the correlation matrix and the matrix inversion formula to avoid the computation of a matrix inversion per symbol and per iteration. We propose a new algorithm that can maintain the performance gain per iteration while greatly reducing the computational complexity associated with matrix inversion. Compared with other simplified algorithms, our proposed algorithm can be applied to high order constellations.

According to Gresset (2004) and Wang *et al*. (2007), the matrix inversion in Eq.(6) can be rewritten as

$$\begin{aligned}&(\boldsymbol{HR}_{\text{ss},i}\boldsymbol{H}^{\text{H}}+\sigma_{\text{n}}^2\boldsymbol{I}_{N_\text{r}})^{-1}\\&=\left(\sum_{j\neq i}var\{s_j\}\boldsymbol{h}_j\boldsymbol{h}_j^{\text{H}}+E_\text{s}\boldsymbol{h}_i\boldsymbol{h}_i^{\text{H}}+\sigma_{\text{n}}^2\boldsymbol{I}_{N_\text{r}}\right)^{-1}\\&=\left(\sum_{j=1}^{N_\text{t}}var\{s_j\}\boldsymbol{h}_j\boldsymbol{h}_j^{\text{H}}+(E_\text{s}-var\{s_i\})\boldsymbol{h}_i\boldsymbol{h}_i^{\text{H}}+\sigma_{\text{n}}^2\boldsymbol{I}_{N_\text{r}}\right)^{-1}.\end{aligned}\tag{12}$$

Denote $\boldsymbol{B}=\sum_{j=1}^{N_\text{t}}var\{s_j\}\boldsymbol{h}_j\boldsymbol{h}_j^{\text{H}}+\sigma_{\text{n}}^2\boldsymbol{I}_{N_\text{r}}$ and $\boldsymbol{a}_i=\boldsymbol{B}^{-1}\boldsymbol{h}_i$. By applying the matrix inversion formula (Zhang, 2004), we obtain

$$(\boldsymbol{HR}_{\text{ss},i}\boldsymbol{H}^{\text{H}}+\sigma_{\text{n}}^2\boldsymbol{I}_{N_\text{r}})^{-1}=\boldsymbol{B}^{-1}-\frac{(E_\text{s}-var\{s_i\})\boldsymbol{a}_i\boldsymbol{a}_i^{\text{H}}}{1+(E_\text{s}-var\{s_i\})\boldsymbol{h}_i^{\text{H}}\boldsymbol{a}_i}.\tag{13}$$

From Eq.(13), we can see that we need to compute only one matrix inversion in each iteration. For each antenna, we have to compute one 'matrix by vector' multiplication and two 'vector by vector' multiplications. Let

$$\begin{aligned}\boldsymbol{R}_{\text{ss}}=\text{diag}\big\{&var\{s_1\},\ \cdots,\ var\{s_{i-1}\},\ var\{s_i\},\\&var\{s_{i+1}\},\ \cdots,\ var\{s_{N_\text{t}}\}\big\}.\end{aligned}\tag{14}$$

Then $\boldsymbol{B}^{-1}$ can be rewritten as

$$\boldsymbol{B}^{-1}=\left(\boldsymbol{HR}_{\text{ss},i}\boldsymbol{H}^{\text{H}}+\sigma_{\text{n}}^2\boldsymbol{I}_{N_\text{r}}\right)^{-1}.\tag{15}$$

Without loss of generality, we assume $N_\text{t}=N_\text{r}=N$. The SVD of the channel matrix $\boldsymbol{H}$ (Liang *et al*., 2006) is $\boldsymbol{H}=\boldsymbol{USV}^{\text{H}}$, where

$$\begin{aligned}\boldsymbol{S}&=\text{diag}\{\delta_1,\ \delta_2,\ \cdots,\ \delta_N\},\\\boldsymbol{U}&=[\boldsymbol{u}_1\quad\boldsymbol{u}_2\quad\cdots\quad\boldsymbol{u}_N],\\\boldsymbol{V}&=[\boldsymbol{v}_1\quad\boldsymbol{v}_2\quad\cdots\quad\boldsymbol{v}_N].\end{aligned}$$

Substituting the decomposed matrix $\boldsymbol{H}$ into $\boldsymbol{HR}_{\text{ss}}\boldsymbol{H}^{\text{H}}$, we obtain

$$\begin{aligned}\boldsymbol{HR}_{\text{ss}}\boldsymbol{H}^{\text{H}}&=\boldsymbol{USV}^{\text{H}}\boldsymbol{R}_{\text{ss}}\boldsymbol{VSU}^{\text{H}}\\&=\boldsymbol{U}\begin{bmatrix}\delta_1&&&\\&\delta_2&&\\&&\ddots&\\&&&\delta_N\end{bmatrix}\begin{bmatrix}\boldsymbol{v}_1^{\text{H}}\\\boldsymbol{v}_2^{\text{H}}\\\vdots\\\boldsymbol{v}_N^{\text{H}}\end{bmatrix}\\&\quad\cdot\begin{bmatrix}var\{s_1\}&&&\\&var\{s_2\}&&\\&&\ddots&\\&&&var\{s_N\}\end{bmatrix}\\&\quad\cdot[\boldsymbol{v}_1\quad\boldsymbol{v}_2\quad\cdots\quad\boldsymbol{v}_N]\begin{bmatrix}\delta_1&&&\\&\delta_2&&\\&&\ddots&\\&&&\delta_N\end{bmatrix}\boldsymbol{U}^{\text{H}}\\&=\boldsymbol{U}\begin{bmatrix}\delta_1\boldsymbol{v}_1^{\text{H}}\\\delta_2\boldsymbol{v}_2^{\text{H}}\\\vdots\\\delta_N\boldsymbol{v}_N^{\text{H}}\end{bmatrix}\begin{bmatrix}var\{s_1\}&&&\\&var\{s_2\}&&\\&&\ddots&\\&&&var\{s_N\}\end{bmatrix}\\&\quad\cdot[\delta_1\boldsymbol{v}_1\quad\delta_2\boldsymbol{v}_2\quad\cdots\quad\delta_N\boldsymbol{v}_N]\boldsymbol{U}^{\text{H}}.\end{aligned}\tag{16}$$

When MMSE is implemented, it can be observed that when the detector works at the first iteration, the variance of all symbols is one. When the SNR is high and the iteration number increases, the variance on all antennas approaches zero. If we assume that the variance of each signal of the same iteration is equal, that is $var\{s_1\}=var\{s_2\}=\cdots=var\{s_N\}=c$, then Eq.(16) can be simplified as

$$\boldsymbol{HR}_{\text{ss}}\boldsymbol{H}^{\text{H}}=\boldsymbol{U}\begin{bmatrix}c\delta_1^2&&&\\&c\delta_2^2&&\\&&\ddots&\\&&&c\delta_N^2\end{bmatrix}\boldsymbol{U}^{\text{H}}.\tag{17}$$

We can evaluate the calculation of $\boldsymbol{B}^{-1}$ as

$$\boldsymbol{B}^{-1}=\sum_{j=1}^{N}\frac{1}{c\delta_j^2+\sigma_\mathrm{n}^2}\boldsymbol{u}_j\boldsymbol{u}_j^\mathrm{H}\ . \tag{18}$$

In fact, the variance of each signal is different in most cases. So we let

$$c=\frac{1}{N_\mathrm{t}}\sum_{j=1}^{N_\mathrm{t}}var\{s_j\}\ . \tag{19}$$

Then we can use Eq.(18) to calculate $\boldsymbol{B}^{-1}$. The computational complexity associated with the matrix inversion is thus greatly reduced. If the channel is block fading (i.e., the channel is constant during L frames of time), the algorithm can be further simplified. We use the mean over L frames:

$$c=\frac{1}{LN_\mathrm{t}}\sum_{i=1}^{L}\sum_{j=1}^{N_\mathrm{t}}var\{s_j\}\ . \tag{20}$$

Then for the L frames, the calculation of matrix inversion $\boldsymbol{B}^{-1}$ is performed only once. Thus, the calculation of N_t matrix inversions per iteration and time period is avoided.

When constant module constellation is employed, we have

$$c=\frac{1}{LN_\mathrm{t}}\sum_{i=1}^{L}\sum_{j=1}^{N_\mathrm{t}}var\{s_j\}=\frac{1}{LN_\mathrm{t}}\sum_{i=1}^{L}\sum_{j=1}^{N_\mathrm{t}}(1-\overline{s}_j^2)\ . \tag{21}$$

Compared with Choi's algorithm (Choi, 2006), our proposed algorithm can be applied to various constellations and its usage is not limited to constant modulus constellations. Choi's algorithm assumes a constant module transmit signal and this is the basis for its derivation. However, there is no such assumption in this paper.

In the quasi-static channel, channel varies per L frames of time. Then, in the original soft-input soft-output MMSE, the complexity of performing matrix inversions per L frames of time is $\mathcal{O}(N_\mathrm{r}^3N_\mathrm{t}L)$. Using our simplified algorithm, only one SVD of channel matrix is performed and its complexity is $\mathcal{O}(N_\mathrm{r}^3)$. Using Choi's algorithm, an eigenvalue decomposition (EVD) is performed per L frames and its complexity is also $\mathcal{O}(N_\mathrm{r}^3)$. So our proposed algorithm has almost the same complexity as Choi's algorithm but it can be applied to various constellations. Table 1 shows a comparison of the complexity of the different algorithms in a time period of L frames. Here we consider only the computational complexity associated with the calculation of the MMSE filter $\boldsymbol{w}_i$. Channel is assumed to be constant during L frames and the receiver performs N_iter outer iterations.

Table 1 Complexity comparison between different MMSE algorithms

Algorithm	Multiplication	Matrix inversion	SVD	EVD
Original MMSE	$\mathcal{O}(LN_\mathrm{t}N_\mathrm{iter}\cdot(4N_\mathrm{r}^2N_\mathrm{t}+4N_\mathrm{r}^2+5N_\mathrm{r}N_\mathrm{t}))$	$\mathcal{O}(LN_\mathrm{t}N_\mathrm{iter}N_\mathrm{r}^3)$	–	–
Proposed MMSE	$\mathcal{O}(N_\mathrm{iter}(4N_\mathrm{r}^3+4N_\mathrm{r}^2+8N_\mathrm{r}^2N_\mathrm{t}+8N_\mathrm{t}N_\mathrm{r}))$	–	$\mathcal{O}(N_\mathrm{r}^3)$	–
Choi's	$\mathcal{O}(N_\mathrm{iter}(4N_\mathrm{r}^2N_\mathrm{t}+4N_\mathrm{r}^3+12N_\mathrm{r}N_\mathrm{t}+2N_\mathrm{r}^2))$	–	–	$\mathcal{O}(N_\mathrm{r}^3)$

SVD: singular value decomposition; EDV: eigenvalue decomposition

RESULTS

In this section, we compare the BER performance of the original algorithm, the proposed algorithm, and Choi's algorithm. We considered a MIMO system with four transmit antennas and four receive antennas. Channel was assumed to be block fading and constant during 21 frames (L=21). Each element of the channel matrix $\boldsymbol{H}$ was an independent circularly symmetric complex Gaussian random variable with mean zero and variance unity. We denoted the average transmit power on each antenna as $E_\mathrm{s}/N_\mathrm{t}$ and the average received power on receive antenna as E_s. The SNR per bit on each receive antenna was defined as $E_\mathrm{b}/N_0=E_\mathrm{s}N_\mathrm{r}/(N_\mathrm{t}M_\mathrm{c}RN_0)$, where R is the coding rate. A half-rate (R=1/2) turbo code was used with generating polynomial (5, 7) in octal. For turbo coding, a random bit interleaver with 2048 length was employed. For the receiver, 5 iterations were assumed between the detector and the decoder and 8 iterations for the turbo decoder. For simulations, natural mapping QPSK, 16QAM, and 64QAM were employed.

Fig.1 shows the BER performance of the different algorithms when QPSK modulation was used. Fig.1 shows that the three algorithms had the same performance in the first iteration. This is because when there is no a priori information, the three algorithms use the same formulas. In the fifth iteration, the proposed algorithm showed a slight loss of performance compared with the original algorithm.

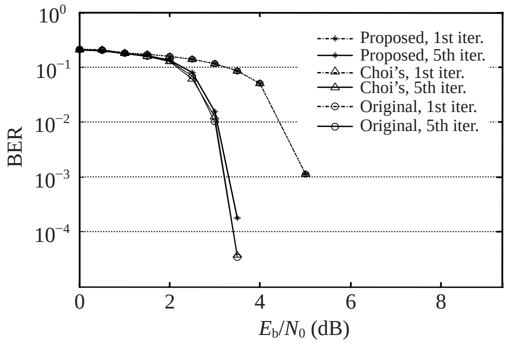

Fig.1 BER performance of QPSK over 4×4 MIMO channel

Fig.2 shows the BER performance of the different algorithms using 16QAM. In the first iteration, the three algorithms had identical performance. In the fifth iteration, our proposed algorithm showed a slight loss of performance compared with the original MMSE. Choi's algorithm showed the worst performance: the BER of Choi's algorithm at the fifth iteration was higher than that of our proposed algorithm even at the first iteration at 5~10 dB. The BER performance of the three algorithms using 64QAM showed a similar result (Fig.3). Choi's algorithm is not applicable to non-constant module modulation and it cannot make the iterative system convergent. Our proposed algorithm can be employed with various modulations and has better applicability.

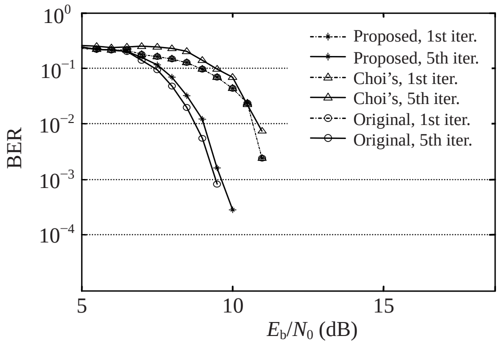

Fig.2 BER performance of 16QAM over 4×4 MIMO channel

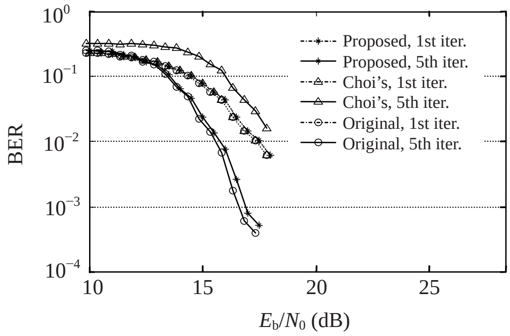

Fig.3 BER performance of 64QAM over 4×4 MIMO channel

CONCLUSION

A simplified MMSE detector is proposed for iterative detection/decoding MIMO systems. Compared with the original algorithm, the proposed algorithm can greatly reduce computational complexity while retaining a good BER performance. Compared with Choi's algorithm, it has better applicability and can be used for various constellations.

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