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A novel differential multiuser detection algorithm for multiuser MIMO-OFDM systems*

Zheng-min KONG[†1,2], Guang-xi ZHU[1,2], Qiao-ling TONG[3], Yan-chun LI[1,2]

([1]*Department of Electronics and Information Engineering, Huazhong University of Science and Technology, Wuhan 430074, China*)

([2]*Wuhan National Lab for Optoelectronics, Huazhong University of Science and Technology, Wuhan 430074, China*)

([3]*Department of Electronic Science and Technology, Huazhong University of Science and Technology, Wuhan 430074, China*)

[†]E-mail: kongzhengmin5@163.com

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Abstract: We propose an efficient low bit error rate (BER) and low complexity multiple-input multiple-output (MIMO) multiuser detection (MUD) method for use with multiuser MIMO orthogonal frequency division multiplexing (OFDM) systems. It is a hybrid method combining a multiuser-interference-cancellation-based decision feedback equalizer using error feedback filter (MIMO MIC DFE-EFF) and a differential algorithm. The proposed method, termed 'MIMO MIC DFE-EFF with a differential algorithm' for short, has a multiuser feedback structure. We describe the schemes of MIMO MIC DFE-EFF and MIMO MIC DFE-EFF with a differential algorithm, and compare their minimum mean square error (MMSE) performance and computational complexity. Simulation results show that a significant performance gain can be achieved by employing the MIMO MIC DFE-EFF detection algorithm in the context of a multiuser MIMO-OFDM system over frequency selective Rayleigh channel. MIMO MIC DFE-EFF with the differential algorithm improves both computational efficiency and BER performance in a multistage structure relative to conventional DFE-EFF, though there is a small reduction in system performance compared with MIMO MIC DFE-EFF without the differential algorithm.

Key words: Multiuser detection (MUD), Orthogonal frequency division multiplexing (OFDM), Multiple-input multiple-output (MIMO), Decision feedback equalizer (DFE), Error feedback filter (EFF), Differential algorithm
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1 Introduction

Over the past decade, orthogonal frequency division multiplexing (OFDM) and multiple-input multiple-output (MIMO) techniques (Hanzo *et al.*, 2003; 2004; de Flaviis *et al.*, 2008) have attracted strong interest in both academic and industrial sectors. These techniques have played an important role in the recent development of next-generation wireless systems because of their potential for high capacity, interference suppression, and increased diversity. Applications include IEEE 802.11 wireless local area networks (WLANs), ultra wideband (UWB) (Chung *et al.*, 2005) and 3GPP (3rd Generation Partnership Project) long-term evolution (LTE), and worldwide interoperability for microwave access (Wimax) (Yaghoobi, 2004), space division multiple access (SDMA) (Hanzo *et al.*, 2003), and beamforming. Multiuser MIMO-OFDM systems allow a single base station (BS) to communicate with many users simultaneously. As a result, the study of multiuser MIMO-OFDM systems has become a popular research area.

Interest in the study of multiuser MIMO-OFDM systems has increased, especially the development of multiuser detection (MUD) techniques in such sys-

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tems (Shen and Hu, 2005; Zhou *et al.*, 2005; Jiang *et al.*, 2006; Xu *et al.*, 2006; Jiang and Hanzo, 2007; Yoon *et al.*, 2007; Wang *et al.*, 2008), including zero-forcing (ZF) detection, minimum mean square error (MMSE) detection, hybrid detection, sphere detection, decision feedback equalizer (DFE) detection, and genetic algorithm detection. Multiuser detection techniques from code division multiple access (CDMA) systems can also be applied in the context of multiuser MIMO-OFDM systems. An optimal multiuser detector proposed by Verdu (1998) had high computational complexity and was too expensive to handle. Therefore, suboptimal detectors have become a focus of research. However, the bit error rate (BER) of the system is quite high when suboptimal detection techniques like MMSE, interference cancellation (IC), and DFE receivers are used (Abdulrahman *et al.*, 1992; 1994).

In this paper, we propose a novel MUD algorithm based on a decision feedback equalizer using an error feedback filter (DFE-EFF) (Kim *et al.*, 1996; Kong and Zhu, 2007), described in full as a multiuser-interference-cancellation-based DFE using an EFF for a multiuser MIMO-OFDM system (MIMO MIC DFE-EFF). The proposed detection algorithm is employed in a multiuser MIMO-OFDM system to improve the performance of the system with respect to BER. Firstly, based on the MIC scheme, the MIMO MIC DFE-EFF detection algorithm is able to cancel a part of the interference and compensate for interference by adding the feedback of output to the input of DFE-EFF. Secondly, using an EFF, the MIMO MIC DFE-EFF system can reduce the correlation of the error signal, which cannot be reduced by a feedforward filter (FFF) or a feedback filter (FBF). Also, the proposed detection algorithm can reduce the computational complexity of a MUD system by employing a differential algorithm.

2 MIMO-OFDM system model

The MIMO technique offers considerable increase in data throughput without the need for additional bandwidth or transmit power. The major advantages of the OFDM technique are its ability to cope with inter-symbol interference and frequency-selective fading by using cyclic prefix and subchannelization. MIMO-OFDM is now being widely used in modern wideband wireless communication systems (such as LET), and will be used in future wideband wireless communication systems. Fig. 1 shows a basic uplink multiuser MIMO-OFDM system model with our proposed MUD in the frequency domain. At the transmitter end, K users' data are forwarded to inverse fast Fourier transform (IFFT) modulators, and then the outputs of the modulators are sent by the transmit antennas. At the receiver end, signals received by M antennas in the base station pass through fast Fourier transform (FFT) OFDM demodulators, and then the outputs of the demodulators are forwarded to the proposed MUD for obtaining the information of each user. Here, we denote the complex vector $\boldsymbol{r}[j, i]$ as the received signal at the ith subcarrier of the jth OFDM symbol, which is combined by the independently faded signal of the K users and the additive white Gaussian noise (AWGN). It can be expressed as

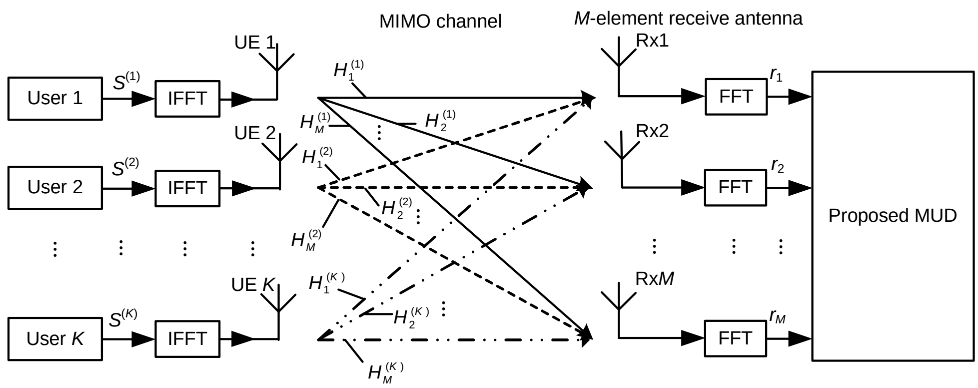

Fig. 1 Scheme of the uplink multiuser MIMO-OFDM system model (Hanzo and Keller, 2006) with the proposed multiuser detection (MUD)

$$\boldsymbol{r}=\boldsymbol{Hs}+\boldsymbol{n}, \tag{1}$$

where $\boldsymbol{r}$ is the received vector of size $M\times1$, $\boldsymbol{s}$ is the transmitted vector of size $K\times1$, and $\boldsymbol{n}$ is the noise signal vector of size $M\times1$. Here, we have dropped the indices $[j, i]$ for notational convenience. Specifically, $\boldsymbol{r}$, $\boldsymbol{s}$, and $\boldsymbol{n}$ are denoted by

$$\boldsymbol{r}=[r_1, r_2, \ldots, r_M]^{\mathrm{T}}, \tag{2}$$
$$\boldsymbol{s}=[s^{(1)}, s^{(2)}, \ldots, s^{(K)}]^{\mathrm{T}}, \tag{3}$$
$$\boldsymbol{n}=[n_1, n_2, \ldots, n_M]^{\mathrm{T}}. \tag{4}$$

$\boldsymbol{H}$ is a matrix of size $M\times K$, which contains the channel frequency response (CFR) of the K users, and is denoted by

$$\boldsymbol{H}=[\boldsymbol{H}^{(1)}, \boldsymbol{H}^{(2)}, \ldots, \boldsymbol{H}^{(K)}], \tag{5}$$

where $\boldsymbol{H}^{(k)}$ (k=1, 2, …, K) is the vector of the CFR for the kth user, expressed as

$$\boldsymbol{H}^{(k)}=[H_1^{(k)}, H_2^{(k)}, \ldots, H_M^{(k)}]^{\mathrm{T}}. \tag{6}$$

In the above expressions, we suppose that the transmitted signal $s^{(k)}$ for the kth user has a zero-mean and a variance of σ_1^2. The AWGN signal at the mth receive antenna, n_m, also has a zero-mean and a variance of σ_n^2. The CFR $H_m^{(k)}$ of different users or receive antennas are independent, have a complex normal distribution, and exhibit a zero-mean and unit variance (Jiang and Hanzo, 2007).

3 Multiuser-interference-cancellation-based decision feedback equalizer using error feedback filter for a multiuser MIMO-OFDM system

3.1 Minimum mean square error performance of MIMO MIC DFE-EFF

Austin (1967) first introduced the DFE and showed that its use can produce a gain in performance. Later, a DFE with error feedback filter (DFE-EFF) for reducing correlations in error signals or for reducing residual error variance was proposed by Kim *et al.* (1996) and improved by Saif (2003) and Kong *et al*. (2007). The scheme of the DFE-EFF is shown in Fig. 2. We denote coefficients of FFF, FBF, and EFF as $\boldsymbol{W}_{\mathrm{f}}$, $\boldsymbol{W}_{\mathrm{b}}$, and $\boldsymbol{W}_{\mathrm{e}}$, respectively. The expressions of these coefficients for K users are given by

$$\boldsymbol{W}_{\mathrm{f}}=\left[\boldsymbol{w}_{\mathrm{f}}^{(1)}, \boldsymbol{w}_{\mathrm{f}}^{(2)}, \ldots, \boldsymbol{w}_{\mathrm{f}}^{(K)}\right]^{\mathrm{T}}, \boldsymbol{w}_{\mathrm{f}}^{(k)}=\left[w_{\mathrm{f}1}^{(k)}, w_{\mathrm{f}2}^{(k)}, \ldots, w_{\mathrm{f}N_{\mathrm{b}}}^{(k)}\right], \tag{7}$$

$$\boldsymbol{W}_{\mathrm{b}}=\left[\boldsymbol{w}_{\mathrm{b}}^{(1)}, \boldsymbol{w}_{\mathrm{b}}^{(2)}, \ldots, \boldsymbol{w}_{\mathrm{b}}^{(K)}\right]^{\mathrm{T}}, \boldsymbol{w}_{\mathrm{b}}^{(k)}=\left[w_{\mathrm{b}1}^{(k)}, w_{\mathrm{b}2}^{(k)}, \ldots, w_{\mathrm{b}N_{\mathrm{b}}}^{(k)}\right], \tag{8}$$

$$\boldsymbol{W}_{\mathrm{e}}=\left[\boldsymbol{w}_{\mathrm{e}}^{(1)}, \boldsymbol{w}_{\mathrm{e}}^{(2)}, \ldots, \boldsymbol{w}_{\mathrm{e}}^{(K)}\right]^{\mathrm{T}}, \boldsymbol{w}_{\mathrm{e}}^{(k)}=\left[w_{\mathrm{e}1}^{(k)}, w_{\mathrm{e}2}^{(k)}, \ldots, w_{\mathrm{e}N_{\mathrm{e}}}^{(k)}\right]. \tag{9}$$

where N_{f}, N_{b}, and N_{e} are the numbers of FFF, FBF, and EFF taps, respectively. The superscript ‘(k)’ is employed to indicate the user k, k=1, 2, ..., K.

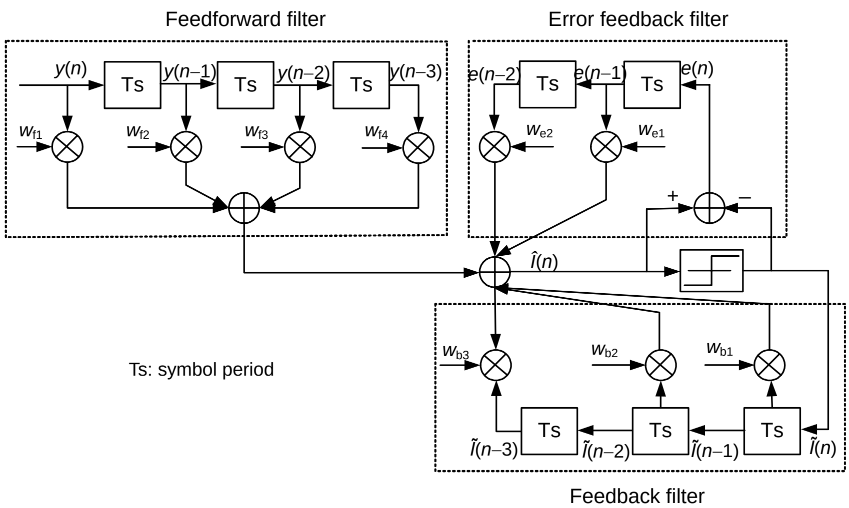

Fig. 2 Scheme of the decision feedback equalizer with error feedback filter (DFE-EFF) (Saif, 2003)

The inputs of FFF, FBF, and EFF are

$$\boldsymbol{Y}_n=\left[\boldsymbol{y}_n^{(1)},\boldsymbol{y}_n^{(2)},...,\boldsymbol{y}_n^{(K)}\right]^{\mathrm{T}},\ \boldsymbol{y}_n^{(k)}=\left[y_n^{(k)},y_{n-1}^{(k)},...,y_{n-N_{\mathrm{f}}+1}^{(k)}\right],\tag{10}$$

$$\tilde{\boldsymbol{I}}_n=\left[\tilde{\boldsymbol{I}}_n^{(1)},\tilde{\boldsymbol{I}}_n^{(2)},...,\tilde{\boldsymbol{I}}_n^{(K)}\right]^{\mathrm{T}},\ \tilde{\boldsymbol{I}}_n^{(k)}=\left[\tilde{I}_{n-1}^{(k)},\tilde{I}_{n-2}^{(k)},...,\tilde{I}_{n-N_{\mathrm{b}}}^{(k)}\right],\tag{11}$$

$$\boldsymbol{E}_n=\left[\boldsymbol{e}_n^{(1)},\boldsymbol{e}_n^{(2)},...,\boldsymbol{e}_n^{(K)}\right]^{\mathrm{T}},\ \boldsymbol{e}_n^{(k)}=\left[e_{n-1}^{(k)},e_{n-2}^{(k)},...,e_{n-N_{\mathrm{e}}}^{(k)}\right].\tag{12}$$

Here, we propose a novel adaptive DFE MUD for a multiuser MIMO-OFDM which has better performance than a conventional DFE or DFE-EFF. It is named a ‘multiuser-interference-cancellation-based decision feedback equalizer with error feedback filter for a multiuser MIMO-OFDM system’, and abbreviated to ‘MIMO MIC DFE-EFF’. As an adaptive DFE, MIMO MIC DFE-EFF is first employed in a multiuser MIMO-OFDM system. Compared with conventional DFE and DFE-EFF, MIMO MIC DFE-EFF not only has the merit of DFE-EFF which can cancel interference in the output of the equalizer, but also can deal with interference in the input of the equalizer. This scheme is shown in Fig. 3. The received signals which come from the effective base-band MIMO channel (Hanzo and Keller, 2006) are sent to multiple access mapping, carrying the information of each user. The information is then sent to the MIMO MIC DFE-EFF multiuser detector for further interference cancellation and for reducing the correlation of the error signal.

The output of the decision device is fed back to the input of DFE-EFF (Fig. 3). Thus, MIMO MIC DFE-EFF can cancel more multiple access interference (MAI) at the input of DFE-EFF. In this scheme, the input of the FFF is no longer represented by Eq. (10), but as

$$\begin{cases}\boldsymbol{Y}_n=\left[\boldsymbol{y}_n^{(1)}-\boldsymbol{q}_{n-1}^{(1)},\boldsymbol{y}_n^{(2)}-\boldsymbol{q}_{n-1}^{(2)},...,\boldsymbol{y}_n^{(K)}-\boldsymbol{q}_{n-1}^{(K)}\right]^{\mathrm{T}},\\ \boldsymbol{y}_n^{(k)}-\boldsymbol{q}_{n-1}^{(k)}=\left[y_n^{(k)}-q_{n-1}^{(k)},y_{n-1}^{(k)}-q_{n-2}^{(k)},...,y_{n-N_{\mathrm{f}}+1}^{(k)}-q_{n-N_{\mathrm{f}}}^{(k)}\right],\end{cases}\tag{13}$$

where $q_n^k=\sum_{i=1,\,i\neq k}^{K}\rho_{ik}\tilde{I}_{n-1}^{(i)}$ expresses the MAI coming from all the users except the desired user k. ρ_{ik} is the spatial signature (Hanzo and Keller, 2006) cross-correlation parameter between any two users i and k.

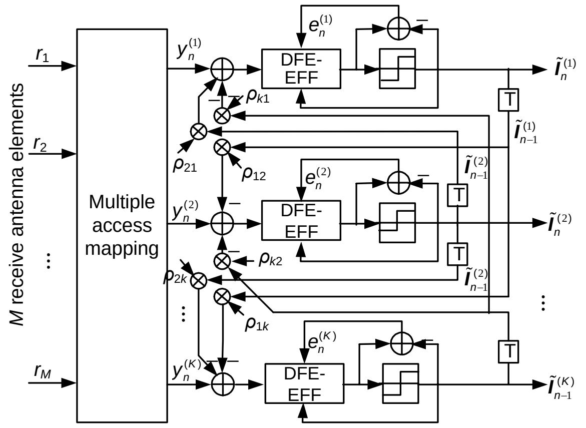

Fig. 3 Scheme of the MIMO MIC DFE-EFF multiuser detector

The output of the conventional DFE without EFF for K users before decision device $\hat{\boldsymbol{I}}_n$, is represented by

$$\hat{\boldsymbol{I}}_n=\mathrm{diag}\left\{[\boldsymbol{W}_{\mathrm{f}}\ \boldsymbol{W}_{\mathrm{b}}][\boldsymbol{Y}_n\ \tilde{\boldsymbol{I}}_n]^{\mathrm{T}}\right\}=\mathrm{diag}\left\{\boldsymbol{W}_{\mathrm{f}}\boldsymbol{Y}_n^{\mathrm{T}}+\boldsymbol{W}_{\mathrm{b}}\tilde{\boldsymbol{I}}_n^{\mathrm{T}}\right\},\tag{14}$$

where $\boldsymbol{Y}_n$ is given by Eq. (10).

For convenience, let

$$\boldsymbol{W}=\left[\boldsymbol{W}_{\mathrm{f}}\ \boldsymbol{W}_{\mathrm{b}}\right]^{\mathrm{T}},\tag{15}$$

$$\boldsymbol{X}=[\boldsymbol{Y}\ \tilde{\boldsymbol{I}}]^{\mathrm{T}}.\tag{16}$$

The error signal vector of all users between $\tilde{\boldsymbol{I}}_n$ and $\hat{\boldsymbol{I}}_n$, the desired signal vector and the DFE output vector respectively, can be defined as

$$\boldsymbol{e}_n=\tilde{\boldsymbol{I}}_n-\hat{\boldsymbol{I}}_n.\tag{17}$$

The mean square error for a single user is represented by

$$\boldsymbol{J}=E[\boldsymbol{e}_n^2]=E[(\tilde{\boldsymbol{I}}_n-\hat{\boldsymbol{I}}_n)^2]=E[\tilde{\boldsymbol{I}}_n^2]-2\boldsymbol{g}^{\mathrm{T}}\boldsymbol{W}+\boldsymbol{W}^{\mathrm{T}}\boldsymbol{R}_{\mathrm{a}}\boldsymbol{W},\tag{18}$$

where $\hat{\boldsymbol{I}}=\boldsymbol{W}^{\mathrm{T}}\boldsymbol{X}=\boldsymbol{X}\boldsymbol{W}^{\mathrm{T}}$, $\boldsymbol{g}$ is an $(N_{\mathrm{f}}+N_{\mathrm{b}})\times 1$ vector which contains the cross-correlation between the desired signal and the input signal, and $\boldsymbol{R}_{\mathrm{a}}$ is an $(N_{\mathrm{f}}+N_{\mathrm{b}})\times(N_{\mathrm{f}}+N_{\mathrm{b}})$ matrix which contains the auto-correlation of the input signal, that is,

$$\boldsymbol{g}=E[\tilde{\boldsymbol{I}}_n\boldsymbol{X}],\ \boldsymbol{R}_{\mathrm{a}}=E[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}].\tag{19}$$

The gradient of the mean square error performance function, assigned by ∇, can be obtained by differentiating Eq. (18) with respect to the weight coefficient $\boldsymbol{W}$:

$$\nabla = \frac{\partial J}{\partial \boldsymbol{W}} = 2\boldsymbol{R}_{\mathrm{a}}\boldsymbol{W} - 2\boldsymbol{g}. \tag{20}$$

The optimal weight coefficient $\boldsymbol{W}_{\mathrm{opt}}$ is then obtained by setting Eq. (20) to zero:

$$\boldsymbol{W}_{\mathrm{opt}} = \boldsymbol{R}_{\mathrm{a}}^{-1}\boldsymbol{g}. \tag{21}$$

Finally, MMSE is obtained by substituting Eq. (21) into Eq. (18) as follows:

$$J_{\min} = E[\tilde{\boldsymbol{I}}_n^2] - \boldsymbol{g}^{\mathrm{T}}\boldsymbol{R}_{\mathrm{a}}^{-1}\boldsymbol{g}. \tag{22}$$

Similarly, the output vector of our proposed MIMO MIC DFE-EFF for all users before the decision device $\hat{\boldsymbol{I}}_n$, is represented by

$$\begin{aligned}\hat{\boldsymbol{I}}_n &= \operatorname{diag}\left\{[\boldsymbol{W}_{\mathrm{f}}\ \boldsymbol{W}_{\mathrm{b}}\ \boldsymbol{W}_{\mathrm{e}}][\boldsymbol{Y}_n\ \tilde{\boldsymbol{I}}_n\ \boldsymbol{E}_n]^{\mathrm{T}}\right\} \\ &= \operatorname{diag}\left\{\boldsymbol{W}_{\mathrm{f}}\boldsymbol{Y}_n^{\mathrm{T}} + \boldsymbol{W}_{\mathrm{b}}\tilde{\boldsymbol{I}}_n^{\mathrm{T}} + \boldsymbol{W}_{\mathrm{e}}\boldsymbol{E}_n^{\mathrm{T}}\right\},\end{aligned} \tag{23}$$

where $\boldsymbol{Y}_n$ is not given by Eq. (10), but by Eq.(13).

For convenience, let

$$\hat{\boldsymbol{W}} = [\boldsymbol{W}_{\mathrm{f}}\ \boldsymbol{W}_{\mathrm{b}}\ \boldsymbol{W}_{\mathrm{e}}]^{\mathrm{T}}, \tag{24}$$

$$\hat{\boldsymbol{X}} = [\boldsymbol{Y}\ \tilde{\boldsymbol{I}}\ \boldsymbol{E}]^{\mathrm{T}}. \tag{25}$$

In this case, $\hat{\boldsymbol{I}} = \hat{\boldsymbol{W}}^{\mathrm{T}}\hat{\boldsymbol{X}} = \hat{\boldsymbol{X}}^{\mathrm{T}}\hat{\boldsymbol{W}}$. Similar to Eq. (18), we can figure out the mean square error of MIMO MIC DFE-EFF for a single user, as

$$J = E[\tilde{\boldsymbol{I}}_n^2] - 2\hat{\boldsymbol{g}}^{\mathrm{T}}\hat{\boldsymbol{W}} + \hat{\boldsymbol{W}}^{\mathrm{T}}\hat{\boldsymbol{R}}_{\mathrm{a}}\hat{\boldsymbol{W}}, \tag{26}$$

where $\hat{\boldsymbol{g}}$ is an $(N_{\mathrm{f}}+N_{\mathrm{b}}+N_{\mathrm{e}})\times 1$ vector which contains cross-correlation between the desired signal and the input signal, and $\hat{\boldsymbol{R}}_a$ is an $(N_{\mathrm{f}}+N_{\mathrm{b}}+N_{\mathrm{e}})\times(N_{\mathrm{f}}+N_{\mathrm{b}}+N_{\mathrm{e}})$ matrix which contains the auto-correlation of the input signal, that is,

$$\hat{\boldsymbol{g}} = E[\tilde{\boldsymbol{I}}_n\hat{\boldsymbol{X}}] = E\begin{bmatrix}\tilde{\boldsymbol{I}}_n\boldsymbol{X} \\ \tilde{\boldsymbol{I}}_n\boldsymbol{e}_{n-1}\end{bmatrix},\ \hat{\boldsymbol{R}}_{\mathrm{a}} = E[\hat{\boldsymbol{X}}\hat{\boldsymbol{X}}^{\mathrm{T}}] = \begin{bmatrix}\boldsymbol{R}_{\mathrm{a}} & \boldsymbol{C} \\ \boldsymbol{C}^{\mathrm{T}} & \sigma_{\boldsymbol{e}}^2\end{bmatrix}, \tag{27}$$

where $\boldsymbol{C} = E[\boldsymbol{e}_{n-1}\boldsymbol{X}]$ and $\sigma_{\boldsymbol{e}}^2 = E[\boldsymbol{e}_{n-1}^2]$.

As in Eq. (21), the optimal weight coefficient $\hat{\boldsymbol{W}}_{\mathrm{opt}}$ can be found by

$$\hat{\boldsymbol{W}}_{\mathrm{opt}} = \hat{\boldsymbol{R}}_{\mathrm{a}}^{-1}\hat{\boldsymbol{g}}. \tag{28}$$

Then, the MMSE is obtained by

$$J_{\min} = E[\tilde{\boldsymbol{I}}_n^2] - \hat{\boldsymbol{g}}^{\mathrm{T}}\hat{\boldsymbol{R}}_{\mathrm{a}}^{-1}\hat{\boldsymbol{g}}. \tag{29}$$

Using the block matrix inverse method, we can obtain

$$\begin{cases}\hat{\boldsymbol{R}}_{\mathrm{a}}^{-1} = \begin{bmatrix}\boldsymbol{R}_{\mathrm{a}}^{-1} + \boldsymbol{A}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{R}_{\mathrm{a}}^{-1} & -\boldsymbol{R}_{\mathrm{a}}^{-1}\boldsymbol{C}(\sigma_{\boldsymbol{e}}^2\boldsymbol{B})^{-1} \\ -(\sigma_{\boldsymbol{e}}^2\boldsymbol{B})^{-1}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{R}_{\mathrm{a}}^{-1} & (\sigma_{\boldsymbol{e}}^2 - \boldsymbol{B})^{-1}\end{bmatrix}, \\ \boldsymbol{A} = \boldsymbol{R}_{\mathrm{a}}^{-1}\boldsymbol{C}(\sigma_{\boldsymbol{e}}^2 - \boldsymbol{C}^{\mathrm{T}}\boldsymbol{R}_{\mathrm{a}}^{-1}\boldsymbol{C})^{-1},\ \boldsymbol{B} = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{R}_{\mathrm{a}}^{-1}\boldsymbol{C}.\end{cases} \tag{30}$$

Thus, the MMSE of our proposed MIMO MIC DFE-EFF is obtained by substituting Eqs. (27) and (30) into Eq. (29). Because $\boldsymbol{X}$ and $\boldsymbol{e}_n$ are orthogonal, and $\boldsymbol{C}$ decreases to a zero vector, the MMSE of MIMO MIC DFE-EFF can be rewritten as

$$J_{\min} = E[\tilde{\boldsymbol{I}}_n^2] - \boldsymbol{g}^{\mathrm{T}}\boldsymbol{R}_{\mathrm{a}}^{-1}\boldsymbol{g} - (\sigma_{\boldsymbol{e}}^2)^{-1}\left\{E[\boldsymbol{e}_n\boldsymbol{e}_{n-1}]\right\}^2. \tag{31}$$

Notice that the first two terms of Eq. (31) correspond to the MMSE of a conventional DFE described in Eq. (22). Since the last term of Eq. (31) is always negative, the MMSE of our proposed MIMO MIC DFE-EFF is always smaller than that of a DFE by $(\sigma_{\boldsymbol{e}}^2)^{-1}\left\{E[\boldsymbol{e}_n\boldsymbol{e}_{n-1}]\right\}^2$.

From the investigation above, the optimal weight coefficient $\hat{\boldsymbol{W}}_{\mathrm{opt}}$ of MIMO MIC DFE-EFF is as shown in Eq. (28). But it is only a theoretical value, and is hardly obtained in practice. Thus, we assign a realizable optimum value. We extend the least-mean-square (LMS) algorithm to calculate the filter weight coefficients updating formula for time-varying channel, as

$$\boldsymbol{W}(n) = \boldsymbol{W}(n-1) + \mu\boldsymbol{e}(n-1)\boldsymbol{X}(n-1), \tag{32}$$

where μ is a step size, which can be chosen to ensure that the mean square error of our proposed MIMO MIC DFE-EFF algorithm remains bounded. A sufficient condition to guarantee a bounded mean square error is $0<\mu<1/\lambda_{\max}$, where $\lambda_{\max}$ is a maximum eigenvalue of the auto-correlation matrix $\hat{\boldsymbol R}_{\mathrm a}$.

3.2 Computational complexity reduction for a multistage MIMO MIC DFE-EFF system

According to the above numerical analysis, theoretically, MIMO MIC DFE-EFF outperforms DFE and DFE-EFF in BER performance. But MIMO MIC DFE-EFF has the added cost of increased computational complexity. The computational complexity of the MIMO MIC DFE-EFF algorithm adds almost $KN_{\mathrm f}+K(K-1)$ multiplications and $KN_{\mathrm f}+K(K-1)$ additions per bit and per stage compared with DFE-EFF (Figs. 2 and 3).

We use differential theory analysis as proposed by Nahler *et al.* (2002) and Xu *et al.* (2002) and improved by our group for the beyond 3G system (Kong *et al.*, 2007; 2008), and propose a new approach for solving the computational complexity problem of multistage MIMO MIC DFE-EFF. We exploit the differential structure in the multistage detection of MIMO MIC DFE-EFF, avoiding unnecessary repeated calculations of certain terms in consecutive stages. In this structure, each stage detector employs a MIMO MIC DFE-EFF detection algorithm as described above. The input of each stage in this structure is a differential vector, which is generated by subtracting the input decision from the previous decision (Fig. 4). Let us analyze the principle of a differential algorithm. In the multistage detection system, the output of the lth iteration is

$$\begin{cases}\hat{\boldsymbol I}^{(l)}=\hat{\boldsymbol W}^{\mathrm T}\hat{\boldsymbol X}^{(l-1)},\ \tilde{\boldsymbol I}^{(0)}=\mathrm{sign}(y),\ \tilde{\boldsymbol I}^{(l)}=\mathrm{sign}(\hat{\boldsymbol I}^{(l)}),\\ \hat{\boldsymbol X}^{(0)}=[\mathrm{sign}(y)\ \tilde{\boldsymbol I}^{(1)}\ \boldsymbol E],\ \hat{\boldsymbol X}^{(l-1)}=[\mathrm{sign}(\hat{\boldsymbol I}^{(l-1)})\ \tilde{\boldsymbol I}^{(l)}\ \boldsymbol E].\end{cases} \tag{33}$$

After l iterations, we are more likely to find $\tilde{\boldsymbol I}^{(l)}=\tilde{\boldsymbol I}^{(l-1)}$, reflecting the convergence of the iterative method. We also find that, instead of dealing with each vector $\tilde{\boldsymbol I}^{(l)}$, as in Eq. (33), we can deal with the difference of the estimated bits vector in two consecutive stages. In other words, the input of each stage becomes $\tilde{\boldsymbol m}^{(l)}=\tilde{\boldsymbol I}^{(l)}-\tilde{\boldsymbol I}^{(l-1)}$, the differential vector. Thus, Eq. (33) can be rewritten as

$$\hat{\boldsymbol I}^{(l)}=\hat{\boldsymbol W}^{\mathrm T}\hat{\boldsymbol X}^{(l-1)}=\hat{\boldsymbol I}^{(l-1)}+\hat{\boldsymbol W}^{\mathrm T}[\tilde{\boldsymbol m}^{(l-1)}\ \tilde{\boldsymbol m}^{(l)}\ \boldsymbol E], \tag{34}$$

$$\begin{cases}\tilde{\boldsymbol I}^{(l)}=\mathrm{sign}(\hat{\boldsymbol I}^{(l)}),\\ \hat{\boldsymbol X}^{(l-1)}=[\mathrm{sign}(\hat{\boldsymbol I}^{(l-1)})\ \tilde{\boldsymbol I}^{(l)}\ \boldsymbol E],\\ \tilde{\boldsymbol m}^{(l)}=\tilde{\boldsymbol I}^{(l)}-\tilde{\boldsymbol I}^{(l-1)}.\end{cases} \tag{35}$$

Employing this differential algorithm, many more computations can be saved by dealing with Eq. (34) instead of a set of formulas as in Eq. (33), since more terms in the vector $\tilde{\boldsymbol m}^{(l)}$ tend to be zero after several stages. Furthermore, all the non-zero elements in $\tilde{\boldsymbol m}^{(l)}$ are equal to +2 or −2. The constant multiplication by +2 or −2 can be carried out by arithmetic shifts. Thus, the complexity of the detection system as well as the power of detector implementation can be reduced by using this differential

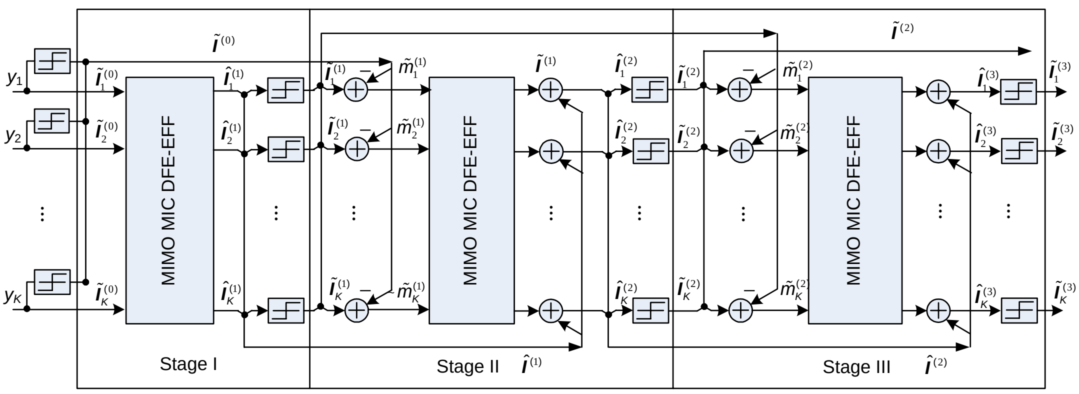

Fig. 4 Differential structure of a multistage MIC DFE-EFF detection system

algorithm detection. Ultimately, because our modification which subtracts two consecutive stages is a linear transformation, the BER after each stage remains almost unchanged, compared with a multistage MIMO MIC DFE-EFF detector without a differential algorithm.

Quantitative analysis can be used to compare the computational complexity of different algorithms. We use the float-point (flop) operation method. We define addition and multiplication operations (ops) as add and mul, respectively. The complex number operations are defined as C_{add}=2 add=2 ops, C_{mul}=4 mul+2 add=6 ops, since the signal may be a complex number. The complexity O, measured as the number of operations, is calculated for the DFE, DFE-EFF, MIMO DFE-EFF, and MIMO DFE-EFF with the differential algorithm with one EFF tap in a multistage structure, as follows:

$$\begin{aligned} O_{\text{DFE}} &= SPNMK\left[C_{\text{mul}}(N_{\text{f}}+1)+C_{\text{add}}N_{\text{f}}+C_{\text{mul}}N_{\text{b}}\right.\\ &\quad \left.+C_{\text{add}}(N_{\text{b}}-1)+C_{\text{add}}\right]\\ &= SPNMK(8N_{\text{f}}+8N_{\text{b}}+6)\text{ ops}, \end{aligned} \tag{36}$$

$$\begin{aligned} O_{\text{DFE-EFF}} &= SPNMK\left[C_{\text{mul}}(N_{\text{f}}+1)+C_{\text{add}}N_{\text{f}}+C_{\text{mul}}N_{\text{b}}\right.\\ &\quad \left.+C_{\text{add}}(N_{\text{b}}-1)+C_{\text{mul}}+C_{\text{add}}+2C_{\text{add}}\right]\\ &= SPNMK(8N_{\text{f}}+8N_{\text{b}}+16)\text{ ops}, \end{aligned} \tag{37}$$

$$\begin{aligned} &O_{\text{MIMO DFE-EFF}}\\ &= SPNMK\left\{2C_{\text{add}}+(C_{\text{mul}}+C_{\text{add}})[(N_{\text{f}}+1)\right.\\ &\quad +(K-1)]+C_{\text{mul}}(N_{\text{f}}+1)+C_{\text{add}}N_{\text{f}}\\ &\quad \left.+C_{\text{mul}}N_{\text{b}}+C_{\text{add}}(N_{\text{b}}-1)+C_{\text{mul}}+C_{\text{add}}\right\}\\ &= SPNMK(16N_{\text{f}}+8N_{\text{b}}+8K+16)\text{ ops}, \end{aligned} \tag{38}$$

$$\begin{aligned} &O_{\text{MIMO DFE-EFF with differential algorithm}}\\ &= \sum_{l=1}^{S}(1-\beta_l)PNMK\left\{2C_{\text{add}}+(C_{\text{mul}}+C_{\text{add}})\right.\\ &\quad \cdot[(N_{\text{f}}+1)+(K-1)]+C_{\text{mul}}(N_{\text{f}}+1)+C_{\text{add}}N_{\text{f}}\\ &\quad \left.+C_{\text{mul}}N_{\text{b}}+C_{\text{add}}(N_{\text{b}}-1)+C_{\text{add}}+C_{\text{mul}}\right\}\\ &= PNMK\sum_{l=1}^{S}(1-\beta_l)\left(16N_{\text{f}}+8N_{\text{b}}+8K+16\right)\text{ ops}, \end{aligned} \tag{39}$$

where S, P, N, M, K, N_{f}, and N_{b} are the numbers of stages, multipaths, transmit antennas, receive antennas, users, FFF taps, and FBF taps respectively, and β_l (β_1=0; otherwise, $0<\beta_l\leq 1$) is the ratio of non-changed estimated data to all estimated data in two consecutive stages (i.e., zero-ratio of $\tilde{m}^{(l)}$). Clearly, the complexity of MIMO DFE-EFF with the differential algorithm is smaller than that of MIMO DFE-EFF without the differential algorithm.

4 Simulation

Simulations were performed for a multiuser MIMO-OFDM communication system in which the transmitter has four transmit antennas and the receiver has four receive antennas. We initialized the system with a particular bit allocation and a particular energy allocation by using Chow's algorithm, and by using Campello's algorithm for further efficiency improvement. Linear precoding was used for dealing with inter-user interference (Spencer *et al.*, 2004). To investigate the performance of our proposed MIMO MIC DFE-EFF detector in a multiuser MIMO-OFDM system, we compared our detector with the conventional DFE and DFE-EFF using the system parameters summarized in Table 1. Note that this multiuser MIMO-OFDM system may be viewed as an orthogonal frequency division multiple access (OFDMA) or orthogonal frequency and code division multiplexing (OFCDM or MC-CDMA) scenario (Hu *et al.*, 2005; Zhou *et al.*, 2005).

Fig. 5 compares the BER performance of DFE, DFE-EFF, and MIMO MIC DFE-EFF in the MIMO OFDMA system over frequency selective Rayleigh channel (Kondo and Milstein, 1996; Doukas and Kalivas, 2005). The MIMO MIC DFE-EFF system has the best BER performance among the detection algorithms (Fig. 5). At a BER of 2×10^{-5}, MIMO MIC DFE-EFF has a performance more than 1 dB BER better than DFE-EFF. When the SNR is 20 dB, the BER of the proposed MIMO MIC DFE-EFF is nearly 10^{-6}.

We compared the BER performance of DFE, DFE-EFF, and MIMO MIC DFE-EFF in MIMO OFCDM and MIMO MC-CMDA systems over frequency selective Rayleigh channel with different spread factors. The MIMO MIC DFE-EFF system exhibits a relatively low BER at a high system load and outperforms DFE and DFE-EFF (Fig. 6a). At a BER of 4×10^{-5}, the performance of MIMO MIC DFE-EFF is more than 1 dB BER better than that of

DFE-EFF. The MIMO MIC DFE-EFF system has a much lower BER with a spread factor of N_S=8, nearly 10^{-7} BER when the SNR is 18 dB (Fig. 6b).

Table 1 Summary of system parameters

Configuration	Parameter	Value
MIMO	Number of users, K	4
	Number of transmit antennas, N	4
	Number of receive antennas, M	4
OFDM	Number of subcarriers, N_c	64
	Cyclic prefix length, GI	16
	Modulation	16-QAM*
Taps of equalizer	Number of FFFs, N_f	4
	Number of FBFs, N_b	3
	Number of EFFs, N_e	1
MIMO frequency selective Rayleigh channel	Number of multipaths	3
	Power delay profile	$[1, e^{-1}, e^{-2}]$
	White noise variance, σ	0.01
Multiple access	OFDMA	
	OFCDM or MC-CMDA	Walsh code**
System load	Capacity	K/N_S

* Maximum constellation number is 16; ** Number of spread factors N_S=4 and 8. FFF: feedforward filter; FBF: feedback filter; EFF: error feedback filter. OFDMA: orthogonal frequency division multiple access; OFCDM or MC-CMDA: orthogonal frequency and code division multiplexing

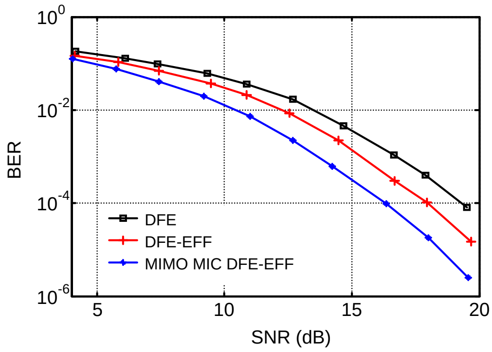

Fig. 5 BER performance of the proposed MIMO MIC DFE-EFF compared with conventional DFE, and DFE-EFF over MIMO frequency selective Rayleigh channel

Although MIMO MIC DFE-EFF has the best BER performance, the MIMO MIC DFE-EFF with and without the differential algorithm have almost the same BER performance (Fig. 7). Also, because of the linear transformation of subtraction in two consecutive stages, the BER after each stage of MIMO MIC DFE-EFF with the differential algorithm remains almost unchanged, compared with the BER after each stage of the multistage MIMO MIC DFE-EFF detector without the differential algorithm.

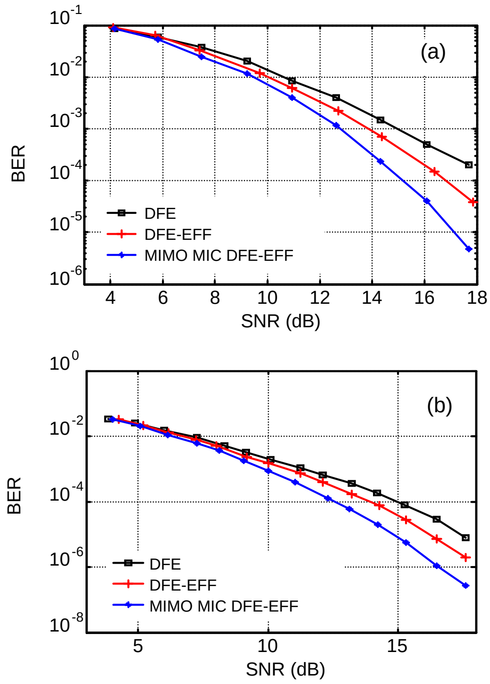

Fig. 6 BER performance of our proposed MIMO MIC DFE-EFF compared with conventional DFE and DFE-EFF over MIMO frequency selective Rayleigh channel, for four (a) and eight (b) spread factors

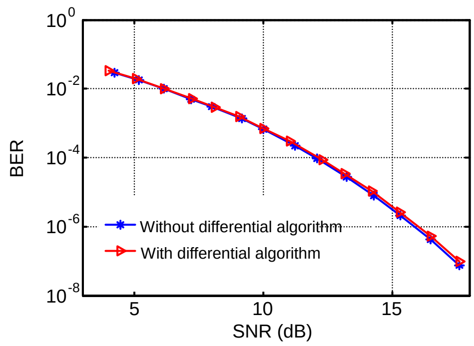

Fig. 7 BER performance of multistage MIMO MIC DFE-EFF with and without the differential algorithm, for eight spread factors and three iterations

Fig. 8 shows the BER performance as a function of the system load K/N_S for DFE, DFE-EFF, and MIMO MIC DFE-EFF with and without the differential algorithm. The SNR is fixed at 10 dB. With various system loads, the proposed two kinds of MIMO MIC DFE-EFF detection show much better performance than the other detectors. Though MIMO MIC DFE-EFF has the best performance, MIMO MIC DFE-EFF with and without the differential algorithm have almost the same performance. Compared to DFE-EFF, both MIMO MIC DFE-EFF detection algorithms show a gain in performance of 5×10^{-3} BER, even when the system load is close to 1.

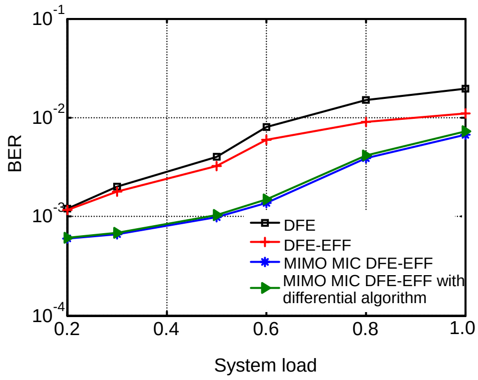

Fig. 8 System performance as a function of the system load (K/N_S) at an SNR of 10 dB

Fig. 9 shows the percentage of zeros and indicates a reduction in computational complexity when using the differential vector of MIMO MIC DFE-EFF with the differential algorithm. The iterations converge progressively and after the third stage, the proportion of zeros is close to 96%. This result suggests that if we use the conventional multistage MIMO MIC DFE-EFF detector, almost 96% of the computations are wasted after the third stage. In other words, we can employ more stages for detection using the differential algorithm to obtain better BER performance.

Fig. 10 compares the computational efficiency of DFE, DFE-EFF, MIMO MIC DFE-EFF, and MIMO MIC DFE-EFF with the differential algorithm. We used the float-point (flop) method to show clearly how much computation could possibly be saved in a real system. The computational numbers of MIMO MIC DFE-EFF increase exponentially. But after the third stage, the computational numbers of MIMO MIC DFE-EFF with the differential algorithm increase much more slowly, giving a nearly horizontal line.

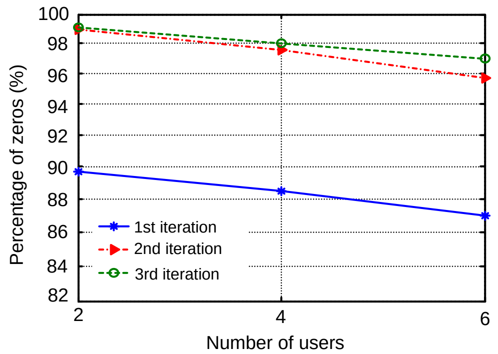

Fig. 9 Percentage of zeros in the differential vectors of MIMO MIC DFE-EFF with the differential algorithm at an SNR of 5 dB

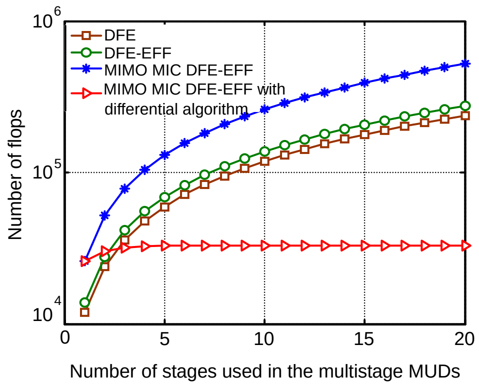

Fig. 10 Computational efficiency of MIMO MIC DFE-EFF with and without the differential algorithm, for four users

5 Conclusions

In this paper, we proposed a novel multiuser detection algorithm, termed the 'multiuser-interference-cancellation-based decision feedback equalizer using error feedback filter' for a multiuser MIMO-OFDM system (MIMO MIC DFE-EFF). In a multiuser MIMO-ODFM system, we analyzed the MMSE performance, calculated the weight coefficients, and presented a new multistage structure MIMO MIC DFE-EFF with the differential algorithm to reduce computational complexity. Simulation results showed that the MIMO MIC DFE-EFF with and without the differential algorithm can achieve significant performance gains under both low and high system loads. The MIMO MIC DFE-EFF has almost the same BER performance either with or without the differential algorithm. Compared to MIMO MIC

DFE-EFF without the differential algorithm, MIMO MIC DFE-EFF with the differential algorithm shows a low computation complexity in multistage detection. Thus, with a small tradeoff in BER performance, a great deal of computation cost can be saved. However, there are still several issues that need to be investigated further, such as the optimization of coefficients.

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