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Controlling traffic jams on a two-lane road using delayed-feedback signals*

Liang ZHENG†, Shi-quan ZHONG†‡, Shou-feng MA

(*Institute of Systems Engineering, College of Management and Economics, Tianjin University, Tianjin 300072, China*)

†E-mail: liangzheng_tju@hotmail.com; shiquan_tju@163.com

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Abstract: This paper focuses mainly on the stability analysis of two-lane traffic flow with lateral friction, which may be caused by irregular driving behavior or poorly visible road markings, and also attempts to reveal the formation mechanism of traffic jams. Firstly, a two-lane optimal velocity (OV) model without control signals is proposed and its stability condition is obtained from the viewpoint of control theory. Then delayed-feedback control signals composed of distance headway information from both lanes are added to each vehicle and a vehicular control system is designed to suppress the traffic jams. Lane change behaviors are also incorporated into the two-lane OV model and the corresponding information about distance headway and feedback signals is revised. Finally, the results of numerical experiments are shown to verify that when the stability condition is not met, the position disturbances and resulting lane change behaviors do indeed deteriorate traffic performance and cause serious traffic jams. However, once the proper delayed-feedback control signals are implemented, the traffic jams can be suppressed efficiently.

Key words: Optimal velocity (OV) model, Two-lane traffic flow, Lateral friction, Stability analysis, Delayed-feedback signals
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1 Introduction

Traffic problems have attracted much attention for decades. To understand the complex phenomena of traffic flow, various traffic flow models have been proposed in recent decades. Based on the level of detail described, these models can be classified into three major categories (Hoogendoorn and Bovy, 2001): macroscopic, mesoscopic, and microscopic models. Here, we concentrate mainly on one of the most representative microscopic models, the optimal velocity (OV) model proposed by Bando *et al.* (1995), in which each vehicle is described by a simple differential model and each driver controls the velocity based on an OV function. Their paper presented the traffic congestion under periodic conditions and derived a simple stability condition for the OV model. Since then, researchers have explored the OV model from different aspects and have used it to analyze various traffic density waves so as to obtain the stability conditions in different situations. These developments included deducing a modified Korteweg-de Vries (mKdv) equation (Komatsu and Sasa, 1995) from the OV model, the introduction of delay time (Davis, 2002), incorporation of relative speed (Jiang *et al.*, 2001), and the development of intelligent transportation systems (ITSs), taking into account the stimulus of not only the vehicle ahead but also many vehicles ahead of and (or) behind the driver (Lenz *et al.*, 1999; Nagatani, 1999; Nakayama *et al.*, 2001; Hasebe *et al.*, 2003; Ge *et al.*, 2006; Sun *et al.*, 2011). Such OV models are found to be a rich source of dynamic behaviors, which are the key to explaining wave features in highway traffic (Gasser *et al.*, 2004; Schönhof and Helbing, 2007).

‡ Corresponding author

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With regard to the stability analysis of two-lane or multi-lane traffic flows, Kurata and Nagatani (2003) and Nagai *et al.* (2005) studied the spatial-temporal dynamics of jam transitions induced by a bus or blockage on a two-lane highway. Tang *et al.* (2005; 2008) recently put forward new two-lane car-following models with consideration of the lateral distance and potential lane changes, which have improved the stability of traffic flow. When analysing the stability of two-lane traffic flows, it is necessary to consider the lateral discomfort (i.e., lateral friction) caused by the weak discipline of lane-based driving (Gunay, 2007). In this paper, we concentrate on the internal friction existing between vehicles moving in the same direction, which is one of four types of friction (May, 1959). Recently, a non-lane-based full velocity difference (FVD) car-following model (Jin *et al.*, 2010) was proposed and used to analyze the influence of lateral friction on the stability of traffic flow. This model inspired our study of the lateral friction in two-lane or multi-lane traffic flows. The question of how to control traffic jams has attracted much attention in the field of transportation. Kerner (2005) proposed a congested pattern control approach to alleviate traffic congestion at highway bottlenecks and drew some important conclusions. With the development of ITS, advanced traffic control systems, such as ramp metering (Papageorgiou *et al.*, 1997; Smaragdis and Papageorgiou, 2003), variable speed limit (Alessandri *et al.*, 1999; Papageorgiou *et al.*, 2008) and coordinated traffic control strategies (Kotsialos *et al.*, 2002; Hegyi *et al.*, 2005) have developed greatly. In particular, adaptive cruise control (ACC) systems have been implemented for improving road capacity and decreasing traffic congestion (Davis, 2004; Zhou and Peng, 2005; van Arem *et al.*, 2006; Kesting *et al.*, 2008). The control of traffic congestion has also attracted the attention of researchers in nonlinear science during recent decades and various control methods have been implemented to suppress chaotic behavior in traffic flows. For instance, Konishi *et al.* (1999; 2000) proposed various versions of the decentralized delayed-feedback control (DDFC) method and applied them to alleviate traffic jams on the road.

However, little attention has been paid recently to the design of vehicular control systems for two-lane traffic flows from the viewpoint of control theory. Chen *et al.* (2007) extended a single-lane feedback control model proposed by Zhao and Gao (2005) to a two-lane optimal velocity feedback control (OVFC) model and made some impressive conclusions. However, they did not give mathematical equations to describe two-lane traffic flows and neglected lateral friction and lane change behaviors. The stability analysis of two-lane traffic flows from the viewpoint of control theory should be regarded as an important issue because: (1) Two-lane traffic flow better accords with real traffic. (2) It is crucial to reveal the mechanism of interaction between vehicles from different lanes, which is an important factor in the formation of traffic jams and the design of vehicular control systems. Thus, there is a need for a mathematical model for two-lane traffic flow that takes account of lateral friction and lane change behaviors, and for an analysis of the stability conditions based on the well-known DDFC method.

This paper is organized as follows: Section 2 explains the OV model for two-lane traffic flow and analyzes the stability conditions. In Section 3, the DDFC method is extended to the two-lane case and utilized to suppress traffic jams, and then a simple procedure for the design of control systems is introduced. Section 4 introduces lane change rules and the corresponding modifications of the comprehensive distance headway and feedback control signals. Numerical simulations are provided in Section 5 to confirm the theoretical results. Finally, conclusions are presented in Section 6.

2 Two-lane optimal velocity model

2.1 Description of the model

Usually, because of irregular driver attitudes or poor road surfaces or markings, vehicles are not always positioned in the centre of a lane, and once some are off centre and close to the neighboring lane they may cause lateral friction (i.e., lateral discomfort) to drivers in the neighboring lane and influence their driving behavior (Gunay, 2007). In this condition, any vehicle causing lateral friction to drivers in the neighboring lane should be considered in studies of vehicle systems. Such a vehicle and its movement are influenced not only by the nearest preceding vehicle in its own lane but also by the lateral friction caused by the nearest preceding vehicle in the neighboring lane. If the lateral separation between two vehicle groups (denoted by LS) is smaller than the width of a typical lane, indicated by $LS_{\max}$, each driver will experience lateral discomfort from the neighboring lane and the smaller is LS the more serious is the lateral discomfort perceived. Therefore, the driver of vehicle n_1 in lane 1 should not only pay attention to the behaviors of vehicle n_1-1 in front, so as to avoid a rear-end accident, but should also notice the potential safety threat resulting from the irregular driving or lane change behaviors of vehicle n_2 in lane 2 (Fig. 1). These behaviors should not be neglected when two-lane or multi-lane traffic flows are modeled.

Therefore, based on the OV model and under the condition of $LS<LS_{\max}$, the vehicle dynamic in lane l is given as

$$\begin{cases} \dfrac{\mathrm{d}v_{l,n_l}(t)}{\mathrm{d}t} = a_l[F_l(\bar{y}_{l,n_l}(t)) - v_{l,n_l}(t)], \\ \dfrac{\mathrm{d}y_{l,n_l}(t)}{\mathrm{d}t} = v_{l,n_l-1}(t) - v_{l,n_l}(t), \\ \dfrac{\mathrm{d}q_{l,n_l}(t)}{\mathrm{d}t} = v^{\mathrm{f}}_{l,n_l}(t) - v_{l,n_l}(t), \end{cases} \tag{1}$$

where n_l=1,2,…,N_l, and N_l is the total number of all vehicles in lane l, $v_{l,n_l}(t)$ and $F_l(\bar{y}_{l,n_l}(t))$ are the velocity of vehicle n_l and the OV function in lane l, respectively, and $y_{l,n_l}(t)$ and $q_{l,n_l}(t)$ are the longitudinal distance (i.e., the distance between two vehicles n_l-1 and n_l in lane l at time t) and the lateral distance (i.e., the distance between the vehicle n_l in lane l and the closest preceding vehicle in the neighboring lane) respectively. $\bar{y}_{l,n_l}(t) = \alpha_l^y y_{l,n_l}(t) + \alpha_l^q \cdot q_{l,n_l}(t)$ is the comprehensive distance headway, where $\alpha_l^y = LS/LS_{\max}$ and $\alpha_l^q = 1 - LS/LS_{\max}$ denote the weights of the information about the distance headway from lane l and the neighboring lane, respectively. $v^{\mathrm{f}}_{l,n_l}(t)$ is the velocity of the nearest preceding vehicle in the neighboring lane. a_l>0 is the driver's sensitivity with respect to the difference between the optimal and current velocity in lane l.

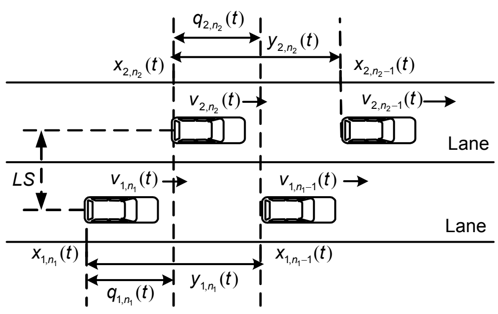

Fig. 1 Interaction between two vehicle groups on a two-lane road

From Fig. 1 and vehicular dynamics Eq. (1), it is known that the behavior of vehicle n_l is determined by two factors: one is the block effect from the nearest vehicle in front in lane l, i.e., if the longitudinal distance $y_{l,n_l}(t)$ decreases, vehicle n_l will decelerate to avoid a rear-end collision; the other is the potential safety threat from the neighboring lane. This occurs when the lateral distance $q_{l,n_l}(t)$ decreases and the psychological state of the n_lth driver becomes nervous and the driver slows down to reduce the uncomfortable friction. Based on the assumption that the stable velocity of all vehicles is v_0, the whole vehicular system has the following steady state:

$$(v^*_{l,n_l}, \bar{y}^*_l)^T = (v_0, F_l^{-1}(v_0))^T.$$

Once weight coefficients α_l^y, α_l^q and the desired longitudinal distance y_l^* are determined, the lateral distance q_l^* can be designed and the final steady state of the whole vehicular system can be expressed as follows:

$$(v_0, y_l^*, q_l^*)^T. \tag{2}$$

2.2 Stability analysis

To obtain the stability conditions of such a two-lane traffic flow model, firstly let the vehicular dynamics Eq. (1) be linearized around a steady state Eq. (2), and then the linearized vehicular dynamics in lane l can be written as

$$\begin{cases} \dfrac{\mathrm{d}v_{l,n_l}(t)^{\mathrm{o}}}{\mathrm{d}t} = a_l\{\Lambda_l^y y_{l,n_l}(t)^{\mathrm{o}} + \Lambda_l^q q_{l,n_l}(t)^{\mathrm{o}} - v_{l,n_l}(t)^{\mathrm{o}}\}, \\ \dfrac{\mathrm{d}y_{l,n_l}(t)^{\mathrm{o}}}{\mathrm{d}t} = v_{l,n_l-1}(t)^{\mathrm{o}} - v_{l,n_l}(t)^{\mathrm{o}}, \\ \dfrac{\mathrm{d}q_{l,n_l}(t)^{\mathrm{o}}}{\mathrm{d}t} = v^{\mathrm{f}}_{l,n_l}(t)^{\mathrm{o}} - v_{l,n_l}(t)^{\mathrm{o}}, \end{cases} \tag{3}$$

where $v_{l,n_l}(t)^{\mathrm{o}} = v_{l,n_l}(t) - v_0$, $v_{l,n_l-1}(t)^{\mathrm{o}} = v_{l,n_l-1}(t) - v_0$, $y_{l,n_l}(t)^{\mathrm{o}} = y_{l,n_l}(t) - y_l^*$, $q_{l,n_l}(t)^{\mathrm{o}} = q_{l,n_l}(t) - q_l^*$, $v_{l,n_l}^{\mathrm{f}}(t)^{\mathrm{o}} = v_{l,n_l}^{\mathrm{f}}(t) - v_0$. Λ_l^y and Λ_l^q are partial derivatives of $F_l(y_{l,n_l}(t), q_{l,n_l}(t))$ in the directions $y_{l,n_l}(t)$ and $q_{l,n_l}(t)$, respectively.

$$\Lambda_l^y = \left. \frac{\partial F_l(y_{l,n_l}(t), q_{l,n_l}(t))}{\partial y_{l,n_l}(t)} \right|_{y_{l,n_l}(t)=y_l^*, q_{l,n_l}(t)=q_l^*},$$

$$\Lambda_l^q = \left. \frac{\partial F_l(y_{l,n_l}(t), q_{l,n_l}(t))}{\partial q_{l,n_l}(t)} \right|_{y_{l,n_l}(t)=y_l^*, q_{l,n_l}(t)=q_l^*}.$$

From the viewpoint of control theory, vehicular dynamics Eq. (3) can be rewritten as a linear time-invariant system, that is

$$\begin{bmatrix} \frac{\mathrm{d}v_{l,n_l}(t)^{\mathrm{o}}}{\mathrm{d}t} \\ \frac{\mathrm{d}y_{l,n_l}(t)^{\mathrm{o}}}{\mathrm{d}t} \\ \frac{\mathrm{d}q_{l,n_l}(t)^{\mathrm{o}}}{\mathrm{d}t} \end{bmatrix} = \begin{bmatrix} -a_l & a_l\Lambda_l^y & a_l\Lambda_l^q \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_{l,n_l}(t)^{\mathrm{o}} \\ y_{l,n_l}(t)^{\mathrm{o}} \\ q_{l,n_l}(t)^{\mathrm{o}} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} v_{l,n_l-1}(t)^{\mathrm{o}} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_{l,n_l}^{\mathrm{f}}(t)^{\mathrm{o}}.$$

After Laplace transformation, we have

$$s\begin{bmatrix} V_{l,n_l}(s)^{\mathrm{o}} \\ Y_{l,n_l}(s)^{\mathrm{o}} \\ Q_{l,n_l}(s)^{\mathrm{o}} \end{bmatrix} = \begin{bmatrix} -a_l & a_l\Lambda_l^y & a_l\Lambda_l^q \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{l,n_l}(s)^{\mathrm{o}} \\ Y_{l,n_l}(s)^{\mathrm{o}} \\ Q_{l,n_l}(s)^{\mathrm{o}} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} V_{l,n_l-1}(s)^{\mathrm{o}} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V_{l,n_l}^{\mathrm{f}}(s)^{\mathrm{o}}, \tag{4}$$

where $V_{l,n_l}(s)^{\mathrm{o}} := L[v_{l,n_l}(t)^{\mathrm{o}}]$, $Y_{l,n_l}(s)^{\mathrm{o}} := L[y_{l,n_l}(t)^{\mathrm{o}}]$, $Q_{l,n_l}(s)^{\mathrm{o}} := L[q_{l,n_l}(t)^{\mathrm{o}}]$, $V_{l,n_l-1}(s)^{\mathrm{o}} := L[v_{l,n_l-1}(t)^{\mathrm{o}}]$, $V_{l,n_l}^{\mathrm{f}}(s)^{\mathrm{o}} := L[v_{l,n_l}^{\mathrm{f}}(t)^{\mathrm{o}}]$. L denotes the Laplace transformation, s=jω where j is the imaginary unit and ω is the frequency. From Eq. (4) we can take the following forms:

$$V_{l,n_l}(s)^{\mathrm{o}} = \frac{a_l\Lambda_l^y}{s^2 + a_l s + a_l(\Lambda_l^y + \Lambda_l^q)} V_{l,n_l-1}(s)^{\mathrm{o}} + \frac{a_l\Lambda_l^q}{s^2 + a_l s + a_l(\Lambda_l^y + \Lambda_l^q)} V_{l,n_l}^{\mathrm{f}}(s)^{\mathrm{o}}. \tag{5}$$

Therefore, the main relationship of backward propagation of velocity fluctuation (i.e., the velocity fluctuations of the vehicle n_l-1 in lane l and the closest preceding vehicle in the neighboring lane propagated to the velocity fluctuation of vehicle n_l in lane l) can be analyzed as follows.

Based on the assumption that the maximum disturbance of vehicle n_l-1 in lane l (i.e., denoted by δ_{l,n_l-1}) is equal to that of the nearest preceding vehicle in the neighboring lane (indicated by $\delta_{l,n_l}^{\mathrm{f}}$), then the propagation relationship from δ_{l,n_l-1} or $\delta_{l,n_l}^{\mathrm{f}}$ to the velocity disturbance of vehicle n_l (denoted by δ_{l,n_l}) can be expressed as

$$\frac{\delta_{l,n_l}}{\delta_{l,n_l-1}} = \left| \frac{a_l(\Lambda_l^y + \Lambda_l^q)}{s^2 + a_l s + a_l(\Lambda_l^y + \Lambda_l^q)} \right|.$$

Moreover, only if $\delta_{l,n_l} / \delta_{l,n_l-1} \le 1$, would the velocity disturbance not be amplified when propagated backwards. Based on the above analysis, the brief Eq. (5) can be rewritten in the following form:

$$V_{l,n_l}(s)^{\mathrm{o}} = G_l(s) V_{l,n_l-1}(s)^{\mathrm{o}}, \tag{6}$$

where the transfer function in lane l is $G_l(s)$= $a_l(\Lambda_l^y + \Lambda_l^q)/d_l(s)$ and $d_l(s) = s^2 + a_l s + a_l(\Lambda_l^y + \Lambda_l^q)$ is the characteristic polynomial of $G_l(s)$.

From definition 1 of Konishi *et al.* (2000), it is known that if the characteristic polynomial is stable and the H_∞-norm of transfer function is equal to or less than 1, then traffic jams will not occur. That is, the stability of the characteristic polynomials is the necessary and sufficient condition for the steady state to exist, and the H_∞-norm of transfer functions should not be larger than 1 to guarantee that the velocity fluctuation will not be amplified when propagated backwards. Therefore, after some algebraic operations, the stability condition in lane l is given as

$$a_l \geq 2(\Lambda_l^y + \Lambda_l^q). \tag{7}$$

Thus, the stability analysis can be summarized as follows:

Lemma a If the stability condition (i.e., Eq. (7)) is met for both lanes, traffic jams never occur.

Lemma b If the stability condition (i.e., Eq. (7)) is not met for either or both lanes, the velocity disturbance in one lane will be propagated backwards with growing amplitude to the neighboring lane because of the transfer effect of the lateral friction, which results finally in traffic jams.

3 Suppressing traffic jams

3.1 Description of the model with control signals

To alleviate traffic jams (e.g., **Lemma b**), a delayed-feedback control signal, $u_{l,n_l}(t)$, is added to the vehicular dynamics Eq. (1).

$$\begin{cases} \dfrac{\mathrm{d}v_{l,n_l}(t)}{\mathrm{d}t} = a_l[F_l(\bar{y}_{l,n_l}(t)) - v_{l,n_l}(t)] + u_{l,n_l}(t), \\ \dfrac{\mathrm{d}y_{l,n_l}(t)}{\mathrm{d}t} = v_{l,n_l-1}(t) - v_{l,n_l}(t), \\ \dfrac{\mathrm{d}q_{l,n_l}(t)}{\mathrm{d}t} = v_{l,n_l}^{\mathrm{f}}(t) - v_{l,n_l}(t), \end{cases} \tag{8}$$

where the control signal $u_{l,n_l}(t)$ is

$$\begin{aligned} u_{l,n_l}(t) = k_l^y[y_{l,n_l}(t) - y_{l,n_l}(t-\tau_l)] \\ + k_l^q[q_{l,n_l}(t) - q_{l,n_l}(t-\tau_l)], \end{aligned} \tag{9}$$

where k_l^y, k_l^q are the feedback gains, and τ_l is the delay time in lane l. The control signal $u_{l,n_l}(t)$ not only involves the difference between the current and past longitudinal distances (i.e., $y_{l,n_l}(t)$ and $y_{l,n_l}(t-\tau_l)$), which can be seen as the feedback information from lane l, but also relates to the difference between the current and past lateral distances (i.e., $q_{l,n_l}(t)$ and $u_{l,n_l}(t-\tau_l)$), which denote the feedback information about lateral friction from the neighboring lane. Note that if all vehicles are running with a stable velocity, the control signals $u_{l,n_l}(t)$ will vanish.

Obviously, the vehicular dynamics Eq. (8) involves a continuous-time version of the DDFC method (Konishi *et al.*, 2000). Around the steady state Eq. (2), the vehicle dynamics Eq. (8) with control signals Eq. (9) can be rewritten as a linear time-invariant system, that is

$$\begin{bmatrix} \dfrac{\mathrm{d}v_{l,n_l}(t)^{\mathrm{o}}}{\mathrm{d}t} \\ \dfrac{\mathrm{d}y_{l,n_l}(t)^{\mathrm{o}}}{\mathrm{d}t} \\ \dfrac{\mathrm{d}q_{l,n_l}(t)^{\mathrm{o}}}{\mathrm{d}t} \end{bmatrix} = \begin{bmatrix} -a_l & a_l\Lambda_l^y & a_l\Lambda_l^q \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_{l,n_l}(t)^{\mathrm{o}} \\ y_{l,n_l}(t)^{\mathrm{o}} \\ q_{l,n_l}(t)^{\mathrm{o}} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} v_{l,n_l-1}(t)^{\mathrm{o}} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_{l,n_l}^{\mathrm{f}}(t)^{\mathrm{o}} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_{l,n_l}(t), \tag{10}$$

where

$$u_{l,n_l}(t) = k_l^y[y_{l,n_l}(t)^{\mathrm{o}} - y_{l,n_l}(t-\tau_l)^{\mathrm{o}}] + k_l^q[q_{l,n_l}(t)^{\mathrm{o}} - q_{l,n_l}(t-\tau_l)^{\mathrm{o}}].$$

After the Laplace transformation for such a linear time-invariant system Eq. (10), we have

$$s\begin{bmatrix} V_{l,n_l}(s)^{\mathrm{o}} \\ Y_{l,n_l}(s)^{\mathrm{o}} \\ Q_{l,n_l}(s)^{\mathrm{o}} \end{bmatrix} = \begin{bmatrix} -a_l & a_l\Lambda_l^y & a_l\Lambda_l^q \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{l,n_l}(s)^{\mathrm{o}} \\ Y_{l,n_l}(s)^{\mathrm{o}} \\ Q_{l,n_l}(s)^{\mathrm{o}} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} V_{l,n_l-1}(s)^{\mathrm{o}} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V_{l,n_l}^{\mathrm{f}}(s)^{\mathrm{o}} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U_{l,n_l}(s), \tag{11}$$

where

$$U_{l,n_l}(s) := L(u_{l,n_l}(t)),$$

$$U_{l,n_l}(s) = k_l^y(1-\mathrm{e}^{-s\tau_l})Y_{l,n_l}(s)^{\mathrm{o}} + k_l^q(1-\mathrm{e}^{-s\tau_l})Q_{l,n_l}(s)^{\mathrm{o}}.$$

In system Eq. (11), it is also assumed that the maximum velocity disturbance for each lane is equal, i.e., $V_{l,n_l-1}(s)^{\mathrm{o}} = V_{l,n_l}^{\mathrm{f}}(s)^{\mathrm{o}}$. Then, after derivation we obtain the following transfer relationship of velocity disturbance in lane l:

$$V_{l,n_l}(s)^{\mathrm{o}} = G_l(s)^* V_{l,n_l-1}(s)^{\mathrm{o}}, \tag{12}$$

where the transfer function in lane l is $G_l(s)^* = a_l(\Lambda_l^y + \Lambda_l^q) + (k_l^y + k_l^q)(1 - \mathrm{e}^{-s\tau_l}) / d_l(s)^*$ and the characteristic polynomial of $G_l(s)^*$ is $d_l(s)^* = d_l(s) + k_l^y(1 - \mathrm{e}^{-s\tau_l}) + k_l^q(1 - \mathrm{e}^{-s\tau_l})$.

3.2 Design of the control system

After derivation of system Eq. (11), we can design a delayed-feedback control system for each vehicle so as to suppress or avoid traffic jams on a two-lane road, as shown in Fig. 2, where $H_{11} = k_l^y(1 - \mathrm{e}^{-s\tau_l})$, $H_{12} = k_l^q(1 - \mathrm{e}^{-s\tau_l})$, and the expression of $R(s)$ refers to Appendix A.

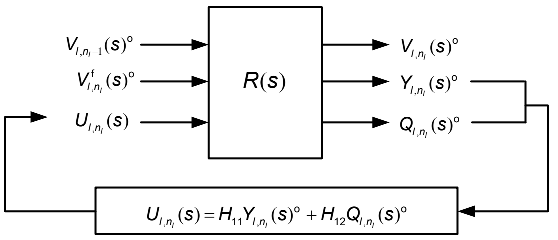

Fig. 2 Block diagram of the control system for the n_lth vehicle of lane l

From Fig. 2, it is clear that this system is a typical closed-loop feedback control system. In such a control system, velocity disturbances of vehicle n_l-1 and the closest preceding vehicle in the neighboring lane are the input signals, the comprehensive distance headway information from both lanes (i.e., $U_{l,n_l}(s)$) is the feedback control signal, and the output signals include the vehicle n_l's disturbances of velocity, longitudinal distance and lateral distance. In the following, we attempt to acquire the stability conditions for such a control system so as to make the three output signals vanish gradually. First of all, to make $d_l(s)^*$ stable, the well-known small gain theorem guarantees that $d_l(s)^*$ is stable if

$$\begin{cases} \| G_{33}(s) \|_\infty \| H_{12}(s) \|_\infty < 1, \\ \| G_{23}(s) \|_\infty \| H_{11}(s) \|_\infty < 1, \\ \| G_{23}(s)H_{11}(s) \|_\infty + \| G_{23}(s)H_{12}(s) \\ \quad \cdot [1 - G_{33}(s)H_{12}(s)]^{-1} G_{33}(s)H_{11}(s) \|_\infty < 1, \\ \| G_{33}(s)H_{12}(s) \|_\infty + \| G_{33}(s)H_{11}(s) \\ \quad \cdot [1 - G_{23}(s)H_{11}(s)]^{-1} G_{23}(s)H_{12}(s) \|_\infty < 1. \end{cases} \tag{13}$$

Moreover, under the condition of $a_l < 2(\Lambda_l^y + \Lambda_l^q)$, which does not agree with the stability condition Eq. (7), we can rewrite Eq. (13) after some algebraic operations as follows:

$$\begin{cases} \dfrac{a_l\sqrt{a_l[4(\Lambda_l^y + \Lambda_l^q) - a_l]}}{2} > |k_l^y| + \sqrt{(k_l^y)^2 + 4|k_l^y||k_l^q|}, \\ \dfrac{a_l\sqrt{a_l[4(\Lambda_l^y + \Lambda_l^q) - a_l]}}{2} > |k_l^q| + \sqrt{(k_l^q)^2 + 4|k_l^y||k_l^q|}. \end{cases} \tag{14}$$

Under the condition that $d_l(s)^*$ is stable, it should be confirmed that $\|G_l(s)^*\|_\infty$ is not larger than 1 if we want to guarantee that traffic jams never occur or would be suppressed on the road. Therefore, in the following, we try to design k_l^y, k_l^q and τ_l so as to make $\|G_l(s)^*\|_\infty$ be 1 or less. Therefore, from the viewpoint of frequency domain, we set $s=\mathrm{j}\omega$, then the absolute value of $G_l(s)^*$ can be written as

$$|G_l(j\omega)^*| = B_1 / B_2, \tag{15}$$

where $B_1 = \{a_l(\Lambda_l^y + \Lambda_l^q) + (k_l^y + k_l^q)[1 - \cos(\omega\tau_l)]\}^2 + [(k_l^y + k_l^q)\sin(\omega\tau_l)]^2$ and $B_2 = \{a_l(\Lambda_l^y + \Lambda_l^q) - \omega^2 + (k_l^y + k_l^q)[1 - \cos(\omega\tau_l)]\}^2 + [a_l\omega + (k_l^y + k_l^q)\sin(\omega\tau_l)]^2$.

Obviously, it is difficult to derive an analytical expression for $\|G_l(s)^*\|_\infty$ to be 1 or less, but through numerical experiments we can find the proper feedback gains k_l^y, k_l^q and delay time τ_l so that maximum values of $|G_l(\mathrm{j}\omega)^*|$ are not larger than 1 for all $\omega \in [0, +\infty)$ (Section 5.2).

From the above analysis, a theorem is derived as follows:

Theorem 1 If Eq. (14) is satisfied and $|G_l(\mathrm{j}\omega)^*|$ is not larger than 1 for all $\omega \in [0, +\infty)$ (i.e., $\|G_l(s)^*\|_\infty \le 1$), traffic jams can be suppressed efficiently once such delayed-feedback control signals are added.

4 Introduction of lane change rules

In the above sections, the comprehensive distance headway $\bar{y}_{l,n_l}(t)$ and feedback control signals $u_{l,n_l}(t)$ are formulated in the context of a relatively homogeneous traffic state, which would be broken by the lane change behaviors of any vehicle meeting the lane change conditions. Therefore, to demonstrate the influence of lane change behaviors on choosing the comprehensive distance headway and feedback control signals, firstly the selected lane change rules (Kurata and Nagatani, 2003; Nagai *et al*., 2005) can be described as follows:

$$y_{l,n_l}(t) < 2h_l^{\mathrm{f}}, \tag{16}$$

$$y_{l,n_l}(t) < q_{l,n_l}(t), \tag{17}$$

$$b_{l,n_l}(t) > h_l^{\mathrm{b}}, \tag{18}$$

where inequalities Eqs. (16) and (17) are incentive criteria for a lane change decision, inequality Eq. (18) is the security criterion, $b_{l,n_l}(t)$ is the distance between the vehicle n_l in lane l and the closest following vehicle in the neighboring lane at time t, h_l^{f} is the front safety distance in lane l, and h_l^{b} is the back safety distance in the neighboring lane.

If the lane change conditions for vehicle n_2 are met, it will change its lane to lane 1 just in front of vehicle n_1 (Fig. 3). After that, vehicle n_1's nearest preceding vehicle in the neighboring lane becomes vehicle n_2-1 and its lateral distance (i.e., the distance between vehicles n_1 and n_2-1) is significantly larger than the longitudinal distance (i.e., the distance between vehicles n_1 and n_2). If the inequalities Eqs. (16) and (18) are not met for vehicle n_1 in the next period of time, it will not change its lane to lane 2 and the relatively homogeneous traffic state will be broken. In this condition, vehicle n_1 is influenced only by the nearest preceding vehicle in its own lane (i.e., vehicle n_2). The reason can be explained as follows.

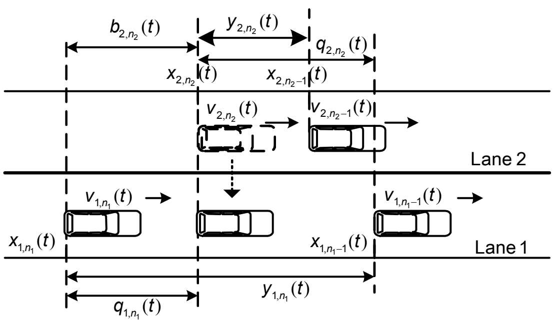

Fig. 3 Situation of lane change

If the lateral distance between the current vehicle and its nearest preceding vehicle in the neighboring lane is very large and the lane change conditions are not met for the current vehicle, according to the dynamic systems Eqs. (1) and (8), the current vehicle has to accelerate even though the longitudinal distance is not large enough. This violates the reality and may cause a rear-end accident. In this situation, the lateral friction from the neighboring lane should be removed, which means neglecting that part of the lateral distance in the comprehensive headway distance and the feedback control signals from the neighboring lane. Therefore, when introducing the lane change rules, the comprehensive headway distance and feedback control signals can be modified as follows:

(1) The comprehensive headway distance:

$$\bar{y}_{l,n_l}(t) = \alpha_l^y y_{l,n_l}(t) + \alpha_l^q f[q_{l,n_l}(t), y_{l,n_l}(t)],$$

where

$$f[q_{l,n_l}(t), y_{l,n_l}(t)] = \begin{cases} q_{l,n_l}(t), & \text{if } q_{l,n_l}(t) \le y_{l,n_l}(t), \\ y_{l,n_l}(t), & \text{if } q_{l,n_l}(t) > y_{l,n_l}(t). \end{cases}$$

(2) The feedback control signals:

$$u_{l,n_l}(t) = k_l^y(y_{l,n_l}(t) - y_{l,n_l}(t-\tau_l)) + k_l^q g[q_{l,n_l}(t), y_{l,n_l}(t)],$$

where $$g[q_{l,n_l}(t), y_{l,n_l}(t)] = \begin{cases} q_{l,n_l}(t) - q_{l,n_l}(t-\tau_l), \\ \qquad \text{if } q_{l,n_l}(t) \le y_{l,n_l}(t); \\ y_{l,n_l}(t) - y_{l,n_l}(t-\tau_l), \\ \qquad \text{if } q_{l,n_l}(t) > y_{l,n_l}(t). \end{cases}$$

5 Numerical simulations

In the numerical simulations, 100 vehicles were distributed on each lane on a road under open boundary condition (i.e., $N_1=N_2=100$). The OV function was $F_l(\bar{y}_{l,n_l}(t)) = \tanh[\bar{y}_{l,n_l}(t) - y_\mathrm{c}] + \tanh y_\mathrm{c}$ and the steady state was set as follows: $v_0=0.9354$ and $y_\mathrm{c}=1.7$ were desired velocity and distance respectively. $y_l^*=2$, $q_l^*=1$, $\alpha_l^y=0.7$, $\alpha_l^q=0.3$, $h_l^{\mathrm{f}}=0.7$, $h_l^{\mathrm{b}}=0.5$ for $l=1$ and 2. Therefore, the initial condition was chosen as follows: as for lane 1, $x_{1,100}(0)=2$,

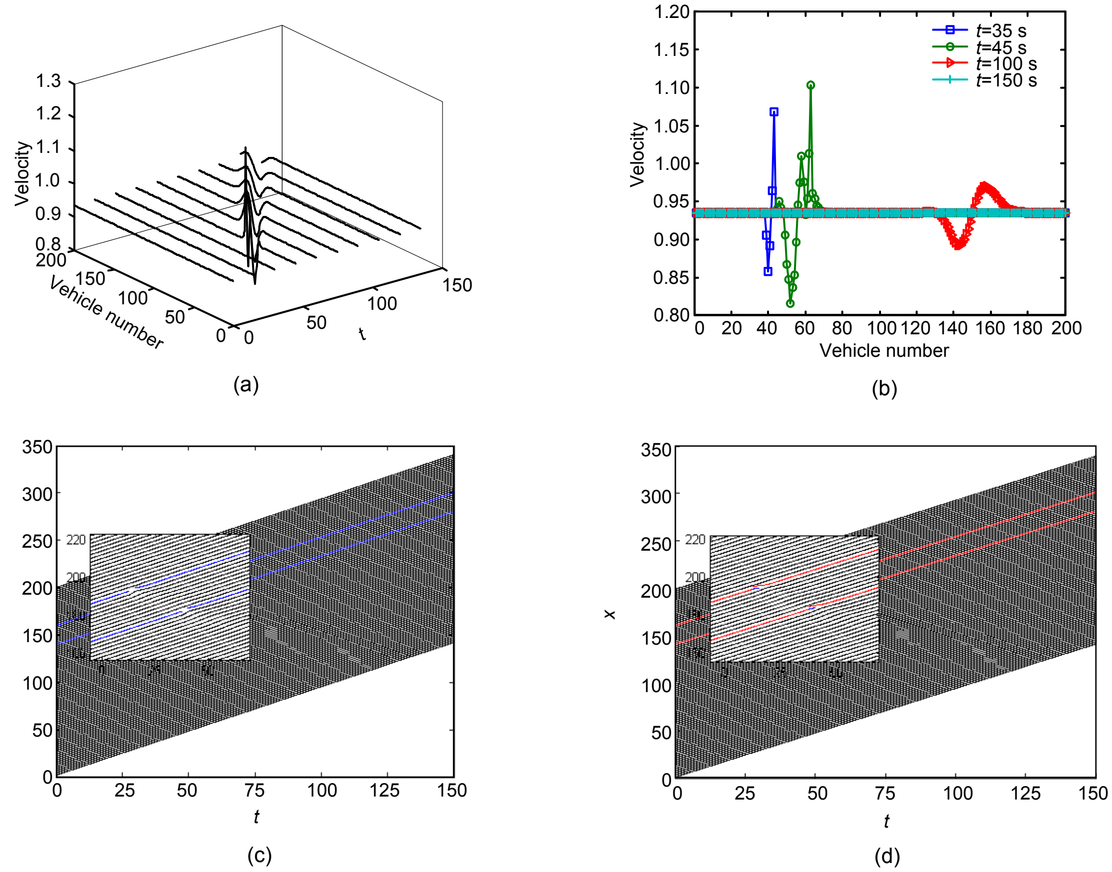

Fig. 4 Numerical simulation for two-lane traffic flow, a_1=3 and a_2=2

(a) Spatial-temporal evolution of the velocity fluctuation; (b) Profiles of the velocity fluctuation; (c) Spatial-temporal trajectories in lane 1; (d) Spatial-temporal trajectories in lane 2

Note: in (a) and (b) vehicles are numbered consecutively according to their initial positions. Blank areas in (c) denote that lane change behaviors occur

$x_{1,n_1-1}(0)=x_{1,n_1}(0)+y_1^*$ for n_1=100, 99, …, 2, and $v_{1,n_1}(0)=v_0$; as for lane 2, $x_{2,100}(0)=1$, $x_{2,n_1-2}(0)=x_{2,n_2}(0)+y_2^*$ for n_2=100, 99, …, 2, and $v_{2,n_2}(0)=v_0$. The time step Δt of simulation was 0.05 s.

To analyze the stability performance of traffic flow with and without feedback control signals, the following position disturbances could be added to some vehicles at some times, causing these vehicles to deviate from their steady states and trigger some lane change behaviors.

$$\text{If } t=35\text{ s}, \begin{cases} x_{2,20}(t)=x_{2,20}(t)+y_l^*\cdot 2/3, \\ x_{1,21}(t)=x_{1,21}(t)+q_l^*; \end{cases}$$

$$\text{if } t=45\text{ s}, \begin{cases} x_{2,30}(t)=x_{2,30}(t)+y_l^*\cdot 2/3, \\ x_{1,31}(t)=x_{1,31}(t)+q_l^*. \end{cases}$$

5.1 Performance of traffic flow without control signals

The initial velocity fluctuation is due to position disturbances and the resulting lane change behaviors but it gradually dissolves (Figs. 4a and 4b). This results from the sensitivities of drivers in the two lanes being 3 and 2, respectively (i.e., a_1=3 and a_2=2), which definitely meets **Lemma a**. Moreover, from the spatial-temporal trajectories of all vehicles in both lanes (Figs. 4c and 4d) it is further verified that the traffic flow finally returns to its original steady state, although there are two vehicles conducting lane change behaviors. However, when the two sensitivities a_1 and a_2 decrease to 1.0 and 1.5, respectively, which obviously satisfies **Lemma b**, the velocity fluctuation would be propagated backwards with growing vibration amplitude (Figs. 5a and 5b) and eventually cause serious traffic jams propagated upstream. This is also shown by the spatial-temporal trajectories of all vehicles on the road (Figs. 5c and 5d).

5.2 Suppressing the traffic jams with control signals

5.2.1 Determination of control parameters

To suppress the traffic jams shown in Fig. 5 efficiently, we have to determine the proper delay time τ_l and feedback gains k_l^y, k_l^q so as to meet **Theorem 1**.

As for lane 1 (i.e., l=1), based on the assumption of a_1=1.0 and $|k_1^y|=|k_1^q|$, and substituting these parameters into condition Eq. (14), we get $|k_1^y|<0.2676$ and $|k_1^q|<0.2676$. Then, fixing the feedback gains at $k_1^y=k_1^q=0.25$, Fig. 6a shows the absolute values of the transfer function (i.e., $|G_1(\mathrm{j}\omega)^*|$) for τ_1=0, 1 and 2 s. $|G_1(\mathrm{j}\omega)^*|$ has peaks greater than 1 when τ=0, which accords with the situation without control signals. $|G_1(\mathrm{j}\omega)^*|$ also has peaks greater than 1 when τ_1=2 s. However, when τ_1 is 1 s, the values are not all larger than 1 for all $\omega\in[0, +\infty)$. Therefore, only when the delay time τ_1 is 1 s do the conditions satisfy **Theorem 1**.

Under the condition of τ_1=1, we obtain the values of $|G_1(\mathrm{j}\omega)^*|$ when $k_1^y=k_1^q=-0.25$, $k_1^y=k_1^q=0$ and $k_1^y=k_1^q=0.25$ (Fig. 6b). The values when $k_1^y=k_1^q=0.25$ are equal to or less than 1 for all $\omega\in[0, +\infty)$. These numerical calculations guarantee that the traffic jams never occur or would be suppressed in lane 1 when $k_1^y=k_1^q=0.25$ and τ_1=1 s.

As for lane 2, (i.e., l=2), set a_1=1.5 and $|k_2^y|=|k_2^q|$, the process of determining feedback gains k_2^y, k_2^q and delay time τ_2 is similar to that for lane 1. Therefore, we find that traffic jams in lane 2 never occur or would vanish if the feedback gains

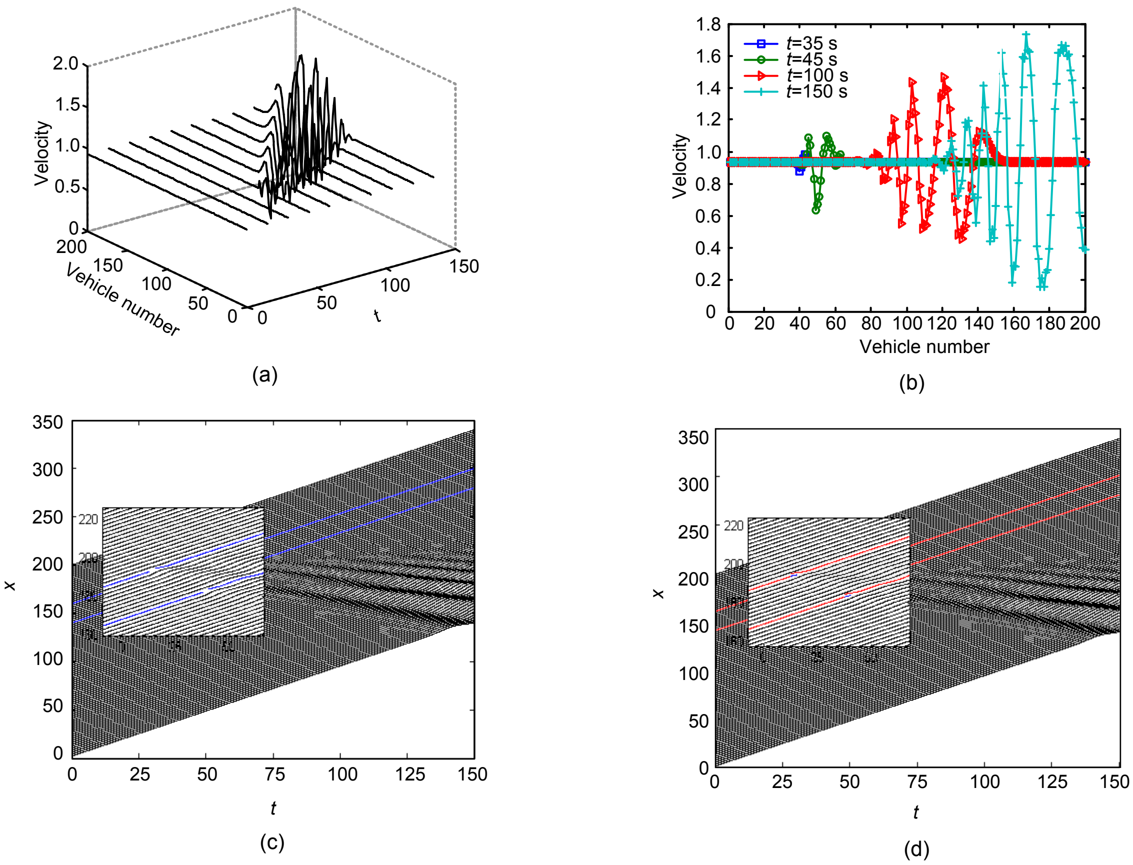

Fig. 5 Numerical simulation for two-lane traffic flow, a_1=1.0 and a_2=1.5

(a) Spatial-temporal evolution of the velocity fluctuation; (b) Profiles of the velocity fluctuation; (c) Spatial-temporal trajectories in lane 1; (d) Spatial-temporal trajectories in lane 2

Note: in (a) and (b) vehicles are numbered consecutively according to their initial positions. Blank areas in (c) denote that lane change behaviors occur (i.e., k_2^y and k_2^q) are both chosen as 0.4 and the delay time τ_2 is fixed at 1 s (Fig. 7).

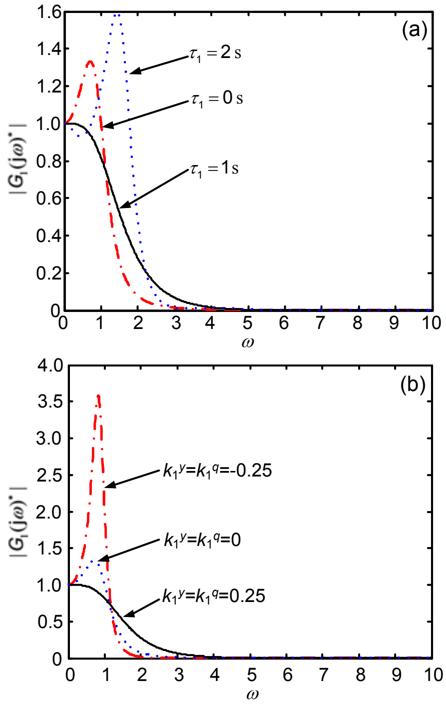

Fig. 6 Absolute value of $|G_1(j\omega)^*|$
(a) $k_1^y = k_1^q = 0.25$; (b) τ_1=1 s

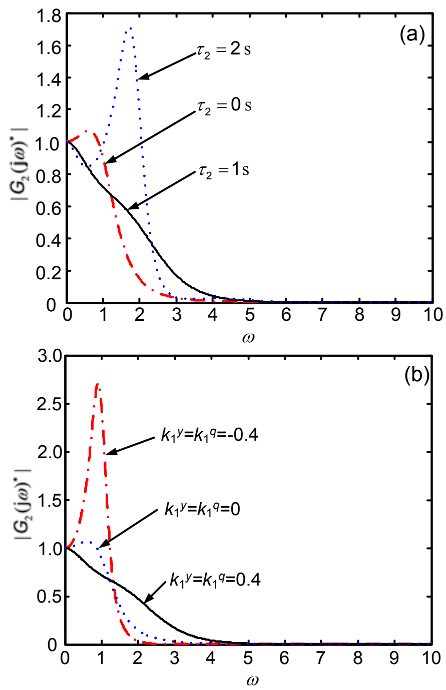

Fig. 7 Absolute value of $|G_2(j\omega)^*|$
(a) $k_1^y = k_1^q = 0.4$; (b) τ_2=1 s

5.2.2 Performance of traffic flow with control signals

Some conclusions can be drawn by comparing Fig. 5 with Fig. 8: when **Lemma b** is satisfied on the two-lane road (e.g., when a_1=1.0 and a_2=1.5), serious traffic jams appear due to position disturbances and the resulting lane change behaviors (Fig. 5). But when the proper delayed-feedback control signals are added, each vehicle can be treated as a feedback control system and adjusts its own acceleration or deceleration according to the difference between the current and past traffic conditions, which involves using the information about the longitudinal and lateral distances. Therefore, the traffic jams would dissolve gradually and traffic flow would return to the original steady state (Figs. 8a and 8b). This can also be verified by the spatial-temporal trajectories of all vehicles on the road (Figs. 8e and 8f). Comparing Figs. 8a and 8b and Figs. 8c and 8d shows that the stronger feedback signals correspond to the larger amplitudes of velocity fluctuation. All these results demonstrate that the feedback control scheme proposed in this study is a useful way to alleviate traffic jams, i.e., a two-lane traffic flow with delayed-feedback control signals has strong robustness to traffic perturbation.

6 Conclusions

This paper proposes a two-lane OV model, which involves lateral friction from the neighboring lane, to describe the dynamic behaviors of two vehicle groups and their correlations under open boundary condition. Firstly, from the viewpoint of control theory we derive the stability conditions of a two-lane OV model with the method of H_∞-norm. Then, in the unstable traffic condition, the delayed feedback control signals can be added to each vehicle and corresponding stability conditions are obtained, where the appropriate control parameters can be solved by a numerical procedure. When incorporating lane change behaviors, the relatively homogeneous state of two-lane traffic flow would be broken and therefore the comprehensive headway distance and feedback control signals should be revised to neglect the traffic information from the neighboring lane. Finally, through numerical experiments, we can draw some important conclusions as follows: (a) When **Lemma a**

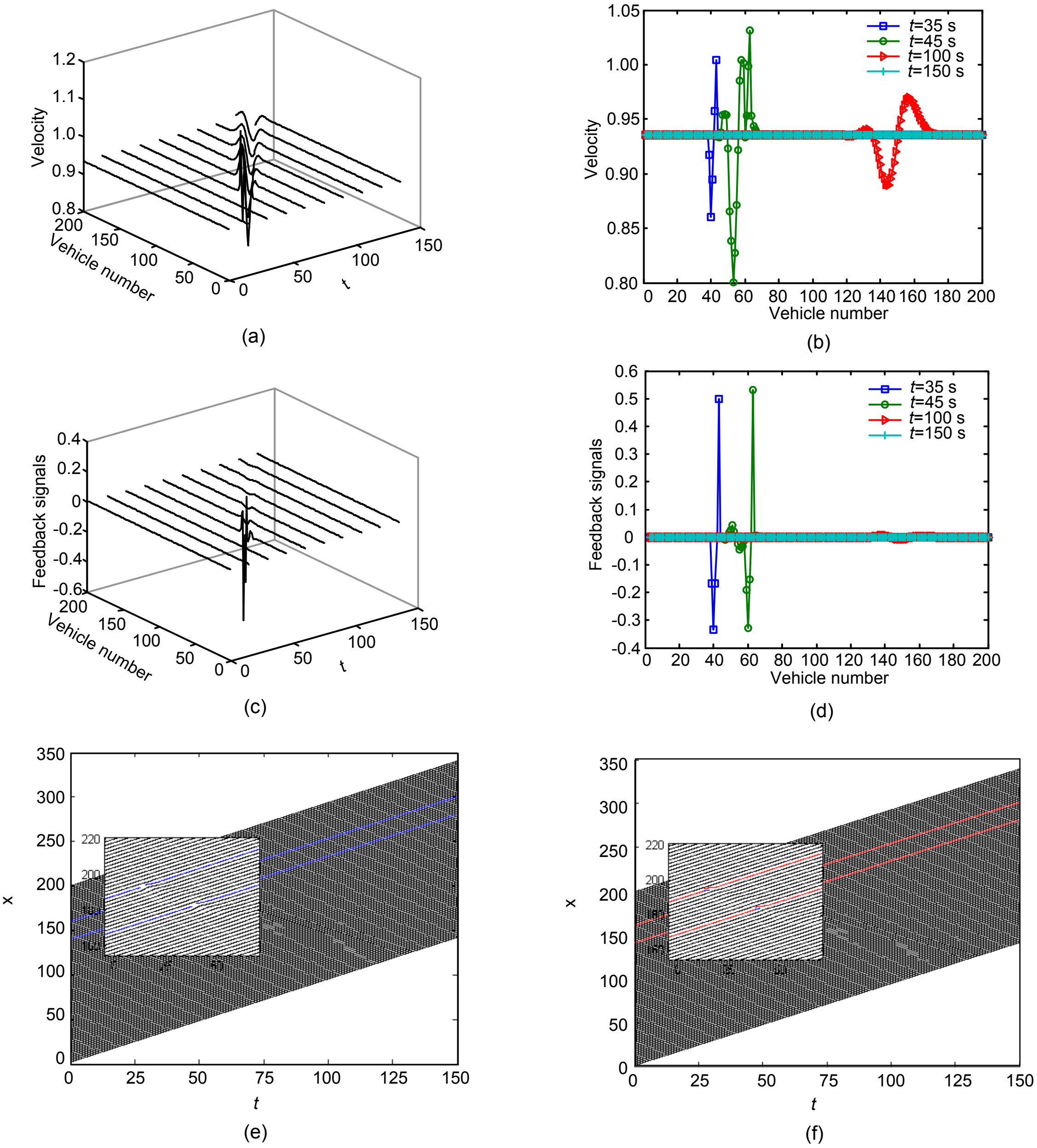

Fig. 8 Numerical simulation for two-lane traffic flow with feedback control signals, a_1=1.0 and a_2=1.5

(a) Spatial-temporal evolution of the velocity fluctuation; (b) Profiles of the velocity fluctuation; (c) Spatial-temporal evolution of the feedback signals; (d) Profiles of the feedback signals fluctuation; (e) Spatial-temporal trajectories in lane 1; (f) Spatial-temporal trajectories in lane 2

Note: in (a)–(d) vehicles are numbered consecutively according to their initial positions. Blank areas in (e) denote that lane change behaviors occur

is met, traffic jams do not happen, although there are position disturbances and lane change behaviors in the beginning. (b) Once **Lemma b** is satisfied, the small velocity fluctuation would be propagated backwards with increasing amplitude, finally resulting in serious traffic jams on the road. (c) Under the prerequisite of **Lemma b**, the feedback control scheme which meets **Theorem 1** can successfully suppress traffic jams resulting from small position disturbances and lane change behaviors. Therefore, it is useful to design a proper vehicular control system, which helps to maintain the stability of traffic flow.

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Appendix A

$$R(s)=\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix},$$

where

$$G_{11}=\frac{a_l\Lambda_l^y}{s^2+a_ls+a_l(\Lambda_l^y+\Lambda_l^q)},$$

$$G_{12}=\frac{a_l\Lambda_l^q}{s^2+a_ls+a_l(\Lambda_l^y+\Lambda_l^q)},$$

$$G_{13}=\frac{s}{s^2+a_ls+a_l(\Lambda_l^y+\Lambda_l^q)},$$

$$G_{21}=\frac{s^2+a_ls+a_l\Lambda_l^q}{s[s^2+a_ls+a_l(\Lambda_l^y+\Lambda_l^q)]},$$

$$G_{22}=\frac{-a_l\Lambda_l^q}{s[s^2+a_ls+a_l(\Lambda_l^y+\Lambda_l^q)]},$$

$$G_{23}=\frac{-1}{s^2+a_ls+a_l(\Lambda_l^y+\Lambda_l^q)},$$

$$G_{31}=\frac{-a_l\Lambda_l^y}{s[s^2+a_ls+a_l(\Lambda_l^y+\Lambda_l^q)]},$$

$$G_{32}=\frac{(s^2+a_ls+a_l\Lambda_l^y)}{s[s^2+a_ls+a_l(\Lambda_l^y+\Lambda_l^q)]},$$

$$G_{33}=\frac{-1}{s^2+a_ls+a_l(\Lambda_l^y+\Lambda_l^q)}.$$