# Feedback analysis and design of inductive power links driven by Class-E amplifiers with variable coupling coefficients* 

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#### Abstract

The efficiency of inductive power links driven by Class-E amplifiers may deteriorate due to variation in the coupling coefficient when the relative position of the radio frequency (RF) coils changes. To solve this problem, a new design methodology of power links is presented in this paper. The aim of the new design is to use the feedback signal, which is a phase difference between the driving signal and the output current of the Class-E amplifier, to adjust the duty cycle and angular frequency of the driving signal to maintain the optimum state of the inductive power link, and to adjust the supply voltage to keep the output power constant when the coupling coefficient of the RF coils changes. The parameter adjustments with respect to the coupling coefficient and the feedback signal are derived from the design equation of the inductive power link. To validate the feedback control rules, a prototype of the inductive power link was constructed, and its performance validated with the coupling coefficient set at 0.2 and a duty cycle of 0.5 . The experimental results showed that, by adjusting the duty cycle, the angular frequency, and the supply voltage, the power link can be kept in optimal operation with a constant output power when the coupling coefficient changes from 0.2 to 0.1 to 0.25 .


Key words: Inductive power link, Closed-loop Class-E amplifier, High efficient, Variable coupling coefficient doi:10.1631/jzus.C0910607

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## 1 Introduction

Inductive power links driven by Class-E amplifiers are commonly used to power implantable biomedical devices (Sarpeshkar et al., 2005; Lenaerts and Puers, 2007). An increasing trend is to use rechargeable implanted batteries to power such devices, which require periodical recharging through a power link. A basic structure of the inductive power links is shown in Fig. 1 (Baker and Rahul, 2007).

The Class-E amplifier drives the primary radio frequency (RF) coil. The coil then sends inductive power to the secondary RF coil through the skin of the patient. A power supply can be created by rec-

[^0]tifying the RF signal on the secondary coil, which can then be used to charge the implanted battery. The center distance between the RF coil pair is $d$. Variation in $d$ and displacement of the RF coil pair can affect the coupling coefficient $k$, and hence change the operating state of the power link.

The inductive power link is expected to be of high efficiency to prolong the feeding duration of the battery and reduce the possibility of tissue damage caused by overheating. It is also expected to have constant output power or constant output voltage so that the implanted devices can work in the normal state. However, because of body movements, displacement of the inductive coils or variation in the distance between them may occur leading to variation in $k$. Consequently, the efficiency may decrease and the output power may change. Previous studies reported that when $k$ changes, the Class-E amplifier may be returned to a better or optimum operating state
by adjusting the frequency (Kendir et al., 2005; Zan et al., 2007). However, the frequency adjustment ranges have not yet been determined theoretically. When the transmitting coil is flexible or deformable, an electrically controllable inductance is employed to compensate for changes in the transmitting coil (Lenaerts and Puers, 2008). However, variation in $k$ will lead to variation in the impedance that the secondary circuit reflects to the primary circuit, and the inductance compensation cannot retune the power link. The output power will vary due to the variation in $k$. It can be kept constant or be varied as designated by increasing or decreasing the supply voltage of the primary circuit according to the data sent back by the secondary circuit (Wang et al., 2005). However, the efficiency may deteriorate due to the loss of the optimum state.


Fig. 1 Example of an inductive power link for implanted devices

In this paper, a design methodology is presented for an inductive power link driven by a Class-E amplifier with the aim of improving efficiency and keeping the output power constant when $k$ changes. Analytical expressions are derived as functions of the duty cycle $D$ and the angular frequency $\omega_{0}$ of the driving signal. The phase difference $\theta_{\mathrm{PD}}$ between the output current of the Class-E amplifier and the driving signal is used as a feedback quantity; $D, \omega_{0}$, and $V_{D D}$ are designated as the adjusted parameters. The relations between the adjusted parameters and $k$ as well as $\theta_{\mathrm{PD}}$ are obtained from the design equations of the inductive link with $k=0.2$ and $D=0.5$. Experiments were conducted to validate the theoretical design when $k$ is changed from 0.2 to 0.1 to 0.25 .

## 2 Determination of component values

To simplify the derivation of the analytical equation parameter of each component, the load re-
sistances of the secondary circuit are equivalent to one resistance $R_{\mathrm{L}}$. The nonlinear secondary circuit is converted to a linear model by transferring the DC load to an equivalent AC load (Fig. 2) (Ko et al., 1977; Donaldson and Perkins, 1983).


Fig. 2 Approximated linear model of the secondary circuit $L_{2}$ is the secondary RF coil and $C_{2}$ is the resonant capacitance

The RF coil pair links the primary and secondary circuits, and the reflected impedance of the secondary circuit appearing at the primary circuit, $Z^{\prime}{ }_{\text {eL }}$, is defined by

$$
\begin{align*}
Z_{\mathrm{eL}}^{\prime}= & \omega_{0}^{2} M^{2} / Z_{2} \\
= & \frac{\omega_{0}^{2} k^{2} L_{1} L_{2}\left(4+\omega_{0}^{2} C_{2}^{2} R_{\mathrm{L}}^{2}\right)}{4 R_{\mathrm{L}}^{2}+\left[\omega_{0} L_{2}\left(4+\omega_{0}^{2} C_{2}^{2} R_{\mathrm{L}}^{2}\right)-\omega_{0} C_{2} R_{\mathrm{L}}^{2}\right]^{2}}  \tag{1}\\
& \cdot\left\{2 R_{\mathrm{L}}-\mathrm{j}\left[\omega_{0} L_{2}\left(4+\omega_{0}^{2} C_{2}^{2} R_{\mathrm{L}}^{2}\right)-\omega_{0} C_{2} R_{\mathrm{L}}^{2}\right]\right\} \\
= & R_{\mathrm{eL}}^{\prime}+\mathrm{j} X_{\mathrm{eL}}^{\prime},
\end{align*}
$$

where $\omega_{0}$ is the angular frequency, $M=k \sqrt{L_{1} L_{2}}$ is the mutual inductance between the transmitter coils $L_{1}$ and $L_{2}, k$ is the coupling coefficient, $Z_{2}$ is the secondary impedance from the mutual inductance, $R_{\text {eL }}^{\prime}$ is the reflected resistance, and $X^{\prime}{ }_{\text {eL }}$ is the reflected reactance.

The primary circuit is a basic Class-E amplifier. Fig. 3 shows the schematic diagram of the primary circuit with an equivalent impedance $Z_{\text {eL }}$.

To enable the inductive power link to operate in the optimum state, the reflected impedance of the secondary circuit is required to equal the equivalent impedance of the Class-E amplifier; i.e., the design equation of the inductive power link can be expressed by

$$
\begin{equation*}
Z_{\mathrm{eL}}^{\prime}=Z_{\mathrm{eL}} . \tag{2}
\end{equation*}
$$

$Z^{\prime}{ }_{\text {eL }}$ can be derived from Eq. (1) and $Z_{\text {eL }}$ can be obtained from the design equations of the Class-E amplifier. To derive the design equations, the following analysis is performed on the assumptions that
$L_{\mathrm{f}}$ is lossless and large enough to neglect its ripple current, and that the load quality factor $Q$ of the se-ries-resonant circuit is high enough so that the output current can be considered a sine wave.


Fig. 3 Schematic diagram of the primary circuit with an equivalent impedance
A basic Class-E amplifier consists of an ideal switch, a shunt capacitance $C$, an RF choke coil $L_{\mathrm{f}}$, and a series-resonant circuit comprising $L_{1}$, a capacitance $C_{1}$, and an equivalent load impedance $Z_{\mathrm{eL}} . Z_{\mathrm{eL}}$, which is composed of an equivalent resistance $R_{\mathrm{eL}}$ and an equivalent reactance $X_{\mathrm{eL}}$, is defined as an equivalent impedance that lets the Class-E amplifier operate in the optimum state. $v_{\mathrm{g}}$ is the driving signal, which has a duty cycle $D$ and an angular frequency $\omega_{0}$. $v_{\mathrm{S}}$ is the switch voltage

The output current $i_{01}$ of the Class-E amplifier can be expressed as

$$
\begin{equation*}
i_{\mathrm{o1}}(\theta)=I_{\mathrm{olm}} \sin \theta \tag{3}
\end{equation*}
$$

where $I_{01 \mathrm{~m}}$ is the current amplitude and $\theta=\omega_{0} t$ is the phase.

To satisfy the zero-derivative switching (ZDS) condition (Sokal and Sokal, 1975; Raab, 1977), the switch should turn on at the instant that $i_{01}$ is equal to the supply current $I_{\mathrm{DD}}$. The relationship between $I_{\mathrm{DD}}$ and $i_{01}$ can be expressed as

$$
\begin{equation*}
I_{\mathrm{DD}}=I_{\mathrm{olm}} \sin \theta_{\mathrm{x}} \tag{4}
\end{equation*}
$$

where $\theta_{\mathrm{X}}=\pi-\theta_{\mathrm{PD}}$, and $\theta_{\mathrm{PD}}$ is the phase difference between $v_{\mathrm{g}}$ and $i_{01}$. $\theta_{\mathrm{X}}$ can be determined by the zero-voltage switching (ZVS) condition (Sokal and Sokal, 1975; Raab, 1977) as

$$
\begin{equation*}
\theta_{\mathrm{x}}=\arctan \left(\frac{1-\cos \theta_{\mathrm{D}}}{2 \pi-\theta_{\mathrm{D}}+\sin \theta_{\mathrm{D}}}\right) \tag{5}
\end{equation*}
$$

where $\theta_{D}=2 \pi D$ is the switch-on phase duration.
In the switch-off duration, the current of shunt capacitance $i_{\mathrm{C}}=I_{\mathrm{DD}}-i_{\mathrm{o1}}$, and in the switch-on duration,
$i_{\mathrm{C}}=0$. In a switch-off and switch-on cycle, the DC component of $i_{\mathrm{C}}$ is zero because of the ZVS condition. The switch voltage $v_{\mathrm{S}}$ can be expressed as $V_{\mathrm{S}}(\mathrm{j} \omega)=$ $I_{C}(\mathrm{j} \omega) /(\mathrm{j} \omega C)$ in the frequency domain, where $V_{\mathrm{S}}$ and $I_{\mathrm{C}}$ are the Fourier transforms of $v_{\mathrm{S}}$ and $i_{\mathrm{C}}$, respectively. The voltage drop of $L_{\mathrm{f}}$ is zero since $L_{\mathrm{f}}$ is lossless. Hence the supply voltage $V_{\mathrm{DD}}$ equals the DC component of $V_{S}(\mathrm{j} \omega)$, i.e., $V_{\mathrm{DD}}=V_{\mathrm{S}}(0)$. $V_{\mathrm{DD}}$ can be expressed as

$$
\begin{equation*}
V_{\mathrm{DD}}=\frac{I_{01 \mathrm{~m}}}{2 \pi \omega_{0} C} \alpha \tag{6}
\end{equation*}
$$

where $\alpha=-\theta_{\mathrm{F}} \theta_{\mathrm{C}} \sin \theta_{\mathrm{X}}+\theta_{\mathrm{F}} \cos \left(\theta_{\mathrm{D}} / 2\right) \cos \theta_{\mathrm{C}}+2 \sin \left(\theta_{\mathrm{D}} / 2\right) \times$ $\cos \theta_{\mathrm{c}}+2 \theta_{\mathrm{C}} \sin \left(\theta_{\mathrm{D}} / 2\right) \sin \theta_{\mathrm{c}}, \theta_{\mathrm{F}}=2 \pi-\theta_{\mathrm{D}}$, and $\theta_{\mathrm{C}}=\theta_{\mathrm{D}}-\theta_{\mathrm{X}}$.

The load impedance of the Class-E amplifier at $\omega_{0}$ can be expressed as $Z_{1 \mathrm{~L}}=R_{\text {eL }}+\mathrm{j}\left[X_{\text {eL }}+\omega_{0} L_{1}-\right.$ $\left.1 /\left(\omega_{0} C_{1}\right)\right]$. It can also be obtained by $Z_{1 L}=$ $V_{\mathrm{s}}\left(\omega_{0}\right) / I_{01}\left(\omega_{0}\right)$, where $V_{\mathrm{s}}\left(\omega_{0}\right)$ and $I_{01}\left(\omega_{0}\right)$ are the spectral components of $V_{\mathrm{S}}$ and $I_{01}$ respectively at $\omega_{0}$. Comparing the two expressions of $Z_{1 \mathrm{~L}}$, the real and imaginary parts of $Z_{1 L}$ can be obtained by

$$
\begin{equation*}
R_{\mathrm{eL}}=\beta /\left(2 \pi \omega_{0} C\right) \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
X_{\mathrm{eL}}=\gamma /\left(2 \pi \omega_{0} C\right)-\omega_{0} L_{1}+1 /\left(\omega_{0} C_{1}\right) \tag{8}
\end{equation*}
$$

where $\beta=\left[\sin \theta_{\mathrm{X}}+\sin \left(\theta_{\mathrm{D}}-\theta_{\mathrm{X}}\right)\right]^{2}$ and $\gamma=\theta_{\mathrm{F}} \cos \left(2 \theta_{\mathrm{X}}\right)+$ $\sin \theta_{\mathrm{D}} \cos \left(2 \theta_{\mathrm{c}}\right) . \alpha, \beta$, and $\gamma$ are functions of $D$. Eqs. (7) and (8) are the design equations of the Class-E amplifier. When the component values satisfy the design equations, the Class-E amplifier will operate in the optimum state.

In the power link, $D, V_{\mathrm{DD}}$, and the output power $P_{\text {out }}$ are always designated as the given parameters. $P_{\text {out }}$ is equal to both the transmitting power $P_{\mathrm{RF}}$ and the input power $P_{\text {in }}$ if the parasitic resistance of each component is zero. From Eqs. (4), (6), and (7), $R_{\text {eL }}$ can be expressed in terms of $P_{\mathrm{in}}, V_{\mathrm{DD}}$, and $D$ as

$$
\begin{equation*}
R_{\mathrm{eL}}=\frac{\beta \sin \theta_{\mathrm{X}} V_{\mathrm{DD}}^{2}}{\alpha P_{\mathrm{in}}} \tag{9}
\end{equation*}
$$

With the calculated $R_{\text {eL }}, C$ can be determined by Eq. (7). $L_{1}$ can be obtained by $L_{1}=Q R_{\mathrm{eL}} / \omega_{0}$ with a given Q. $L_{\mathrm{f}}$ is expected to be large enough, but it will take a long time to reach steady state if $L_{\mathrm{f}}$ is too large. $L_{\mathrm{f}}$ can be determined by $L_{\mathrm{f}}=4 \pi\left(\pi^{2} / 4+1\right) R_{\mathrm{eL}} / \omega_{0}$ (Kazimierczuk
and Czarkowski, 1995).
Combining Eqs. (1), (7), and (8), the resonant capacitance $C_{2}$ can be determined by the resonant condition ( $\omega_{0}=1 / \sqrt{L_{2} C_{2}}$ ) as

$$
\begin{equation*}
C_{2}=\frac{2 R_{\mathrm{eL}}}{\omega_{0}^{2} k^{2} L_{1} R_{\mathrm{L}}} . \tag{10}
\end{equation*}
$$

$L_{2}$ can then be obtained by the resonant condition. $X_{\mathrm{eL}}$ can be obtained from Eq. (1). Finally, $C_{1}$ is determined by Eq. (8).

In previous studies, $X_{\text {eL }}$ was always designated as 0 to facilitate the component determination of the Class-E amplifier, and the operating frequency was slightly smaller than the resonant frequency of $L_{2}$ and $C_{2}$ (Donaldson and Perkins, 1983). In our design, $X_{\mathrm{eL}}$ is a capacity reactance. From Eq. (1), $X_{\text {eL }}$ is derived by

$$
\begin{equation*}
X_{\mathrm{eLL}}=-k^{2} \omega_{0} L_{1} . \tag{11}
\end{equation*}
$$

The power link with the calculated component values will meet the design Eq. (2) and then will operate in the optimum state when $k$ is equal to the designated values.

## 3 Feedback analysis

### 3.1 Variation in the coupling coefficient

When $k$ varies, the power link will deviate from its optimum state. The efficiency will deteriorate and the output power will change. Adjustments are required to improve the efficiency and keep the output power constant. In the following analysis, $D$ and $\omega_{0}$ are employed as adjustments for re-tuning the power link, and the adjustment of $V_{\mathrm{DD}}$ is used to keep $P_{\text {out }}$ constant. By using the experimental parameters shown later in Table 2, $D, \omega_{0}$, and $V_{\mathrm{DD}}$ with respect to $k$ are obtained from the design Eq. (2).

Substituting Eqs. (1), (7), and (8) into Eq. (2), the relation between $\omega_{0}$ and $D$ can be expressed by

$$
\begin{equation*}
\frac{L_{2} C_{2}^{2} R_{\mathrm{L}}}{2} \omega_{0}^{3}-\frac{L_{1} C}{\beta} \omega_{0}^{2}+\left(\frac{2 L_{2}}{R_{\mathrm{L}}}-\frac{C_{2} R_{\mathrm{L}}}{2}\right) \omega_{0}+\frac{C}{C_{1} \beta}+\frac{\gamma}{\beta}=0 . \tag{12}
\end{equation*}
$$

This is a cubic equation with a given $D$, and thus has three roots. Eq. (12) can be solved by MATLAB,
and it can be seen that two roots are a pair of complex conjugates and one root is real. Frequencies are real numbers, so the complex roots are ineffective. The effective real root $\omega_{0}$ with respect to $D$ can be defined as $\omega_{0}=f(D)$ (Fig. 4). It can be seen that a pair of $D$ and $\omega_{0}$ will satisfy Eq. (2) for a certain $k$. Therefore, the power link using the component values of Table 2 will operate in the optimum state.

Substituting $f(D)$ into Eqs. (1), (7), and (8), $R_{\mathrm{eL}}$, $R^{\prime}{ }_{\mathrm{eL}}$ and $X_{\mathrm{eL}}, X^{\prime}{ }_{\mathrm{eL}}$ with respect to $D$ are obtained, as shown in Figs. 5a and 5b, respectively. We can see that $R^{\prime}{ }_{\text {eL }}$ and $X_{\text {eL }}^{\prime}$ are in direct proportion to $k^{2}$, and intersect with $R_{\mathrm{eL}}$ and $X_{\mathrm{eL}}$ at two points when $k<0.28$, at one point when $k=0.28$, and at no point when $k>0.28$. At those intersections, design Eq. (2) is satisfied and the inductive power link will operate in the optimum state.


Fig. 4 Angular frequency with respect to duty cycle for the optimum state


Fig. 5 Equivalent resistance and reflected resistance with respect to duty cycle
(a) Equivalent resistance and reflected resistance; (b) Equivalent reactance and reflected reactance

Combining Eqs. (1) and (7), $k$ can be expressed as

$$
\begin{equation*}
k=\sqrt{\frac{\beta\left\{4 R_{\mathrm{L}}^{2}+\left[\omega_{0} L_{2}\left(4+\omega_{0}^{2} C_{2}^{2} R_{\mathrm{L}}^{2}\right)-\omega_{0} C_{2} R_{\mathrm{L}}^{2}\right]^{2}\right\}}{4 \pi \omega_{0}^{3} L_{1} L_{2} R_{\mathrm{L}} C\left(4+\omega_{0}^{2} C_{2}^{2} R_{\mathrm{L}}^{2}\right)}} \tag{13}
\end{equation*}
$$

Substituting $f(D)$ into Eq. (13), $k$ as a function of $D$ and $\omega_{0}$ is obtained. For a pair of $D$ and $\omega_{0}, k$ is determined and in turn, for a certain $k$, there is a pair of $D$ and $\omega_{0}$ which will satisfy design Eq. (2). $D$ and $\omega_{0}$ with respect to $k$ are shown in Figs. 6a and 6b, respectively. Both $D$ and $\omega_{0}$ need adjustment to maintain the optimum state of the power link when $k$ varies.

Although the power link can be kept in the optimum state by adjusting $D$ and $\omega_{0}$ when $k$ changes, $R^{\prime}{ }_{\text {eL }}$ is charged. Thus $P_{\text {out }}$ will be changed. To keep $P_{\text {out }}$ constant, $V_{\mathrm{DD}}$ needs to be adjusted.

Combining Eqs. (7) and (9), $V_{\mathrm{DD}}$ can be expressed as

$$
\begin{equation*}
V_{\mathrm{DD}}=\sqrt{\frac{\alpha P_{\mathrm{in}}}{2 \pi \omega_{0} C \sin \theta_{\mathrm{x}}}} . \tag{14}
\end{equation*}
$$

Substituting $f(D)$ into Eq. (14), $V_{\text {DD }}$ with respect to $k$ is obtained. This relation is the ideal feedback control rule of $V_{\mathrm{DD}}$ to keep $P_{\text {out }}$ constant when $k$ varies (Fig. 6c). In a real circuit, power loss of each component exists and varies for different $R_{\mathrm{eL}}$ and $D$ (Kessler and Kazimierczuk, 2004; Mury and Fusco, 2006). To keep $P_{\text {out }}$ constant, $V_{\mathrm{DD}}$ needs additional adjustment when $k$ varies.

Substituting $f(D)$ into Eq. (6), $I_{01 \mathrm{~m}}$ with respect to $k$ is obtained (Fig. 6d). $I_{o 1 \mathrm{~m}}$ varies with $k$ to keep $P_{\mathrm{RF}}$ constant due to the variation in $R^{\prime}{ }_{\text {eL }}$. From Eq. (4), $I_{\mathrm{DD}}$ with respect to $k$ is derived. $I_{D D}$ varies with $k$ to keep $P_{\text {in }}$ constant.

### 3.2 Feedback control

The power link will operate in the optimum state with constant output power if $D, \omega_{0}$, and $V_{\mathrm{DD}}$ vary (Figs. 6a-6c). To detect the $k$ variations, the phase difference $\theta_{\mathrm{PD}}$ is employed as the feedback quantity. From Eqs. (5) and (13), $\theta_{\text {PD }}$ with respect to $k$ can be determined (Fig. 6e). The variation range of $k$ is set to $0.1 \leq k \leq 0.25$, and the variation region of the feedback quantity is $2.17 \leq \theta_{\mathrm{PD}} \leq 2.78$ (the bold line in Fig. 6e).


Fig. 6 Main parameters with respect to the coupling coefficient
(a) Duty cycle; (b) Angular frequency; (c) Supply voltage; (d) Input current and the amplitude of the output current; (e) Feedback quantity

From Eqs. (5) and (12), $D$ and $\omega_{0}$ can be expressed as functions of $\theta_{\mathrm{PD}}$ (Figs. 7a and 7b, respectively). From Eqs. (5), (12), and (14), $V_{\mathrm{DD}}$ with respect to $\theta_{\mathrm{PD}}$ is obtained (Fig. 7c).

To implement the control rules, $\theta_{\mathrm{PD}}$ can be detected by a phase detector, and a microprocessor can be used to adjust $D, \omega_{0}$, and $V_{\mathrm{DD}}$.


Fig. 7 Adjusted parameters with respect to feedback quantity
(a) Duty cycl
(b) Angular frequency;
(c) Supply voltage

## 4 Experimental results

To experimentally validate the operating state and the output power of the inductive power link when the relative position between the transmitter coil and the receiver coil changes, an inductive power link was designed which satisfies the parameters given in Table 1. Based on the component determination equations shown in Section 2, the component values are shown in Table 2.

The MOSFET IRF 510 was employed as the switch. The driving signals were derived from a function generator. All capacitors were Silver Mica, and the inductive coils were made of Litz wire, which consists of 200 strands with a diameter of 0.1 mm . The primary and secondary coils were 19 and 18.75 turns respectively, with a diameter of 85 mm . The capacitors, the inductor, and their equivalent series resistances (ESR) were measured using an HP LF

Table 1 Design parameters of the power link

| Parameter | Value |
| :--- | :---: |
| Output power, $P_{\text {out }}$ | 1 W |
| Supply voltage, $V_{\mathrm{DD}}$ | 6 V |
| Quality factor, $Q$ | 10 |
| Duty ratio, $D$ | 0.5 |
| Operating frequency, $f$ | 1 MHz |
| Coupling coefficient, $k$ | 0.2 |
| Load resistance, $R_{\mathrm{L}}$ | $1 \mathrm{k} \Omega$ |

Table 2 Component values of the power link

| Component | Value | ESR $(\Omega)$ |
| :--- | :---: | :---: |
| Choke coil, $L_{\mathrm{f}}$ | $144.0 \mu \mathrm{H}$ | 0.18 |
| Shunt capacitance, $C$ | 1.41 nF | 0.21 |
| Series inductance, $L_{1}$ | $33.0 \mu \mathrm{H}$ | 0.93 |
| Series capacitance, $C_{1}$ | 907.3 pF | 0.16 |
| Resonance inductance, $L_{2}$ | $31.8 \mu \mathrm{H}$ | 0.86 |
| Resonance capacitance, $C_{2}$ | 795.8 pF | 0.14 |
| Equivalent resistance, $R_{\mathrm{eL}}$ | $20.8 \Omega$ | - |
| ESR: equivalent series resistance |  |  |

Impedance Analyzer 4192 A in the laboratory at the intended operating frequency of 1 MHz (Table 2). The coupling coefficients between the RF coil pair were calculated using the equation $U_{2}=k L_{1} L_{2} \omega_{0} I_{1} . I_{1}$ is the current of the primary coil and $U_{2}$ is the induced voltage of the secondary coil. The primary coil was driven by a 1 MHz sine wave, which was derived by the function generator. The voltages of the primary and secondary coils were measured and then $k$ could be calculated. It was found that the coupling coefficients between the coil pair were $0.1,0.2$, and 0.25 when their center distances were 69, 46, and 40 mm , respectively.

The primary circuit adopted the basic Class-E amplifier (Fig. 3) and the secondary circuit was simplified using the linear model (Fig. 2). The minimum and maximum values of $k$ were set to 0.1 and 0.25 , respectively. For different $k$ 's, the adjustments of $D$, frequency $f\left(=\omega_{0} /(2 \pi)\right)$, and $V_{\mathrm{DD}}$ are shown in Table 3. $P_{\text {out }}$ was calculated by $P_{\text {out }}=V_{0}^{2} / R_{\mathrm{L}}$, where $V_{\mathrm{o}}$ is the effective value of $v_{0} . P_{\text {in }}$ is determined by $V_{\mathrm{DD}}$ and $I_{\mathrm{DD}}$. Each component of the power link has parasitic resistance, and power loss will exist. $P_{\text {out }}$ may be smaller than that designated. To satisfy the require-
ment of constant output power of $1 \mathrm{~W}, V_{\mathrm{DD}}$ needed to be adjusted. $V_{\text {DD }}$ was adjusted to $2.6,6.4$, and 10.4 V when $k$ was $0.1,0.2$, or 0.25 , respectively. The adjustments of $V_{D D}$ were slightly larger than the theoretical values of $1.8,6$, and 9.9 V .

The experimental switch voltages $v_{\mathrm{S}}$ and output voltages $v_{0}$ of the secondary circuit are shown in Fig. 8. The waveforms of the power link with a $k$ of 0.2 are shown in Fig. 8b. It can be seen that $v_{\mathrm{S}}$ returned to zero smoothly when the switch turned on. The slope of $v_{\mathrm{S}}$ will be zero at the instant the switch turns on, and thus the power link will operate in the optimum state; the amplitude of $v_{0}$ was 31.6 V and $P_{\text {out }}$ was 1 W . In Figs. 8a and 8c, the waveforms of the power link with and without adjustments are plotted on the left and right when $k$ was 0.1 and 0.25 , respectively. It can be seen that the power link with the adjustments to $D$, $\omega_{0}$, and $V_{D D}$ will operate in the optimum state with constant output power; without any adjustment, the power link will not maintain its optimum state and the output power will reduce. In the no-adjustment condition, the amplitudes of the output voltage were 16.5 V and 24.4 V , and the output powers were 272 mW and 595 mW when $k$ was 0.1 and 0.25 , respectively. These values are much smaller than those designated.

There are parasitic resistances in the components of the primary and secondary circuits. To analyze the efficiency of the circuits, the primary efficiency $\eta_{\mathrm{p}}$ is defined as the total efficiency of the primary circuit except the driving signal, and the secondary efficiency $\eta_{\mathrm{s}}$ is the total efficiency of the secondary circuit. With the ESR of each component, $\eta_{\mathrm{p}}, \eta_{\mathrm{s}}$, and the theoretical total efficiency $\eta_{\mathrm{c}}$ were calculated (Fig. 9) (Yang et al., 2009). The experimental total efficiency with adjustments $\eta_{\text {ac }}$ and the experimental total efficiency without any adjustment $\eta_{\text {nc }}$ are plotted in Fig. 9. It can be seen that $\eta_{\mathrm{ac}}$ closely matches $\eta_{\mathrm{c}}$; without any adjustment, the efficiency would decrease significantly, especially for the small $k$ condition.

The experimental input power, output power, and efficiency are shown in Table 3. $\eta_{\text {ac }}$ was $67.6 \%$ when the coupling coefficient $k$ was 0.1 , which was much smaller than when $k$ was 0.2 or 0.25 . The reason is that the equivalent load resistance $R_{\mathrm{eL}}$ is only $4.6 \Omega$ at $k=0.1$. This is so small that the parasitic resistance
of each component will significantly affect the circuit. A large $R_{\mathrm{eL}}$ will decrease the effect of parasitic resistances and reduce the component power loss, thereby increasing efficiency.


Fig. 8 Waveforms of switch voltage and output voltage
(a) $k=0.1$; (b) $k=0.2$; (c) $k=0.25$


Fig. 9 Computed and measured efficiencies with respect to the coupling coefficient $\boldsymbol{k}$
$\eta_{\mathrm{p}}$ : primary efficiency; $\eta_{\mathrm{s}}$ : secondary efficiency; $\eta_{\mathrm{c}}$ : theoretical total efficiency; $\eta_{\mathrm{ac}}$ : experimental total efficiency with adjustments; $\eta_{\mathrm{nc}}$ : experimental total efficiency without any adjustment

Table 3 Adjusted parameters, power, and efficiency

| Parameter | Value |  |  |
| :--- | :---: | :---: | :---: |
|  | $k=0.1$ | $k=0.2$ | $k=0.25$ |
| Equivalent resistance, $R_{\mathrm{eL}}(\Omega)$ | 4.6 | 20.8 | 25.2 |
| Duty cycle, $D$ | 0.71 | 0.75 | 0.38 |
| Frequency, $f(\mathrm{MHz})$ | 0.925 | 1.000 | 1.113 |
| Supply voltage, $V_{\mathrm{DD}}(\mathrm{V})$ | 2.6 | 6.4 | 10.4 |
| Input power, $P_{\mathrm{in}}(\mathrm{W})$ | 1.48 | 1.09 | 1.06 |
| Output power, $P_{\mathrm{out}}(\mathrm{W})$ | 1.00 | 1.00 | 1.00 |
| Experimental total efficiency <br> with adjustments, $\eta_{\mathrm{ac}}(\%)$ | 67.6 | 91.7 | 94.3 |

## 5 Conclusions

In this paper, a feedback analysis of an inductive power link driven by a Class-E amplifier with a variable coupling coefficient is presented. Analytical design equations are derived at any duty cycle using an impedance-analysis method. Feedback analyses are evolved from the design equations. In feedback control, both the duty cycle and frequency of the driving signal are adjusted to keep the power link tuning, and the supply voltage is controlled to keep the output power constant when the coupling coefficient is changed. The phase difference between the output current of the Class-E amplifier and the driving signal is used as a feedback quantity. The relations between the adjusted parameters and the coupling coefficient, and the feedback quantity are derived in a design example with a coupling coefficient of 0.2 and a duty cycle of 0.5 . An experiment was conducted to validate the control functions. The results showed that the inductive power link operated in the optimum state with constant output voltage by adjustment of the duty cycle, the frequency of the driving signal, and the supply voltage when the coupling coefficients changed from 0.2 to 0.1 to 0.25 .

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