

Thin film bulk acoustic wave filters with ring-dot electrodes^{*}

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Abstract: We propose a new structure of ZnO or AlN thin film bulk acoustic wave filters using a ring-dot driving electrode operating with thickness-extensional modes. A theoretical analysis is performed to show the operating frequencies and modes of the proposed filter. The scalar differential equation by Tiersten and Stevens (1983) is used. An analytical solution is obtained. Numerical calculations based on the solution show that the vibration tends to be trapped in the electroded central and annular areas and decays away from the electrode edges. It is also shown that by properly designing the electrode dimensions and mass density, the nodal line of one of the two operating modes can be adjusted to lie in the gap between the ring and the dot driving electrodes which is ideal for a filter.

Key words: Piezoelectric; Thin film; Vibration; Resonator; Filter

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1 Introduction

Acoustic wave resonators and filters made from piezoelectric crystals are key components in a lot of electronic equipment (Bottom, 1982; Salt, 1987). Electrical circuits such as oscillators with acoustic wave components are widely used as frequency standards and operators for time keeping and signal generation as well as processing. Conventional piezoelectric resonators and filters are made from crystals like quartz and lithium niobate, etc. They may operate with bulk (Bottom, 1982; Salt, 1987) or surface acoustic waves (Campbell, 1998; Hashimoto, 2000). During the last couple of decades, researchers

succeeded in depositing a very thin piezoelectric film of AlN or ZnO on a silicon layer to form thin film bulk acoustic wave resonators (FBARs) operating in the GHz frequency range (Lakin, 1999; Iriarte et al., 2002; Martin et al., 2006). FBARs have several advantages over conventional crystal resonators in filter applications (Lakin et al., 1995; Satoh et al., 2005). They have also been used to make acoustic wave sensors for mass and fluid sensing (Link et al., 2006; Fu et al., 2010; Zhang and Bao, 2014). Structurally, FBARs are multilayered plates with metal electrodes, a piezoelectric film, and an elastic substrate layer. The films can be lifted or solidly mounted on a substrate (Pang et al., 2005; Link et al., 2006). The c-axis of the material may be along the thickness or in-plane direction of the film, inclined in any direction, or zigzag in a multilayered film (Du et al., 2009; Qin et al., 2010; Zhang et al., 2010; Zhang and Kosinski, 2012; Zhang and Bao, 2014). FBAR materials and devices are of current and growing research interest experimentally, numerically, and theoretically (Jung et al., 2003; Gong et al., 2007; Buccella et al., 2008;

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Campanella et al., 2009; de Santis et al., 2009; Katus and Reindl, 2010; Meltaus et al., 2012; Pan et al., 2012; Thakar et al., 2012; Wang et al., 2015).

Square (or rectangular) dielectric and piezoelectric layers as well as square electrodes have been routinely used for FBARs. It is known that electrode corners can cause electric field concentration (Yang et al., 2007) which is associated with degrading effects or even failure of the materials involved. In this paper, fundamentally different from the existing FBAR filters with rectangular electrodes (Tiersten and Stevens, 1983; Zhao et al., 2016a), we propose a new structure for an FBAR filter with circular geometry. It has a ring-dot driving electrode configuration. The proposed filter has a few advantages over filters with rectangular electrodes. The circular driving electrode does not have corners and the associated electric field concentration. A circular dot or ring electrode is also perfectly compatible with the in-plane material isotropy of AlN and ZnO. In addition, there is experimental suggestion that a circular thin film resonator has a higher quality factor (Kumar et al., 2016). We perform a theoretical analysis to obtain the basic vibration characteristics of the proposed filter.

2 Governing equations

Because of the material anisotropy of piezoelectric crystals and their electromechanical coupling, theoretical analyses of piezoelectric devices using the 3D theory of linear piezoelectricity present considerable mathematical challenges. Useful solutions can only be obtained in rare and special cases. To obtain theoretical results needed for the understanding and design of piezoelectric devices, various approximate techniques are necessary. For the most common FBARs with the *c*-axis of the material along the plate thickness, a single scalar differential equation was derived by Tiersten and Stevens (1983) for the operating mode, i.e. the transversely varying thickness-extensional mode in the device. The scalar equation itself is very simple and accurate. However, its derivation was very involved and one needs to go through a significant amount of algebra to calculate the coefficients of the equation in order to use it. As a consequence the equation has only been used in a few

studies (Zhao et al., 2015, 2016a, 2016b; Liu et al., 2017) with great successes. Recently, a summary of the expressions used to calculate the coefficients of the scalar equation was given in the appendix of Zhao et al. (2015), which made the use of the equation more convenient. The scalar equation is slightly different depending on whether there is a top driving electrode or not. For time-harmonic free vibrations of an FBAR at a resonance with a circular frequency ω which is to be determined, all fields have the same time dependence with a common factor $\exp(i\omega t)$ which will be dropped below. When there is no driving electrode, the n th-order thickness-extensional displacement u_3^n is approximately represented by Tiersten and Stevens (1983):

$$u_3^n \cong f^n(x_1, x_2)g^n(x_3), \quad (1)$$

where x_1 and x_2 are the in-plane coordinates of the plate. x_3 is the thickness coordinate. $g^n(x_3)$ is the field variation along the plate thickness. The in-plane field variation is described by $f^n(x_1, x_2)$ which is governed by Eq. (6.2) of Tiersten and Stevens (1983):

$$M_n \left(\frac{\partial^2 f^n}{\partial x_1^2} + \frac{\partial^2 f^n}{\partial x_2^2} \right) - \bar{c}_{33}^f \hat{\eta}_{f^n}^2 f^n + \rho^f \omega^2 f^n = 0. \quad (2)$$

Similarly, when there is a driving electrode, from Eq. (6.1) of Tiersten and Stevens (1983), the in-plane variation of the n th thickness-extensional mode is governed by

$$M_n \left(\frac{\partial^2 f^n}{\partial x_1^2} + \frac{\partial^2 f^n}{\partial x_2^2} \right) - \bar{c}_{33}^f \bar{\eta}_{f^n}^2 f^n + \rho^f \omega^2 f^n = 0. \quad (3)$$

M_n , \bar{c}_{33}^f , $\hat{\eta}_{f^n}^2$, and $\bar{\eta}_{f^n}^2$ in Eqs. (2) and (3) are independent of the spatial coordinates and are calculated according to Tiersten and Stevens (1983). ρ^f is the mass density of the ZnO film.

Along a curve separating a 2D region with a driving electrode and a region without a driving electrode, f^n and its normal derivative perpendicular to the curve must be continuous (Tiersten and Stevens, 1983). Denoting

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}, \quad (4)$$

$$\bar{\omega}_n^2 = \frac{\bar{c}_{33}^f \bar{\eta}_{f^n}^2}{\rho^f}, \quad \hat{\omega}_n^2 = \frac{\bar{c}_{33}^f \hat{\eta}_{f^n}^2}{\rho^f},$$

we rewrite Eqs. (2) and (3) in the following more compact form:

$$M_n \nabla^2 f^n + \rho^f (\omega^2 - \hat{\omega}_n^2) f^n = 0, \quad (5)$$

$$M_n \nabla^2 f^n + \rho^f (\omega^2 - \bar{\omega}_n^2) f^n = 0. \quad (6)$$

For circular devices we use polar coordinates defined by $x_1 = r \cos \theta$ and $x_2 = r \sin \theta$. Let the θ dependence of the fields be $\cos(m\theta)$, where $m=0, 1, 2, \dots$. Because of the in-plane material isotropy of ZnO and AlN, there is no need to consider the case of $\sin(m\theta)$. Then, for the plate with a ring-dot driving electrode configuration in Fig. 1, Eqs. (5) and (6) reduce to

$$M_n \left(\frac{\partial^2 f^n}{\partial r^2} + \frac{1}{r} \frac{\partial f^n}{\partial r} - \frac{m^2}{r^2} f^n \right) + \rho^f (\omega^2 - \bar{\omega}_n^2) f^n = 0, \quad r < R_0,$$

$$M_n \left(\frac{\partial^2 f^n}{\partial r^2} + \frac{1}{r} \frac{\partial f^n}{\partial r} - \frac{m^2}{r^2} f^n \right) + \rho^f (\omega^2 - \hat{\omega}_n^2) f^n = 0, \quad R_0 < r < R_1, \quad (7)$$

$$M_n \left(\frac{\partial^2 f^n}{\partial r^2} + \frac{1}{r} \frac{\partial f^n}{\partial r} - \frac{m^2}{r^2} f^n \right) + \rho^f (\omega^2 - \bar{\omega}_n^2) f^n = 0, \quad R_1 < r < R_2,$$

$$M_n \left(\frac{\partial^2 f^n}{\partial r^2} + \frac{1}{r} \frac{\partial f^n}{\partial r} - \frac{m^2}{r^2} f^n \right) + \rho^f (\omega^2 - \hat{\omega}_n^2) f^n = 0, \quad R_2 < r < \infty,$$

for the central region with a dot electrode, the gap between the dot and the ring electrodes, the annular region with a ring electrode, and the unelectroded outer region, respectively. For the so-called trapped modes we are interested in, the vibration outside an electrode decays away from the electrode edge, in particular when r approaches infinity. The boundary and continuity conditions are

$$f^n \text{ is finite, } r=0, \quad (8a)$$

$$f^n(R_0^-) = f^n(R_0^+), \quad \left. \frac{df^n}{dr} \right|_{R_0^-} = \left. \frac{df^n}{dr} \right|_{R_0^+}, \quad (8b)$$

$$f^n(R_1^-) = f^n(R_1^+), \quad \left. \frac{df^n}{dr} \right|_{R_1^-} = \left. \frac{df^n}{dr} \right|_{R_1^+}, \quad (8c)$$

$$f^n(R_2^-) = f^n(R_2^+), \quad \left. \frac{df^n}{dr} \right|_{R_2^-} = \left. \frac{df^n}{dr} \right|_{R_2^+}, \quad (8d)$$

$$f^n \rightarrow 0, \quad r \rightarrow \infty. \quad (8e)$$

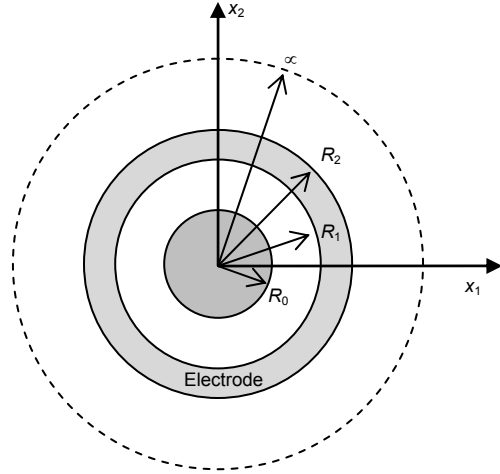


Fig. 1 An FBAR filter with ring-dot driving electrodes

3 Analytical solution

We denote

$$\hat{\alpha}^2 = \frac{\rho^f (\hat{\omega}_n^2 - \omega^2)}{M_n} > 0, \quad \bar{\alpha}^2 = \frac{\rho^f (\omega^2 - \bar{\omega}_n^2)}{M_n} > 0. \quad (9)$$

Eq. (9) defines a frequency interval for the trapped modes of interest. Under the following separate transformations of the radial coordinate for the electroded and unelectroded regions:

$$\begin{aligned} \xi &= \bar{\alpha} r, & r < R_0, \\ \eta &= \hat{\alpha} r, & R_0 < r < R_1, \\ \xi &= \bar{\alpha} r, & R_1 < r < R_2, \\ \eta &= \hat{\alpha} r, & R_2 < r < \infty. \end{aligned} \quad (10)$$

Eq. (7) becomes

$$\frac{\partial^2 f^n}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial f^n}{\partial \xi} + \left(1 - \frac{m^2}{\xi^2} \right) f^n = 0, \quad r < R_0, \quad (11a)$$

$$\frac{\partial^2 f^n}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial f^n}{\partial \eta} - \left(1 + \frac{m^2}{\eta^2}\right) f^n = 0, \quad R_0 < r < R_1, \quad (11b)$$

$$\frac{\partial^2 f^n}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial f^n}{\partial \xi} + \left(1 - \frac{m^2}{\xi^2}\right) f^n = 0, \quad R_1 < r < R_2, \quad (11c)$$

$$\frac{\partial^2 f^n}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial f^n}{\partial \eta} - \left(1 + \frac{m^2}{\eta^2}\right) f^n = 0, \quad R_2 < r < \infty. \quad (11d)$$

Eqs. (11a) and (11c) and Eqs. (11b) and (11d) are the Bessel equations and modified Bessel equations of order m . The general solution to Eq. (11) satisfying Eqs. (8a) and (8e) can be written as

$$f^n = \begin{cases} A_1 J_m(\bar{\alpha}r), & r < R_0, \\ A_2 I_m(\hat{\alpha}r) + A_3 K_m(\hat{\alpha}r), & R_0 < r < R_1, \\ A_4 J_m(\bar{\alpha}r) + A_5 Y_m(\bar{\alpha}r), & R_1 < r < R_2, \\ A_6 K_m(\hat{\alpha}r), & R_2 < r < \infty, \end{cases} \quad (12)$$

where A_1 – A_6 are undetermined constants. J_m and Y_m are the m th-order Bessel functions of the first kind and second kind, respectively. I_m and K_m are the m th-order modified Bessel functions of the first kind and second kind, respectively. The substitution of Eq. (12) into Eqs. (8b)–(8d) yields six linear homogeneous equations for A_1 – A_6 . For nontrivial solutions the determinant of the coefficient matrix of the equations has to vanish, which gives the frequency equation. The corresponding nontrivial solutions of A_1 – A_6 determine the corresponding modes.

4 Numerical results and discussion

As a numerical example, consider a thin ZnO film filter operating with the fundamental thickness-extensional mode with $n=1$ and $m=0$ which are fixed throughout the rest of the paper. The thickness of the ZnO film is $h^f=5 \mu\text{m}$. The thickness of the silicon substrate is $h^s=5 \mu\text{m}$. The thickness of the Au driving electrode is $h'=0.044 \mu\text{m}$ and that of the ground electrode is $h''=0.2 \mu\text{m}$. The corresponding mass ratios (Tiersten and Stevens, 1983) are $R'=0.03$ and $R''=0.1359$. The radii of the electrodes are that $R_0=150 \mu\text{m}$, $R_1=160 \mu\text{m}$, and $R_2=320 \mu\text{m}$. In this case, three roots are found in the frequency interval $(\bar{\omega}_1, \hat{\omega}_1)$ determined by Eq. (9). In terms of the

frequency $f=\omega/(2\pi)$, they are: $f_1=365.5336 \text{ MHz}$, $f_2=367.4778 \text{ MHz}$, and $f_3=372.2517 \text{ MHz}$, very close to each other because they are mainly determined by $n=1$ with a weak dependence on the in-plane mode variation. The three corresponding modes are shown in Figs. 2a–2c, respectively, in the order of increasing frequency. For better visualization, both a 3D plot and its cross-section along $x_3=0$ are shown for each mode. When the displacement of a mode vanishes at a line (point), the line (point) is called a nodal line (point). The three modes in Fig. 2 have zero, one, and two nodal lines, respectively. The thickness-extensional displacement with $n=1$ is essentially the thickness-extensional strain. If a nodal line is present in an electroded region, the strain changes its sign across the nodal line and so does the charge on the electrode produced by the strain through piezoelectric coupling. This causes charge cancellation on the electrode which reduces the capacitance and is usually undesirable for device applications. For the mode in Fig. 2b, the only nodal line is in the gap between the ring and dot electrodes, which is fine and desirable. However, for the mode in Fig. 2c, one of the two nodal lines is within the dot electrode while the other is in the ring electrode. Therefore, the mode in Fig. 2c is not useful for the filter. When the plate is used as a filter, one of the ring-dot electrodes is with an input voltage for driving, and the other produces an output voltage. The filter can operate with the first mode in which the vibration of the entire plate is in phase. The second mode can also be used in which the regions with the dot and ring electrodes vibrate out of phase. When the frequencies of the two modes are close enough and their resonance peaks overlap to some degree, the two modes can be used together as a filter with essentially twice the bandwidth.

For filter design, the distance between the ring and the dot electrodes is crucial. If the distance is too large, the mechanical interaction between the vibrations under the two electrodes becomes less because of the decay of the vibration from electrode edges. In that case there is little output from the filter. In addition, it is desirable that the nodal line of the mode in Fig. 2b lies in the gap between the two electrodes. In Figs. 3 and 4 (p.791), the radius of the dot electrode, R_0 , and the inner radius of the ring electrodes, R_1 , are varied separately and thus changing the distance between the two electrodes. It can be seen that the modes are very sensitive to R_0 and R_1 . Ideally we want

strong vibrations under both electrodes and there is a strong interaction between them. It can be seen that the vibration in the central region under the dot electrode is usually stronger than that in the annular region under the ring electrode. This in fact may be a good thing for the present circular filter because the ring electrode can be designed with a larger area than

the dot electrode so that the charge on the ring electrode is comparable to that on the dot electrode although the vibration under the ring electrode is weaker.

Fig. 5 shows the effect of the outer radius of the ring electrode, R_2 , on the two operating modes of the filter. For all of the modes, the vibration dies out sufficiently far away outside the ring electrode. This

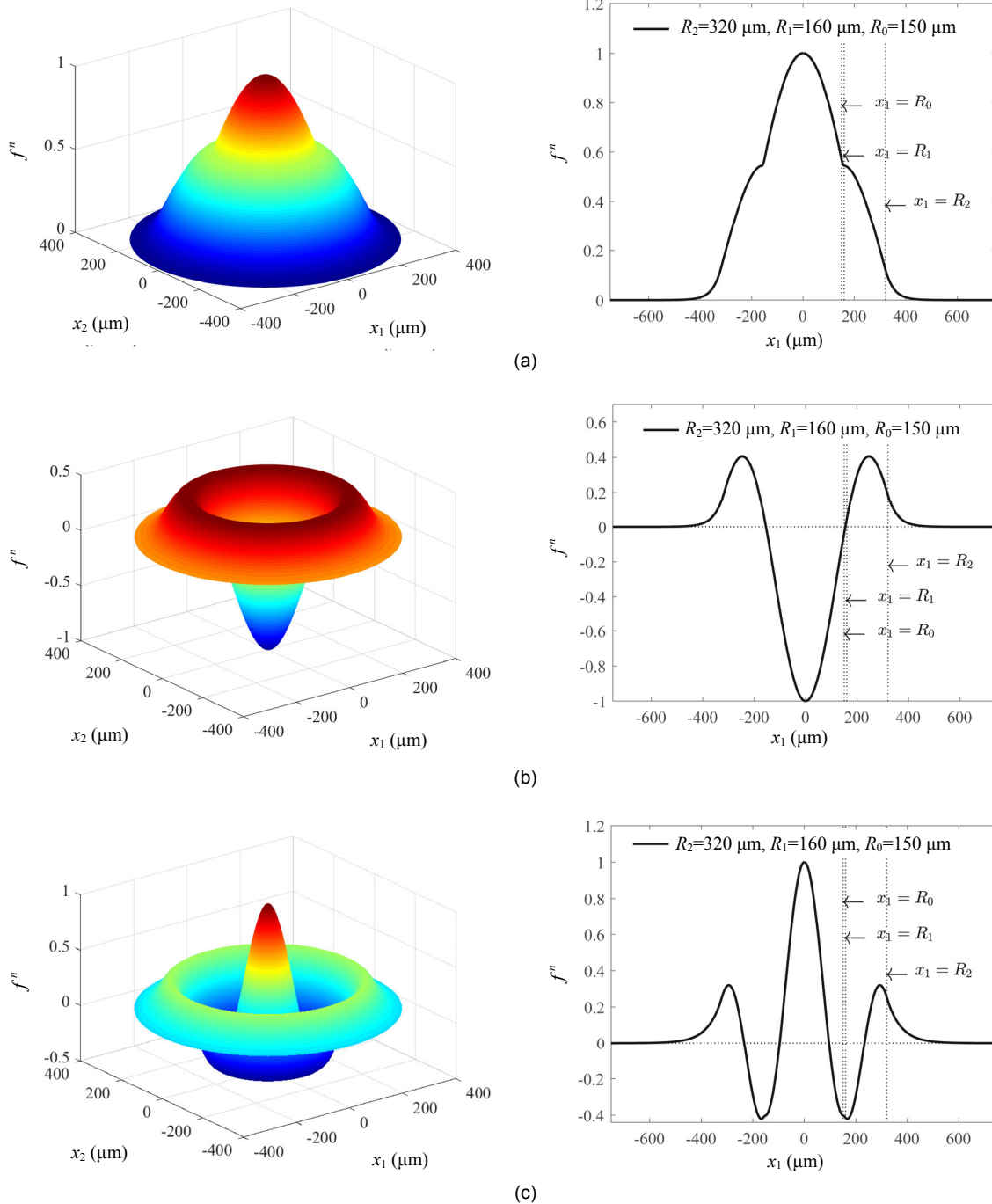


Fig. 2 Three trapped thickness-extensional modes. $h^s = 5 \mu\text{m}$, $h^s = 5 \mu\text{m}$, $h' = 0.044 \mu\text{m}$, $h'' = 0.2 \mu\text{m}$, $R' = 0.03$, $R'' = 0.1359$, $R_0 = 150 \mu\text{m}$, $R_1 = 160 \mu\text{m}$, $R_2 = 320 \mu\text{m}$. (a) $f_1 = 365.5336 \text{ MHz}$; (b) $f_2 = 367.4778 \text{ MHz}$; (c) $f_3 = 372.2517 \text{ MHz}$

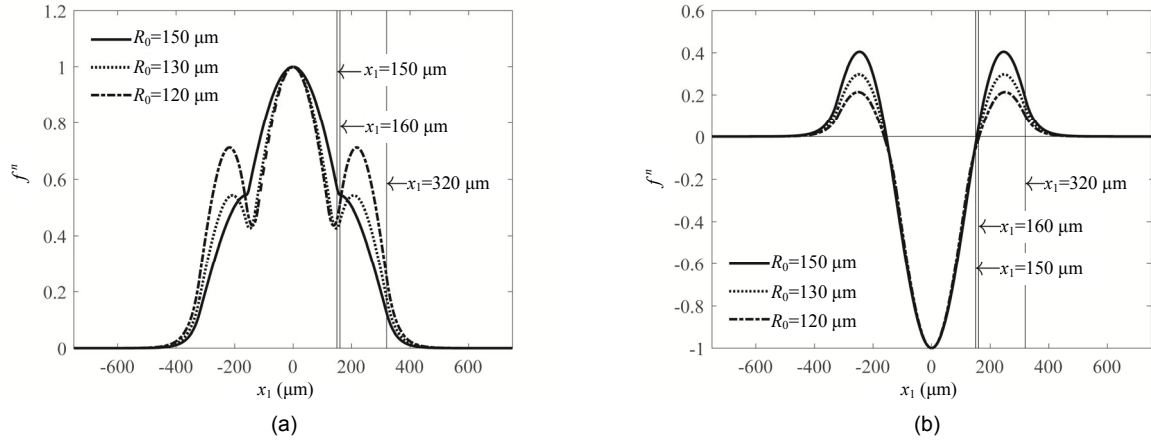


Fig. 3 Effects of the radius of the dot electrode, R_0 , on the two operating modes. $h^f=5\text{ }\mu\text{m}$, $h^s=5\text{ }\mu\text{m}$, $h'=0.044\text{ }\mu\text{m}$, $h''=0.2\text{ }\mu\text{m}$, $R'=0.03$, $R''=0.1359$, $R_1=160\text{ }\mu\text{m}$, $R_2=320\text{ }\mu\text{m}$. (a) $R_0=150\text{ }\mu\text{m}$, $f_1=365.5336\text{ MHz}$; $R_0=130\text{ }\mu\text{m}$, $f_1=366.2273\text{ MHz}$; $R_0=120\text{ }\mu\text{m}$, $f_1=366.4574\text{ MHz}$; (b) $R_0=150\text{ }\mu\text{m}$, $f_2=367.4778\text{ MHz}$; $R_0=130\text{ }\mu\text{m}$, $f_2=367.5586\text{ MHz}$; $R_0=120\text{ }\mu\text{m}$, $f_2=367.7180\text{ MHz}$

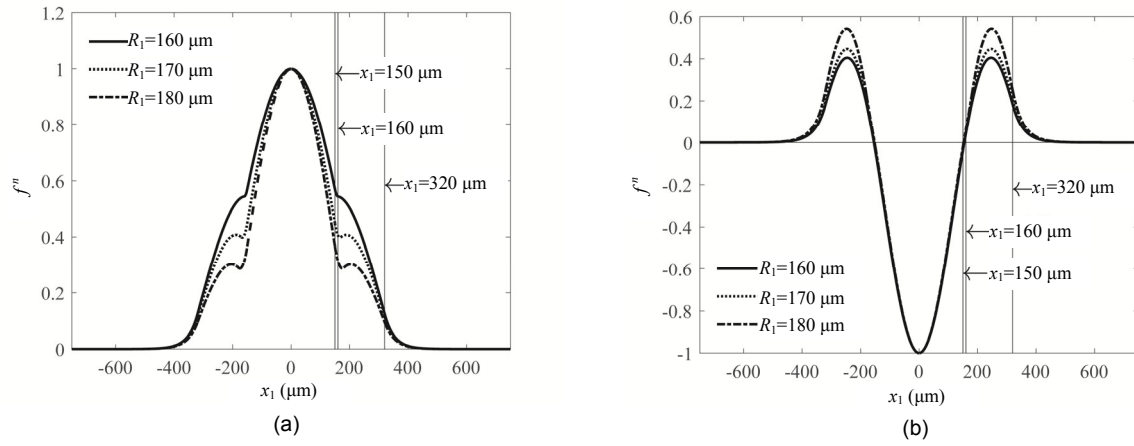


Fig. 4 Effects of the inner radius of the ring electrode, R_1 , on the two operating modes. $h^f=5\text{ }\mu\text{m}$, $h^s=5\text{ }\mu\text{m}$, $h'=0.044\text{ }\mu\text{m}$, $h''=0.2\text{ }\mu\text{m}$, $R'=0.03$, $R''=0.1359$, $R_0=150\text{ }\mu\text{m}$, $R_2=320\text{ }\mu\text{m}$. (a) $R_1=160\text{ }\mu\text{m}$, $f_1=365.5336\text{ MHz}$; $R_1=170\text{ }\mu\text{m}$, $f_1=365.8962\text{ MHz}$; $R_1=180\text{ }\mu\text{m}$, $f_1=366.1621\text{ MHz}$; (b) $R_1=160\text{ }\mu\text{m}$, $f_2=367.4778\text{ MHz}$; $R_1=170\text{ }\mu\text{m}$, $f_2=367.4968\text{ MHz}$; $R_1=180\text{ }\mu\text{m}$, $f_2=367.5646\text{ MHz}$

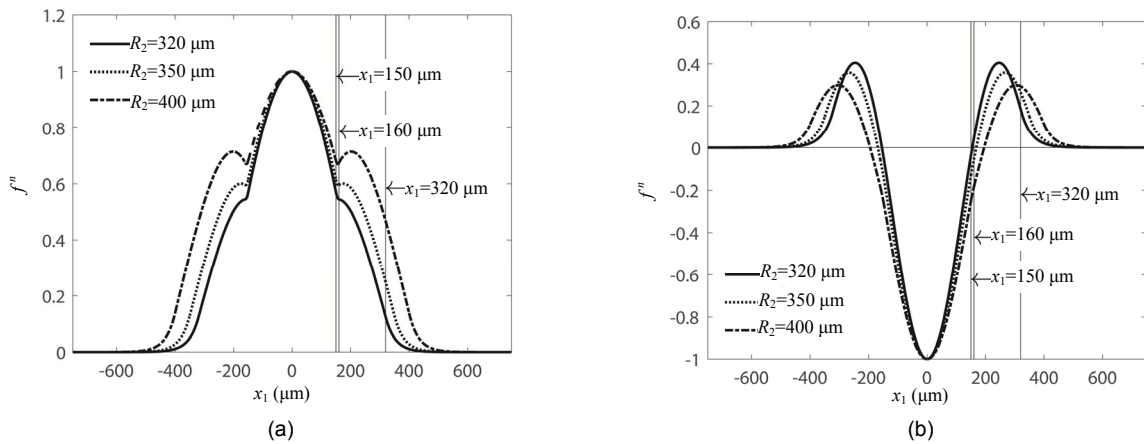


Fig. 5 Effects of the outer radius of the ring electrode, R_2 , on the two operating modes. $h^f=5\text{ }\mu\text{m}$, $h^s=5\text{ }\mu\text{m}$, $h'=0.044\text{ }\mu\text{m}$, $h''=0.2\text{ }\mu\text{m}$, $R'=0.03$, $R''=0.1359$, $R_0=150\text{ }\mu\text{m}$, $R_1=160\text{ }\mu\text{m}$. (a) $R_2=320\text{ }\mu\text{m}$, $f_1=365.5336\text{ MHz}$; $R_2=350\text{ }\mu\text{m}$, $f_1=365.4076\text{ MHz}$; $R_2=400\text{ }\mu\text{m}$, $f_1=365.2253\text{ MHz}$; (b) $R_2=320\text{ }\mu\text{m}$, $f_2=367.4778\text{ MHz}$; $R_2=350\text{ }\mu\text{m}$, $f_2=367.0491\text{ MHz}$; $R_2=400\text{ }\mu\text{m}$, $f_2=366.5753\text{ MHz}$

is the so-called energy trapping of thickness-extensional modes. Energy trapping is crucial to device mounting which can be designed at the plate edges where there is little vibration so that mounting does affect device operation. R_2 is the main design parameter for energy trapping and it determines the size or boundary of a real and finite device. It can be seen that the modes are very sensitive to R_2 .

In Fig. 6, all parameters are kept the same as those in Fig. 2 except that the driving electrode thickness is varied. Similarly, In Fig. 7, only the driving electrode material or effectively the driving electrode mass density is varied. According to Tiersten and Stevens (1983), the two effects in Figs. 6

and 7 affect the behavior of the filter in a combined manner through the mass ratio R' and cannot be separated within the approximate theoretical model of the scalar differential equations in Eqs. (2) and (3). It can be seen that the location of the nodal line and energy trapping of the thickness-extensional modes are both sensitive to the electrode thickness and density. Therefore, the electrode dimension and density are important design parameters.

In filter applications, instead of the frequencies of the two operating modes, i.e. f_1 and f_2 , it is more convenient to use their average and difference. For convenience and comparison, we summarize the average frequency and the frequency difference of the

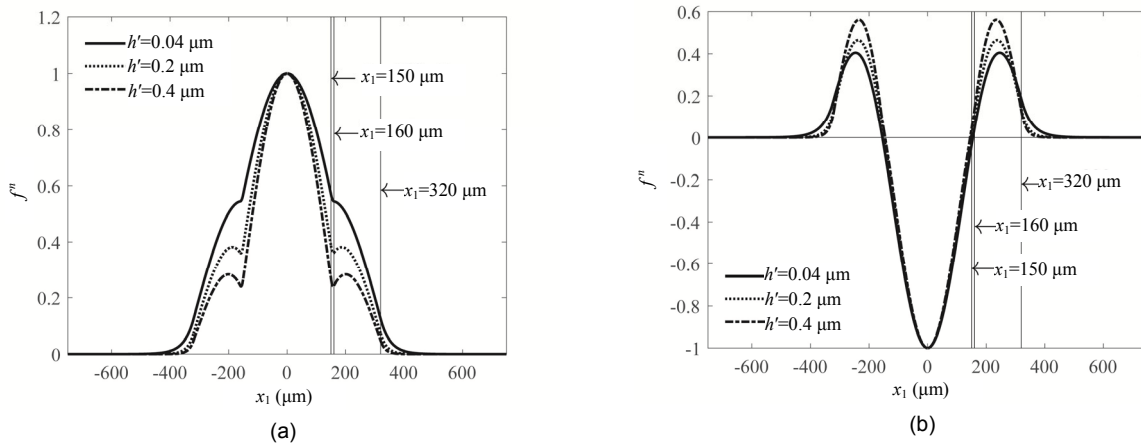


Fig. 6 Effects of the driving electrode thickness, h' , on the two operating modes. $h^f=5 \mu\text{m}$, $h^s=5 \mu\text{m}$, $h''=0.2 \mu\text{m}$, $R''=0.1359$, $R_0=150 \mu\text{m}$, $R_1=160 \mu\text{m}$, $R_2=320 \mu\text{m}$. (a) $h'=0.044 \mu\text{m}$, $f_1=365.5336 \text{ MHz}$; $h'=0.2 \mu\text{m}$, $f_1=348.8182 \text{ MHz}$; $h'=0.4 \mu\text{m}$, $f_1=327.1432 \text{ MHz}$; (b) $h'=0.044 \mu\text{m}$, $f_2=367.4778 \text{ MHz}$; $h'=0.2 \mu\text{m}$, $f_2=350.5370 \text{ MHz}$; $h'=0.4 \mu\text{m}$, $f_2=328.6774 \text{ MHz}$

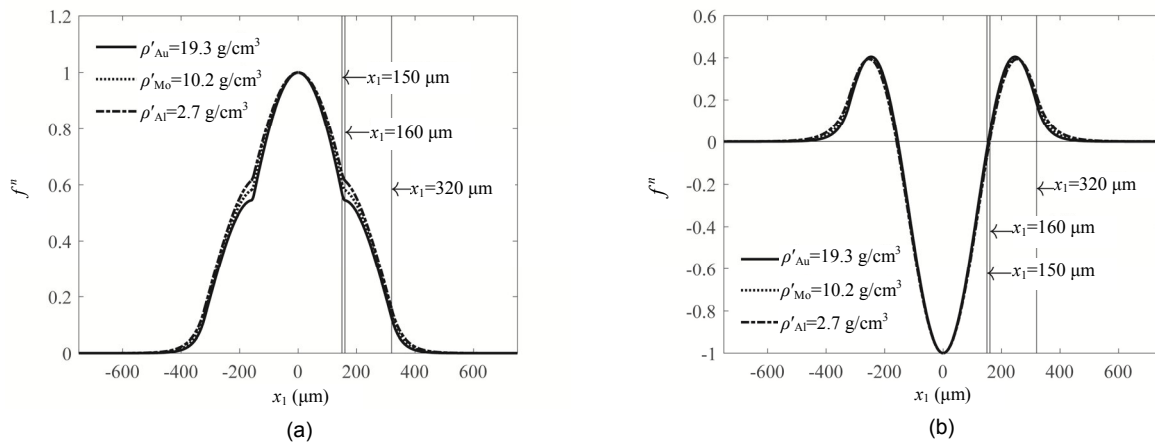


Fig. 7 Effects of the driving electrode mass density, ρ' , on the two operating modes. $h^f=5 \mu\text{m}$, $h^s=5 \mu\text{m}$, $h'=0.044 \mu\text{m}$, $h''=0.2 \mu\text{m}$, $R''=0.1359$, $R_0=150 \mu\text{m}$, $R_1=160 \mu\text{m}$, $R_2=320 \mu\text{m}$. (a) $\rho'=19.3 \text{ g/cm}^3$, $f_1=365.5336 \text{ MHz}$; $\rho'=10.2 \text{ g/cm}^3$, $f_1=367.7326 \text{ MHz}$; $\rho'=2.7 \text{ g/cm}^3$, $f_1=369.5376 \text{ MHz}$; (b) $\rho'=19.3 \text{ g/cm}^3$, $f_2=367.4778 \text{ MHz}$; $\rho'=10.2 \text{ g/cm}^3$, $f_2=369.6926 \text{ MHz}$; $\rho'=2.7 \text{ g/cm}^3$, $f_2=371.4944 \text{ MHz}$

Table 1 Average frequency and frequency difference of the operating modes

R_0 (μm)	R_1 (μm)	R_2 (μm)	h' (μm)	ρ' (g/m^3)	$(f_1+f_2)/2$ (MHz)	f_2-f_1 (kHz)
150	160	320	0.044	19.3	366.5057	1944.2528
130	160	320	0.044	19.3	366.8929	1331.3685
120	160	320	0.044	19.3	367.0877	1260.6349
150	170	320	0.044	19.3	366.6965	1600.5770
150	180	320	0.044	19.3	366.8634	1402.4924
150	160	350	0.044	19.3	366.2283	1641.4400
150	160	400	0.044	19.3	365.9003	1349.9358
150	160	320	0.200	19.3	349.6776	1718.8152
150	160	320	0.400	19.3	327.9103	1534.1840
150	160	320	0.044	10.2	368.7126	1959.9168
150	160	320	0.044	2.7	370.5160	1956.7986

modes in Figs. 2–7 in Table 1. The frequency difference is related to the width of the bandpass of the filter and needs to be designed according to the damping of the specific material used.

5 Conclusions

Within the narrow interval of $(\bar{\omega}_n, \hat{\omega}_n)$, there exist a few trapped modes with vibrations mainly under the dot-ring electrodes. With proper design, the nodal line of the second trapped mode lies in the gap between the ring and the dot electrodes. Then the first two trapped modes of the proposed structure can be used as the ideal operating modes for a filter. The location of the nodal line and the decay rate of the fields away from the electrode edges are sensitive to geometric and physical parameters. Therefore, these filters need to be designed accurately. The analysis in the present paper provides both basic understanding and design tools for these filters. Similar to the filter with interdigital electrodes in (Liu et al., 2017), circular FBARs with multiple ring electrodes can be used to make filters.

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中文概要

题 目: 具有环形-圆形电极的薄膜体声波滤波器

目 的: 研发一种新型氧化锌薄膜体声波滤波器, 建模进行理论分析, 并给出设计工具。

创新点: 提出一种氧化锌薄膜体声波滤波器的新结构。

方 法: 用 Tiersten-Stevens 方程进行理论分析与计算; 方程及其求解和计算简单易行。

结 论: 具有环形-圆形电极的氧化锌薄膜可以作为体声波滤波器。圆形和环形电极具有简单和易加工等优点, 并且工作模态的振动分布与材料的对称性高度一致, 避免了矩形电极角点引起的电场集中等缺点。该滤波器的最优工作状态需要通过精密设计实现, 本文提供了理论和计算设计工具。

关键词: 薄膜; 声波; 滤波器