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Technical note:

Large amplitude free vibration of a flexible panel coupled with a leaking cavity*

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Abstract: This paper addresses the effect of leakage on the natural frequencies of a large amplitude vibrating panel backed by a cavity, which has not been considered in many other related studies. The structural-acoustic governing equations are employed to study this nonlinear problem. An elliptical integral method, which was recently developed for the nonlinear panel cavity problem, is introduced here to solve for the structural-acoustics responses. The present results agree reasonably well with those obtained from the classical harmonic balance method. Modal convergences of the nonlinear solutions are performed to verify the proposed method. The effects of vibration amplitude and leakage size are studied and discussed. It is found that (1) the edge leakages in a panel cavity system significantly affect the natural frequency properties, and (2) the edge leakages induce a low frequency acoustic resonance.

Key words: Large amplitude vibration; Elliptic integral method; Noise and vibration
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1 Introduction

An acoustic cavity is formed by several panels. Leakages are usually caused by imperfect connections between the panels. Fig. 1 shows the schematic diagram of a nonlinear panel backed by a cavity with edge leakages. However, the leakage effect has not been considered in previous panel cavity research. Most researchers, including the author (Pretlove, 1966; Lee and Lee, 2007; Li and Cheng, 2007), adopted the assumption of a perfectly rigid acoustic boundary (or no leakage). In addition, although the related structural acoustic problems or nonlinear panel vibration problems have been of considerable interest to many researchers (Zhang *et al.*, 2007; Xie *et al.*, 2008; Zhu and Bai, 2009), most of them adopted the assumption of linear vibration, and there are still limited studies for nonlinear structural-acoustic problems. That is the motivation in this study about the effect of leakage. In the author's previous study (Lee, 2002), the classical harmonic balance method was employed to obtain the nonlinear solutions. The main drawback of this solution method was the tedious procedures of setting up the full harmonic balance equations. Thus, the elliptical integral method, which was recently employed by Hui *et al.* (2011), is introduced to the nonlinear problem. In the solution procedures of this method, a structural acoustic formulation representing the natural frequency of a nonlinear panel cavity system can be developed, and the elliptical integral solution is obtained by solving only one residual equation.

2 Theory

Fig. 1 shows a nonlinear panel coupled with a cavity with edge leakages. *a*, *b*, and *c* are the cavity

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width, length, and depth, respectively; d is the leakage width. In this study, the nonlinear simply supported panel governing equation (Hui *et al*., 2011) is adopted and incorporated with the acoustic pressure force terms:

$$\rho\frac{\mathrm{d}^2A}{\mathrm{d}t^2}+\rho\omega_{\mathrm{o}}^2A+\beta A^3+\bar{P}_c=0, \tag{1}$$

where $A(t)=\sum_{h=1,3,5,\cdots}^{H}A^h\cos(h\omega t)$ is the panel vibration response, A^h is the hth order harmonic amplitude, ω is the natural frequency to be determined, and H is the number of harmonic terms used. $\bar{P}_c(t)=\sum_{h=1,3,5,\cdots}^{H}\bar{P}_c^h(t)$ is the acoustic pressure force acting on the panel (i.e., $z=c$), and $\bar{P}_c^h(t)$ is the acoustic pressure force induced by the hth order harmonic component of the nonlinear panel vibration. $\omega_{\mathrm{o}}=\sqrt{\frac{E\tau^2}{12\rho(1-v^2)}}\left(\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right)$ is the linear structural natural frequency of the panel, $\beta=\frac{E\tau}{12(1-v^2)}\frac{\gamma}{a^4}$ is nonlinear stiffness coefficient, $\gamma=3\pi^4\left[\left(\frac{3}{4}-\frac{v^2}{4}\right)(1+r^4)+vr^2\right]$, $r=a/b$ is the aspect ratio, E is Young's modulus, v is Poisson's ratio, ρ is the panel density per unit thickness, and τ is the panel thickness.

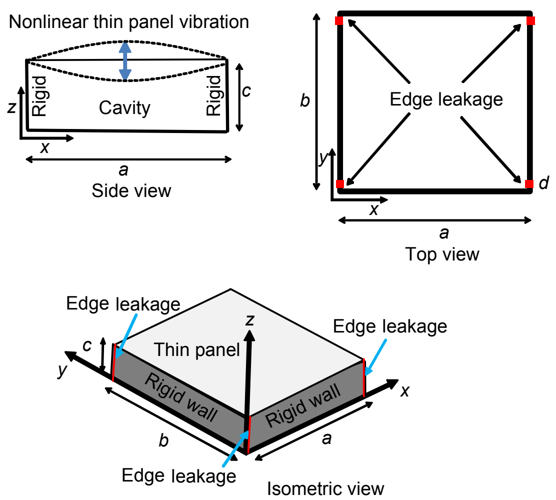

Fig. 1 A panel backed by a cavity with edge leakages

The acoustic pressure within the cavity induced by the panel is given by the following homogeneous wave equation (Lee and Lee, 2007):

$$\nabla^2P^h-\frac{1}{C_{\mathrm{a}}^2}\frac{\partial^2P^h}{\partial t^2}=0, \tag{2}$$

where $P^h(x, y, z, t)$ is the acoustic pressure within the cavity induced by the hth order harmonic component of the nonlinear panel vibration, and C_{a} is the sound speed in air.

The acoustic boundary conditions are given in Eq. (3). In Eqs. (3a), (3c), and (3d), the domains represent the rigid walls at x=0 or a, y=0 or b, and z=0. The air particle velocities on these rigid walls (i.e., $\frac{\partial P^h}{\partial x}$, $\frac{\partial P^h}{\partial y}$, and $\frac{\partial P^h}{\partial z}$) are zero. In Eq. (3b), the domain represents the four edge leakages (or openings) at x=0 or a. The acoustic pressures at the openings (i.e., P^h) are zero. In Eq. (3e), the domain represents the vibrating panel at $z=c$. The air particle velocities are equal to the corresponding panel velocities.

$$\frac{\partial P^h}{\partial x}=0,\ \text{at } x=0 \text{ or } a \text{ and } d<y<b-d, \tag{3a}$$

$$P^h=0,\ \text{at } x=0 \text{ or } a \text{ and } 0\le y\le d \text{ or } b-d\le y\le b, \tag{3b}$$

$$\frac{\partial P^h}{\partial y}=0,\ \text{at } y=0 \text{ or } b, \tag{3c}$$

$$\frac{\partial P^h}{\partial z}=0,\ \text{at } z=0, \tag{3d}$$

$$\frac{\partial P^h}{\partial z}=-\rho_{\mathrm{a}}\frac{\partial^2W^h(x,y,t)}{\partial t^2},\ \text{at } z=c, \tag{3e}$$

where ρ_{a} is the air density, $W^h(x, y, t)$ is the hth order harmonic component of the panel vibration response and is equal to $A^h\cos(h\omega t)\phi(x,y)$, and $\phi(x, y)$ is the double sine panel mode shape, i.e., $\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$.

The acoustic pressure distribution within the x-y plane (or cavity mode) is given by (Lee, 2016a)

$$\varphi_{uv}(x,y)=\cos\left(\frac{u+H(y)\pi}{a}x-H(y)\frac{\pi}{2}\right)\cos\left(\frac{v\pi}{b}y\right), \tag{4}$$

where u and v are the acoustic mode numbers, and $H(y)$ is given by

$$H(y)=1,\quad 0\le y\le d, \tag{5a}$$

$$H(y)=1-\frac{1}{\Delta d}\left[(y-d)-\frac{\sin\left(\frac{2\pi}{\Delta d}(y-d)\right)}{\frac{2\pi}{\Delta d}}\right],\quad d<y\le d+\Delta d, \tag{5b}$$

$$H(y)=0,\quad d+\Delta d<y<b-d-\Delta d, \tag{5c}$$

$$H(y)=\frac{1}{\Delta d}\times\left\{\left[y-(b-d-\Delta d)\right]-\frac{\sin\left[\frac{2\pi}{\Delta d}\left[y-(b-d-\Delta d)\right]\right]}{\frac{2\pi}{\Delta d}}\right\},\quad b-d-\Delta d\le y<b-d, \tag{5d}$$

$$H(y)=1,\quad b-d\le y\le b. \tag{5e}$$

Note that in order to have a 2nd order continuously differentiable function, $H(y)$ is of C^2 continuity at $y=d$, $d+\Delta d$, $b-d-\Delta d$, and $b-d$, and Δd is a very small domain, in which the value of $H(y)$ varies from 0 to 1. In this study, the leakage size concerned is not bigger than 2% of the cavity length. Thus, $\varphi_{uv}(x, y)$ is almost orthogonal. Fig. 2 shows the orthogonality of the acoustic mode function, which is defined in Eq. (6), plotted against the leakage size for different mode numbers.

$$\text{OI}=1-\left|\frac{\int_0^b\int_0^a \varphi_{uv}\varphi_{u'v'}\mathrm{d}x\mathrm{d}y}{\int_0^b\int_0^a (\varphi_{uv})^2\,\mathrm{d}x\mathrm{d}y}\right|, \tag{6}$$

where OI is the orthogonality index. If the acoustic mode function is perfectly orthogonal, the orthogonal index is equal to 1 for $u\neq u'$ and $v\neq v'$.

Theoretically, Δd can be infinitely small to satisfy the boundary conditions. However, in order to avoid a numerical singularity, Δd is set to $d/100$ which is very small when compared with the panel width. In Fig. 2, the three curves are very close. It can be seen that the function is perfectly orthogonal for leakage size of 0; when the leakage is increasing, the index values are also reducing but still very close to 1 for a leakage size less than 2%. On the other hand, according to Lee and Lee (2007), the acoustic pressure solution form in Eq. (2) is

$$P^h(x, y, z, t)=\sum_u^U\sum_v^V\left[L_{uv}^h\sinh(\mu_{uv}^h z)+N_{uv}^h\cosh(\mu_{uv}^h z)\right]\times\varphi_{uv}(x, y)\cos(h\omega t), \tag{7}$$

where L_{uv}^h and N_{uv}^h are the coefficients which depend on the boundary conditions at $z=0$ and $z=c$, $\mu_{uv}^h=\frac{1}{C_\mathrm{a}}\sqrt{\omega_{uv}^2-(h\omega)^2}$, U and V are the numbers of the acoustic modes used, ω_{uv} is the acoustic natural frequency in the case without a flexible panel and can be obtained using Eq. (9), and ω is the natural frequency in the case with flexible panel and leakage (see Appendix A for more details).

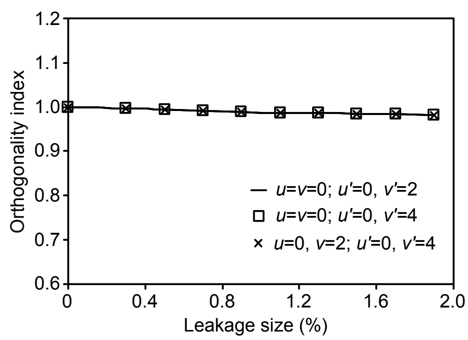

Fig. 2 Orthogonality index versus leakage size for various acoustic modes

Apply the boundary conditions in Eq. (3) to Eq. (7) to obtain

$$P^h(x, y, z, t)=\rho_\mathrm{a}(h\omega)^2\times\sum_u^U\sum_v^V\frac{\cosh(\mu_{uv}^h z)}{\sinh(\mu_{uv}^h c)}\frac{A^h\alpha_{uv}}{\mu_{uv}^h\alpha_\varphi}\varphi_{uv}(x, y)\cos(h\omega t), \tag{8a}$$

$$\overline{P}_c^h(t)=\rho_\mathrm{a}(h\omega)^2\times\sum_u^U\sum_v^V\frac{\coth(\mu_{uv}^h c)}{\mu_{uv}^h}\frac{A^h(\alpha_{uv})^2}{\alpha_\varphi\alpha_\phi}\cos(h\omega t), \tag{8b}$$

where $\alpha_{uv}=\int_0^b\int_0^a\varphi_{uv}\phi\mathrm{d}x\mathrm{d}y$, $\alpha_\varphi=\int_0^b\int_0^a(\varphi_{uv})^2\mathrm{d}x\mathrm{d}y$, and

$\alpha_\phi = \int_0^b \int_0^a \phi^2 \mathrm{d}x\mathrm{d}y$. Hence, the acoustic pressure force term in Eq. (1) can be obtained using Eq. (8b).

Consider the modal residual equation by putting the acoustic pressure solution in Eq. (7) into Eq. (2):

$$\rho_\mathrm{a}(h\omega)^2 \cos(h\omega t)\int_0^b \int_0^a R_{uv}\varphi_{uv}\mathrm{d}x\mathrm{d}y = 0, \tag{9}$$

where

$$R_{uv} = \sum_u^U \sum_v^V \frac{\cosh(\mu_{uv}^h z)}{\sinh(\mu_{uv}^h c)} \frac{A^h \alpha_{uv}}{\mu_{uv}^h \alpha_\varphi} \left[\frac{\mathrm{d}^2\varphi_{uv}}{\mathrm{d}x^2} + \frac{\mathrm{d}^2\varphi_{uv}}{\mathrm{d}y^2} + \frac{\omega_{uv}^2}{C_\mathrm{a}^2}\varphi_{uv} \right].$$

Hence, ω_{uv} can be found using Eq. (9). Note that as aforementioned, φ_{uv} is almost orthogonal when the leakage is small. Thus, $\int_0^b \int_0^a \varphi_{uv}\varphi_{u'v'}\mathrm{d}x\mathrm{d}y$ and $\int_0^b \int_0^a \frac{\mathrm{d}^2\varphi_{uv}}{\mathrm{d}y^2}\varphi_{u'v'}\mathrm{d}x\mathrm{d}y$ are small and ignored for $u \neq u'$ and $v \neq v'$.

According to the solution method of Hui *et al.* (2011), a dummy term $K_\mathrm{a}A$ is considered in Eq. (1), i.e.,

$$\rho \frac{\mathrm{d}^2\overline{A}}{\mathrm{d}t^2} + (\rho\omega_\mathrm{o}^2 + K_\mathrm{a})A + \beta A^3 + \sum_{h=1,3,5,\cdots} \overline{P}_c^h(t) - K_\mathrm{a}A = 0. \tag{10}$$

Then, it is assumed that $\overline{A}$ is the elliptical integral solution to the cubic nonlinear differential equation:

$$\rho \frac{\mathrm{d}^2\overline{A}}{\mathrm{d}t^2} + (\rho\omega_\mathrm{o}^2 + K_\mathrm{a})\overline{A} + \beta\overline{A}^3 = 0, \tag{11}$$

where $\overline{A} = A_\mathrm{o}\mathrm{cn}(\upsilon(\kappa))$, υ is the elliptic integral, $\kappa = \frac{\beta A_\mathrm{o}^2}{2(\rho\omega_\mathrm{o}^2 + K_\mathrm{a} + \beta A_\mathrm{o}^2)}$ is the modulus of υ, cn is the elliptic cosine, and A_o is the modal displacement or the vibration amplitude at t=0. The natural frequency of the nonlinear panel cavity system is given by

$$\omega_\mathrm{n} = 2\pi f_\mathrm{n} = \frac{2\pi}{T_\mathrm{n}}, \tag{12}$$

where

$$T_\mathrm{n} = \frac{4}{\sqrt{\omega_\mathrm{o}^2 + (K_\mathrm{a} + \beta A_\mathrm{o}^2)/\rho}} \int_0^{\pi/2} \frac{1}{\sqrt{1-\kappa^2 \sin(\psi)^2}}\mathrm{d}\psi.$$

Note that the dummy K_a has not been found, and $\overline{A}$ is not the exact solution to Eq. (10) and depends on K_a (i.e., $\overline{A}(K_\mathrm{a})$). In addition, $\overline{P}_c^h$ depends on $\overline{A}(K_\mathrm{a})$ (i.e., $\overline{P}_c^h(K_\mathrm{a})$). If we put $\overline{A}$ into Eq. (10), the residual is given by

$$\sum_{h=1,3,5,\cdots} \overline{P}_c^h(K_\mathrm{a}) - K_\mathrm{a}A(K_\mathrm{a}) = R_\mathrm{o}(K_\mathrm{a}). \tag{13}$$

K_a is the only one unknown in Eq. (13). K_a can be found from the minimization of the residual in Eq. (13).

3 Results and discussion

In this section, the material properties of the panel in the numerical cases are considered as follows: Young's modulus of the beam E=7.1×10^{10} N/m^2, mass density ρ=2700 kg/m^3, Poisson's ratio ν=0.3, and panel dimensions a×b=0.3048 m×0.3048 m. In Tables 1–3 and Figs. 3–7, the panel thickness is 1.2192 mm (except Fig. 5c, where the panel thickness is 0.6096 mm).

Tables 1–3 show the modal convergence of the frequency ratio ω/ω_o for various vibration amplitude ratios, A/τ. It is shown that the approach of four acoustic modes is good enough for obtaining converged and accurate solutions. Fig. 3 shows the difference between the natural frequency results from the present method and the classical harmonic balance method for the panel cavity system without leakage. Although when the cavity depth or the amplitude ratio is larger, the difference is bigger, generally the differences in all cases are less than 1%. Therefore, it can be considered that the frequency ratios for various cavity depths obtained by the elliptical integral method agree reasonably well with those obtained from the classical harmonic balance method. Fig. 4 shows the comparison between the frequency-amplitude results from the proposed method and the multi-level residue harmonic balance method of Lee (2016b) for various panel lengths (other parameter values are the same as those in Fig. 3). The values of the cross symbols are abstracted from Lee (2016b), while the values of the diamond symbols are computed using the multi-level residue harmonic balance method. The results from the proposed method agree well with those from Lee (2016b). In addition, it can be seen that when the panel size is bigger, relatively, the panel stiffness is weaker or the influence of the cavity stiffness on the frequency ratio is bigger. Note that if the cavity stiffness has no influence or the vibration amplitude is zero, the frequency ratio is equal to one.

Table 1 Modal convergence of frequency ratio for various amplitude ratios (leakage size, *d*/*a*=0%)

Approach	Frequency ratio, ω/ω_o			
	A/τ=0.5	A/τ=1	A/τ=1.5	A/τ=2
1 mode	1.5353	1.7540	2.0642	2.4304
4 modes	1.5311	1.7492	2.0585	2.4235
9 modes	1.5310	1.7491	2.0600	2.4234

Table 2 Modal convergence of frequency ratio for various amplitude ratios (leakage size, *d*/*a*=0.5%)

Approach	Frequency ratio, ω/ω_o			
	A/τ=0.5	A/τ=1	A/τ=1.5	A/τ=2
1 mode	1.6007	1.7985	2.0918	2.4474
4 modes	1.5967	1.7938	2.0861	2.4406
9 modes	1.5966	1.7937	2.0900	2.4404

Table 3 Modal convergence of frequency ratio for various amplitude ratios (leakage size, *d*/*a*=1%)

Approach	Frequency ratio, ω/ω_o			
	A/τ=0.5	A/τ=1	A/τ=1.5	A/τ=2
1 mode	1.6764	1.8515	2.1249	2.4679
4 modes	1.6726	1.8468	2.1193	2.4610
9 modes	1.6726	1.8468	2.1200	2.4610

The vibration amplitude ratio is plotted against the frequency ratio for various cavity lengths, panel thicknesses, and leakage sizes as shown in Fig. 5. The dashed line and vertical line represent the natural frequencies in the cases of no leakage and no flexible panel, respectively. It can be seen that the edge leakages induce one more natural frequency solution in all cases. Note that in Figs. 5a and 5b, the wavelengths of the acoustic resonances are longer than 3 m while the cavity length and width are only 0.3048 m. It is because the leakages result in a weaker cavity stiffness or lower acoustic resonant frequency. When the amplitude ratio is smaller or the leakage size is bigger, the difference between the acoustic frequency solution and the natural frequency in the case of no flexible panel, and the difference between the structural frequency solution and the natural frequency in the case of no leakage are larger. As an example, when the amplitude ratio is 0, the differences of 1% and 1.5% leakage sizes are 28% and 33%, respectively. When the amplitude ratio is increasing, the acoustic and structural frequency solutions converge to the natural frequencies in the cases of no flexible panel and no leakage, respectively. In Fig. 5c, the leakage size is 1%, and the dimensionless natural frequency of the no flexible panel case is 1.77, which is much higher than those in Figs. 5a and 5b. The acoustic and structural resonances are significantly affected by each other so that the two natural frequency solution curves are far from the vertical line and dashed line, respectively. It can be seen that the

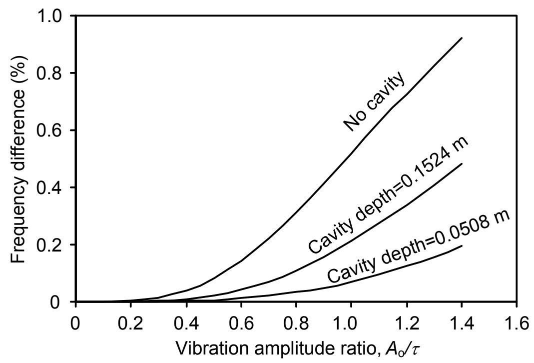

Fig. 3 Difference between the natural frequency results from the elliptic integral and harmonic balance methods versus vibration amplitude ratio

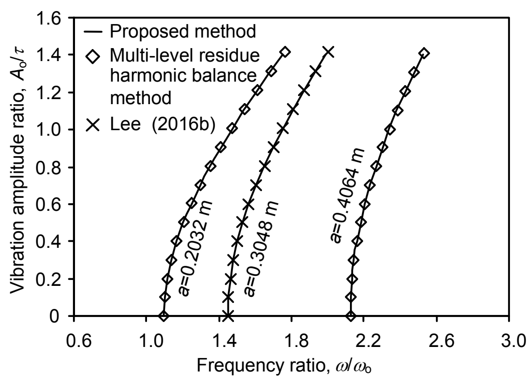

Fig. 4 Comparison between the natural frequency results from the elliptic integral method, multi-level residue harmonic balance method, and Lee (2016b)

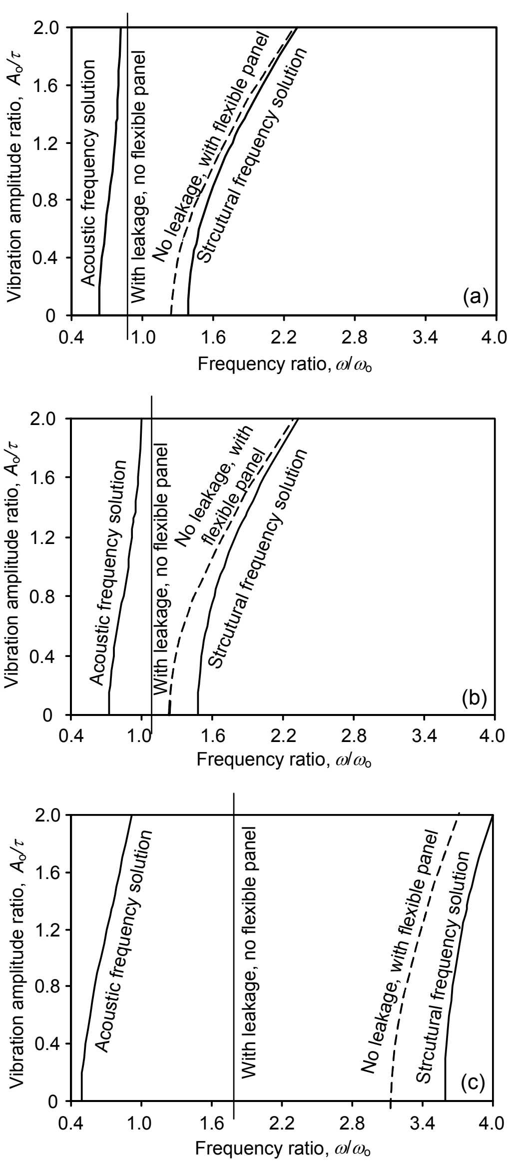

Fig. 5 Vibration amplitude ratio versus frequency ratio
(a) The panel thickness is 1.2192 mm, the cavity depth is 0.3048 m, and the leakage size is 1%; (b) The panel thickness is 1.2192 mm, the cavity depth is 0.3048 m, and the leakage size is 1.5%; (c) The panel thickness is 0.6096 mm, the cavity depth is 0.1524 m, and the leakage size is 1%

assumption of leakage in the panel cavity problems is very important and significantly affects the structural vibration and acoustic pressure behaviors.

In Figs. 6 and 7, the acoustic and structural natural frequencies are plotted against the leakage size, for various amplitude ratios, respectively. In Fig. 6, the three curves are convex. It can be seen that the acoustic natural frequency is monotonically increasing with the leakage size. When the leakage size is smaller, the three curves are closer (or the acoustic natural frequency is less sensitive to the vibration amplitude). When the leakage size is increasing, their slopes are flatter and their differences are larger (or the acoustic natural frequency is more sensitive to the vibration amplitude). In Fig. 7, the three curves are concave. The structural natural frequency is also monotonically increasing with the leakage size. Generally, the slopes of the curves in Fig. 7 are flatter and the differences between the three curves are more significant than those in Fig. 6. It is implied that when compared with the acoustical natural frequency, the structural natural frequency is less sensitive to the leakage size and more sensitive to the vibration amplitude.

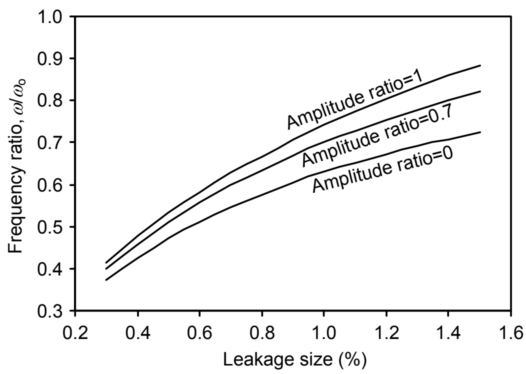

Fig. 6 Acoustic natural frequency versus leakage size for various amplitude ratios

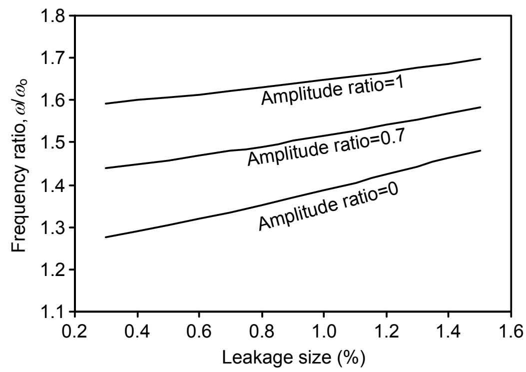

Fig. 7 Structural natural frequency versus leakage size for various amplitude ratios

4 Conclusions

The natural frequencies of a nonlinear panel backed by a cavity with edge leakages have been studied. The nonlinear structural acoustic formulation has been developed and solved by the elliptical integral method. The effect of cavity leakage, which was ignored in much previous structural acoustic research, has been investigated. The present elliptic integral solution agrees reasonably well with those obtained from the classical harmonic balance method. It is concluded that: (1) the edge leakages induce one more low frequency resonances; (2) the edge leakages can significantly affect the structural natural frequency in a panel-cavity system and makes it higher; (3) generally, when the leakage size is larger, its effect on the acoustic and structural natural frequencies is also higher.

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Appendix A

From Eq. (7),

$$P^h(x, y, z, t) = \sum_u^U \sum_v^V \left[L_{uv}^h \sinh(\mu_{uv}^h z) + N_{uv}^h \cosh(\mu_{uv}^h z) \right] \varphi_{uv}(x, y)\cos(h\omega t),$$

hence,

$$\frac{\partial^2 P^h}{\partial x^2} = \sum_u^U \sum_v^V G_{uv}^h(z) \frac{\partial^2 \varphi_{uv}(x,y)}{\partial x^2} \cos(h\omega t), \quad \text{(A1)}$$

$$\frac{\partial^2 P^h}{\partial y^2} = \sum_u^U \sum_v^V G_{uv}^h(z) \frac{\partial^2 \varphi_{uv}(x,y)}{\partial y^2} \cos(h\omega t), \quad \text{(A2)}$$

$$\frac{\partial^2 P^h}{\partial t^2} = -\omega^2 \sum_u^U \sum_v^V G_{uv}^h(z) \varphi_{uv}(x,y) \cos(h\omega t), \quad \text{(A3)}$$

where

$$G_{uv}^h(z) = L_{uv}^h \sinh(\mu_{uv}^h z) + N_{uv}^h \cosh(\mu_{uv}^h z).$$

Then, put the three 2nd derivative terms in Eqs. (A1)–(A3) into the left side of Eq. (2):

$$\sum_u^U \sum_v^V \left[\frac{\partial^2 \varphi_{uv}(x,y)}{\partial x^2} + \frac{\partial^2 \varphi_{uv}(x,y)}{\partial y^2} + \frac{\omega^2}{C_\text{a}^2} \varphi_{uv}(x,y) \right] \times G_{uv}^h(z)\cos(h\omega t) = R, \quad \text{(A4)}$$

where R is the residual in Eq. (2). The cavity resonant frequency of the (u, v) mode, ω_{uv}, can be found by

$$\int_0^b \int_0^a \sum_u^U \sum_v^V \left[\frac{\partial^2 \varphi_{uv}(x, y)}{\partial x^2} + \frac{\partial^2 \varphi_{uv}(x, y)}{\partial y^2} + \frac{\omega_{uv}^2}{C_a^2} \varphi_{uv}(x, y) \right] \times \varphi_{uv}(x, y)\mathrm{d}x\mathrm{d}y G_{uv}^h(z)\cos(h\omega t) = \int_0^b \int_0^a R\varphi_{uv}(x, y)\mathrm{d}x\mathrm{d}y = 0. \quad \text{(A5)}$$

The cavity mode shape is given by (Lee and Lee, 2007)

$$\varphi_{uv}(x, y)=\cos\left(\frac{u\pi}{a}x\right)\cos\left(\frac{v\pi}{b}y\right). \tag{A6}$$

Note that the cavity modes in Eq. (A6) are orthogonal. For $u\neq u'$ and $v\neq v'$,

$$\begin{aligned}&\int_0^b\int_0^a \varphi_{uv}\varphi_{u'v'}\mathrm{d}x\mathrm{d}y=0,\\ &\int_0^b\int_0^a \frac{\partial^2\varphi_{uv}(x, y)}{\partial x^2}\varphi_{u'v'}\mathrm{d}x\mathrm{d}y=0,\\ &\int_0^b\int_0^a \frac{\partial^2\varphi_{uv}(x, y)}{\partial y^2}\varphi_{u'v'}\mathrm{d}x\mathrm{d}y=0.\end{aligned} \tag{A7}$$

Put Eq. (A6) into Eq. (A5) to setup the following equation, and thus the cavity resonant frequency of the (u, v) mode can be found.

$$\nabla^2\phi-\frac{1}{C_\mathrm{a}^2}\frac{\partial^2\phi}{\partial t^2}=0. \tag{A8}$$

In this study, the cavity mode shapes are given in Eq. (4):

$$\varphi_{uv}(x, y)=\cos\left(\frac{u+H(y)\pi}{a}x-H(y)\frac{\pi}{2}\right)\cos\left(\frac{v\pi}{b}y\right), \tag{4}$$

and the orthogonality index of these mode shapes is given in Eq. (6):

$$\mathrm{OI}=1-\left|\frac{\int_0^b\int_0^a \varphi_{uv}\varphi_{u'v'}\mathrm{d}x\mathrm{d}y}{\int_0^b\int_0^a (\varphi_{uv})^2\mathrm{d}x\mathrm{d}y}\right|, \tag{6}$$

where if the acoustic mode function is perfectly orthogonal, the orthogonal index is equal to 1 for $u\neq u'$ and $v\neq v'$. Fig. 2 has shown that the index values are very close to 1 for the leakage size less than 2%, which is the range considered in this study.

中文概要

题　目：与裂缝腔联结的弹性板的大幅自主震动

目　的：板腔系统中的裂缝会导致共振特性发生变化。本文旨在探讨裂缝的影响，推导其中的关系与相关公式，并控制计算精确的程度。

创新点：1. 通过椭圆积分方法破解控制方程，推导裂缝腔联结的弹性板的共振频率；2. 建立理论模型，成功计算不同情况下弹性板的共振频率。

方　法：1. 通过理论推导，计算裂缝腔大小、震动幅度与共振频率之间的关系（公式（10）~（12））；2. 与其它方法得到的数据进行比较，验证所提方法的可行性和有效性（图 3 和 4）；3. 通过仿真模拟，推导裂缝腔对弹性板共振频率的影响（图 5a~5c）。

结　论：1. 板腔系统中的裂缝会导致共振频率出现重大变化；2. 裂缝会导致一个额外的低频率共振点。

关键词：大幅自主震动；椭圆积分方法；声音与震动