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SCKF-STF-CN: a universal nonlinear filter for maneuver target tracking*

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Abstract: Square-root cubature Kalman filter (SCKF) is more effective for nonlinear state estimation than an unscented Kalman filter. In this paper, we study the design of nonlinear filters based on SCKF for the system with one step noise correlation and abrupt state change. First, we give the SCKF that deals with the one step correlation between process and measurement noises, SCKF-CN in short. Second, we introduce the idea of a strong tracking filter to construct the adaptive square-root factor of the prediction error covariance with a fading factor, which makes SCKF-CN obtain outstanding tracking performance to the system with target maneuver or abrupt state change. Accordingly, the tracking performance of SCKF is greatly improved. A universal nonlinear estimator is proposed, which can not only deal with the conventional nonlinear filter problem with high dimensionality and correlated noises, but also achieve an excellent strong tracking performance towards the abrupt change of target state. Three simulation examples with a bearings-only tracking system are illustrated to verify the efficiency of the proposed algorithms.

Key words: Nonlinear system, Maneuver target tracking, Correlated noises, Square-root cubature Kalman filter (SCKF), Strong tracking filtering (STF)

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1 Introduction

Kalman filter presented by Kalman (1960) is a strong tool dealing with the state estimation of the linear dynamic system, and it leads to the establishment of modern filtering theory and is improving rapidly during the past half century. At the same time, as modern control systems become more and more complicated, a large number of nonlinear problems are emerging. Accordingly, the traditional linear filtering techniques are not able to meet the increasing demands from applications.

The classical method for nonlinear estimation is the extended Kalman filter (EKF) based on the linearization theory with Taylor's series expansion, but the application capability is significantly limited because of low accuracy, bad stability and convergence, and high running complexity induced by the computation of the Jacobian matrix. To overcome the shortcomings of EKF on estimation accuracy, stability, convergence, etc., some novel nonlinear filtering approaches were proposed (Zhou *et al.*, 1999; Arulampalam *et al.*, 2002; Julier and Uhlmann, 2004; Deng *et al.*, 2006; Arasaratnam and Haykin, 2008). The first is the strong tracking filtering (STF) established by Zhou *et al.* (1999) and Zhou and Ye (2000). Its kernel is to introduce a fading factor that relates to the newest measurement innovation in the prediction error covariance matrix. Accordingly, the gain matrix can be adjusted adaptively by the current measurement. STF achieves better estimation performance

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than EKF (Julier and Uhlmann, 1997; He *et al.*, 2006; Xu *et al.*, 2009). STF is very effective for the uncertainty of system models, abrupt state change, and target maneuver. Specifically, it can prevent the information loss induced by linearization. However, STF still needs linearization operation. After that, the unscented Kalman filter (UKF) proposed by Julier and Uhlmann (2004) was used. UKF uses the nonlinear model of the system instead of linearization and does not need to calculate the Jacobian matrix. It adopts a nonlinear unscented transformation and a set of sampled points to approximate the probability density of the nonlinear transformation function. Theoretical study shows that this method can achieve at least second-order accuracy. There have been many achievements using UKF (Wan and van der Merwe, 2000; Julier and Uhlmann, 2004; Pan *et al.*, 2005; Xiong *et al.*, 2006; Zhu and Zheng, 2008). However, its accuracy still cannot completely satisfy the requirements of practical applications. More importantly, the performance of UKF will obviously decrease when the system state dimensionality is relatively high, which is called 'the curse of dimensionality' (Daum, 2005). Simultaneously, the absence of positive definition may probably be more hazardous as it prevents UKF from running continuously. Particle filter (PF), a kind of Monte-Carlo method, uses the random sampled points to describe the distribution of random probability (Arulampalam *et al.*, 2002). It approximates the real probability distribution based on the measurement by adjusting the weights of sample points and sample locations. Compared with EKF and UKF, PF greatly improves the estimation precision of the nonlinear filter, but leads to tremendous computational complexity, which restricts its application in real-time systems.

Recently, Arasaratnam and Haykin (2009) proposed the square-root cubature Kalman filter (SCKF). It uses the spherical cubature rule and radial rule to optimize the sigma points and weights. Thus, the ability to deal with the high dimensional state is greatly enhanced for nonlinear estimation. The estimation precision and stability are also clearly improved. Meanwhile, the ***QR*** decomposition avoids the square-root operation of the prediction covariance matrix to ensure the continuity of the filtering process. By analysis, we can see that the existing SCKF does not consider the correlated case between process and measurement noises, and lacks the ability to track sudden state change. Especially in the field of target tracking, this situation often occurs. It is necessary to ensure that SCKF obtains the correlation of noises and has a strong tracking ability in sudden situations.

This paper studies the design of nonlinear filters for the nonlinear system with one step correlation between process and measurement noises. First, the SCKF with correlated noises, SCKF-CN in short, is derived. Next, by introducing the idea of strong tracking filtering to construct a fading square-rooting factor of the prediction error covariance, a universal nonlinear filter SCKF-STF-CN is proposed based on SCKF-CN and STF. The universal filter can not only be used for the conventional nonlinear state estimation but also has better ability to deal with the inaccuracy model and abrupt state change. We present the implementation processes of the filters, and verify their efficiencies by some examples with bearings-only tracking.

2 Problem formulation

2.1 System description

Consider the following discrete nonlinear system:

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{f}(\boldsymbol{x}_{k-1}) + \boldsymbol{w}_{k,k-1}, \\ \boldsymbol{z}_k = \boldsymbol{h}(\boldsymbol{x}_k) + \boldsymbol{v}_k, \end{cases} \tag{1}$$

where $\boldsymbol{x}_k \in \mathbb{R}^{n_x}$ is the state of the system, $\boldsymbol{f}$ and $\boldsymbol{h}$ are known functions, $\boldsymbol{z}_k \in \mathbb{R}^{n_z}$ is measurement to state $\boldsymbol{x}_k$, and $\{\boldsymbol{w}_{k,k-1}\}$ and $\{\boldsymbol{v}_k\}$ are both Gaussian white noise sequences with zero means, satisfying

$$E\left(\begin{pmatrix} \boldsymbol{w}_{k,k-1} \\ \boldsymbol{v}_k \end{pmatrix}\begin{pmatrix} \boldsymbol{w}_{l,l-1}^{\mathrm{T}} & \boldsymbol{v}_l^{\mathrm{T}} \end{pmatrix}\right) = \begin{bmatrix} \boldsymbol{Q}_{k,k-1} & \boldsymbol{D}_k \\ \boldsymbol{D}_k^{\mathrm{T}} & \boldsymbol{R}_k \end{bmatrix}\delta_{k,l}. \tag{2}$$

$\boldsymbol{Q}_{k,k-1}$ and $\boldsymbol{R}_k$ are the variances of process noise $\boldsymbol{w}_{k,k-1}$ and measurement noise $\boldsymbol{v}_k$, respectively, and $\boldsymbol{D}_k$ is the covariance of two noises. The initial state estimation and its covariance are $\boldsymbol{x}_0$ and $\boldsymbol{P}_0$, respectively.

2.2 Problem formulation

For the system (1), we establish a universal nonlinear filter based on SCKF, which should have the following two functions. The first is to estimate the system state with one step correlation between process and measurement noises. The second is to obtain a strong tracking ability towards abrupt state change for maneuver target tracking. By these means, we should derive the SCKF formulae with the noise correlation coefficient, and introduce a strong tracking fading factor to adjust adaptively the square-root factor of the prediction error covariance. Then, the universal nonlinear filter can be obtained.

3 Square-root cubature Kalman filter with correlated noises

Because of the errors introduced by finite word-length, two basic properties of an error covariance matrix, symmetry and positive definition, are often lost. It is important to preserve these two properties in each update cycle, however. The cubature Kalman filter (CKF) may suffer from the computational complexity and cannot run continuously. Thus, the $\boldsymbol{QR}$ decomposition is included in SCKF to compute the prediction error covariance matrices.

In CKF the covariance matrix is $\boldsymbol{P}=\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}$. We easily have $\boldsymbol{A}^{\mathrm{T}}=\boldsymbol{QR}$ in terms of the $\boldsymbol{QR}$ decomposition. $\boldsymbol{Q}$ is an orthogonal matrix and $\boldsymbol{R}$ is an upper triangular matrix. Then, the covariance $\boldsymbol{P}$ becomes

$$\boldsymbol{P}=\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}=\boldsymbol{R}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{R}=\boldsymbol{R}^{\mathrm{T}}\boldsymbol{R}=\boldsymbol{S}\boldsymbol{S}^{\mathrm{T}}, \tag{3}$$

where $\boldsymbol{S}=\boldsymbol{R}^{\mathrm{T}}$.

The $\boldsymbol{QR}$ decomposition can avoid the square-rooting operation of the covariance matrixes. The filtering continues even if the covariance matrix is not positive definite.

Based on the SCKF in Arasaratnam and Haykin (2009), we derive an SCKF with one step correlated noise, SCKF-CN in short.

Here, we introduce the notations involved in SCKF-CN formulation.

1. A general triangularization algorithm is denoted as $\boldsymbol{S}$=Tria($\boldsymbol{A}$), where $\boldsymbol{S}$ and $\boldsymbol{A}$ are as given in Eq. (3).

2. $\boldsymbol{W}_s=\mathrm{diag}\left\{\sqrt{\omega_1}, \sqrt{\omega_2}, ..., \sqrt{\omega_m}\right\}$, where $m=2n_x$, and $\omega_i=1/m$ (i=1, 2, ⋯, m) are the weights of cubature points.

3. $\boldsymbol{S}_{\boldsymbol{Q},\boldsymbol{R}}$ and $\boldsymbol{S}_{\boldsymbol{R},k}$ are square-roots of co-variance matrices $\boldsymbol{Q}_{k,k-1}$ and $\boldsymbol{R}_k$ of the process and measurement noises, respectively.

Theorem 1 (SCKF-CN algorithm) Similar to the Kalman filtering frame, the SCKF-CN algorithm is also composed of two steps, namely time and measurement updates.

1. Time update

1) Compute

$$\boldsymbol{S}_{k-1|k-1}=\mathrm{Tria}([\boldsymbol{X}_{k-1|k-2}-\boldsymbol{K}_{k-1}\boldsymbol{Z}_{k-1|k-2},\ \boldsymbol{K}_{k-1}\boldsymbol{S}_{R,k-1}]), \tag{4}$$

where $\boldsymbol{X}_{k-1|k-2}$, $\boldsymbol{Z}_{k-1|k-2}$, and $\boldsymbol{K}_{k-1}$ can be computed according to Eqs. (17), (15), and (18), respectively.

2) Evaluate the cubature points

$$\boldsymbol{X}_{i,k-1|k-1}=\boldsymbol{S}_{k-1|k-1}\xi_i+\hat{\boldsymbol{x}}_{k-1|k-1},\ i=1,2,\cdots,m, \tag{5}$$

where $m=2n_x$ and $\xi_i=\sqrt{m/2}\cdot[1]$. Here, $[1]\in\mathbb{R}^2$ belongs to the following set of points:

$$\left\{\begin{pmatrix}1\\0\end{pmatrix},\ \begin{pmatrix}0\\1\end{pmatrix},\ \begin{pmatrix}-1\\0\end{pmatrix},\ \begin{pmatrix}0\\-1\end{pmatrix}\right\}. \tag{6}$$

3) Evaluate the propagated cubature points

$$\boldsymbol{X}^*_{i,k|k-1}=\boldsymbol{f}(\boldsymbol{X}_{i,k-1|k-1}),\ i=1,2,\cdots,m, \tag{7}$$

and the predicted state

$$\hat{\boldsymbol{x}}_{k|k-1}=\frac{1}{m}\sum_{i=1}^{m}\boldsymbol{X}^*_{i,k|k-1}. \tag{8}$$

4) The square-root factor of prediction error covariance is

$$\boldsymbol{S}_{k|k-1}=\mathrm{Tria}([\boldsymbol{X}^*_{k|k-1},\ \ \boldsymbol{S}_{Q,k}]), \tag{9}$$

where

$$\begin{aligned}\boldsymbol{X}^*_{k|k-1}=\big[&\boldsymbol{X}^*_{1,k|k-1}-\hat{\boldsymbol{x}}_{k|k-1},\ \boldsymbol{X}^*_{2,k|k-1}-\hat{\boldsymbol{x}}_{k|k-1},\\ &\cdots,\ \boldsymbol{X}^*_{m,k|k-1}-\hat{\boldsymbol{x}}_{k|k-1}\big]\boldsymbol{W}_s.\end{aligned} \tag{10}$$

2. Measurement update

1) Evaluate the cubature points

$$\boldsymbol{X}_{i,k|k-1}=\boldsymbol{S}_{k|k-1}\xi_i+\hat{\boldsymbol{x}}_{k|k-1},\ i=1,2,\cdots,m, \tag{11}$$

and the propagated cubature points

$$Z_{i,k|k-1} = h(X_{i,k|k-1}), \quad i = 1, 2, \cdots, m. \tag{12}$$

2) Estimate the predicted measurement

$$\hat{z}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} Z_{i,k|k-1}. \tag{13}$$

3) Compute the square root of the innovation covariance

$$S_{zz,k|k-1} = \text{Tria}([Z_{k|k-1}, \ S_{R,k}]), \tag{14}$$

where

$$Z_{k|k-1} = \left[Z_{1,k|k-1} - \hat{z}_{k|k-1}, Z_{2,k|k-1} - \hat{z}_{k|k-1}, \cdots, \ Z_{m,k|k-1} - \hat{z}_{k|k-1}\right] W_s. \tag{15}$$

4) Estimate the cross-covariance matrix

$$P_{xz,k|k-1} = X_{k|k-1} Z_{k|k-1} + D_k, \tag{16}$$

where

$$X_{k|k-1} = \left[X_{1,k|k-1} - \hat{x}_{k|k-1}, X_{2,k|k-1} - \hat{x}_{k|k-1}, \cdots, \ X_{m,k|k-1} - \hat{x}_{k|k-1}\right] W_s. \tag{17}$$

5) Compute the Kalman gain

$$K_k = \frac{P_{xz,k|k-1} / S_{zz,k|k-1}^{\mathrm{T}}}{S_{zz,k|k-1}}. \tag{18}$$

6) The final updated state estimation is

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - \hat{z}_{k|k-1}), \tag{19}$$

and the estimation error covariance matrix is

$$P_{k|k} = \left[X_{k|k-1} - K_k Z_{k|k-1}, \ K_k S_{R,k}\right] \cdot \left[X_{k|k-1} - K_k Z_{k|k-1}, \ K_k S_{R,k}\right]^{\mathrm{T}}. \tag{20}$$

According to Wu and Hu (2005), we know that for the systems with additive process and measurement noises, the covariance is a weighted outer product of the transformed points plus the noise covariance. Considering one step noise correlation with $D_k = E(w_{k,k-1} v_k^{\mathrm{T}})$ in Eq. (2), we have

$$P_{xz,k|k-1} = X_{k|k-1} Z_{k|k-1} + E(w_{k|k-1} v_k^{\mathrm{T}}) = X_{k|k-1} Z_{k|k-1} + D_k.$$

Obviously, if the noises are not correlated, we have $D_k = \mathbf{0}$ and $P_{xz,k|k-1} = X_{k|k-1} Z_{k|k-1}$, as in Arasaratnam and Haykin (2009).

Clearly, the difference between SCKF and the proposed SCKF-CN is the D_k in Eq. (16). It means that the SCKF-CN algorithm can operate in the case of one step correlation between process and measurement noises. A way to obtain Eq. (16) with D_k was discussed in Li and Ge (2010) and we also use it in this work.

Like SCKF, SCKF-CN is only a nonlinear filtering algorithm for high-dimensional systems and cannot effectively cope with the abrupt state change. Next, we introduce the idea of strong tracking and propose a novel nonlinear SCKF-CN with strong tracking performance.

4 Square-root cubature Kalman filter and strong tracking filter with correlated noises

4.1 Strong tracking filter with correlated noises

Strong tracking filtering (STF) was proposed to deal with the uncertainty of system models, especially the abrupt change of target state. It is a wise choice for maneuver target tracking. To improve the tracking performance of the SCKF-CN towards target maneuver or abrupt state change, we introduce the 'strong tracking' concept to perfect the function of nonlinear filtering based on the SCKF-CN.

Because of the correlation between the process and measurement noises, consideration should be given to computing of the fading factor. In strong tracking filtering, the orthogonality amongst the measurement innovations at different times must be satisfied. Namely, a filter can have the strong tracking performance only if the filter selects a time-varying filtering gain matrix that satisfies the following conditions:

$$\begin{cases} E((x_{k+1} - \hat{x}_{k+1|k+1})(x_{k+1} - \hat{x}_{k+1|k+1})^{\mathrm{T}}) = \min, \\ E(\gamma_{k+1+j} \gamma_{k+1}^{\mathrm{T}}) = 0, \quad k = 0, 1, 2, \cdots, \end{cases} \tag{21}$$

where the residual error or measurement innovation is

$$\gamma_k = z_k - \hat{z}_{k|k-1}. \tag{22}$$

The second formula in Eq. (21) requires every residual error be orthogonal. In this case, the useful information in the residual error can be extracted to track the actual state. Then, the fading factor can be computed as follows.

Lemma 1 The time-varying sub-optimal regulator λ_k can be calculated as follows:

$$\lambda_k = \begin{cases} c_k, & c_k > 1, \\ 1, & c_k \le 1, \end{cases} \tag{23}$$

where

$$c_k = \mathrm{tr}\, \boldsymbol{N}_k / \mathrm{tr}\, \boldsymbol{M}_k , \tag{24}$$

$$\begin{cases} \boldsymbol{N}_k = \boldsymbol{V}_{0,k} - \beta \boldsymbol{R}_k - \boldsymbol{H}_{k,\hat{x}_{k|k-1}} \cdot (\boldsymbol{Q}_{k,k-1} - \boldsymbol{J}_{k-1}\boldsymbol{R}_{k-1}\boldsymbol{J}^{\mathrm{T}}_{k-1})\boldsymbol{H}^{\mathrm{T}}_{k,\hat{x}_{k|k-1}}, \\ \boldsymbol{M}_k = (\boldsymbol{F}_{k-1,\hat{x}_{k-1|k-1}} - \boldsymbol{J}_{k-1}\boldsymbol{H}_{k-1,\hat{x}_{k-1|k-1}})\boldsymbol{P}_{k-1|k-1} \\ \quad \cdot (\boldsymbol{F}^{\mathrm{T}}_{k-1,\hat{x}_{k-1|k-1}} - \boldsymbol{J}_{k-1}\boldsymbol{H}_{k-1,\hat{x}_{k-1|k-1}})\boldsymbol{H}^{\mathrm{T}}_{k,\hat{x}_{k|k-1}}\boldsymbol{H}_{k,\hat{x}_{k|k-1}}, \end{cases} \tag{25}$$

where

$$\boldsymbol{F}_{k-1,\hat{x}_{k-1|k-1}} = \left.\frac{\partial \boldsymbol{f}(\boldsymbol{x}_k)}{\partial \boldsymbol{x}_k}\right|_{\boldsymbol{x}_k = \hat{\boldsymbol{x}}_{k-1|k-1}}, \tag{26}$$

$$\boldsymbol{H}_{k,\hat{x}_{k|k-1}} = \left.\frac{\partial \boldsymbol{h}(\boldsymbol{x}_k)}{\partial \boldsymbol{x}_k}\right|_{\boldsymbol{x}_k = \hat{\boldsymbol{x}}_{k|k-1}}, \tag{27}$$

$$\boldsymbol{H}_{k-1,\hat{x}_{k-1|k-1}} = \left.\frac{\partial \boldsymbol{h}(\boldsymbol{x}_k)}{\partial \boldsymbol{x}_k}\right|_{\boldsymbol{x}_k = \hat{\boldsymbol{x}}_{k-1|k-1}}, \tag{28}$$

$$\boldsymbol{J}(k) = \boldsymbol{D}_k \boldsymbol{R}_k^{-1}, \tag{29}$$

$$\boldsymbol{V}_{0,k} = \begin{cases} \gamma_1\gamma_1^{\mathrm{T}}, & k=1, \\ (\rho \boldsymbol{V}_{0,k-1} + \gamma_k\gamma_k^{\mathrm{T}})/(1+\rho), & k>1. \end{cases} \tag{30}$$

According to the basic formulation of the STF theory in Zhou and Ye (2000), the forgetting factor ρ should be $0<\rho\le 1$ and is commonly chosen as 0.95. The softening factor β is equal to or greater than 1 for smoothing the state estimation. β can be set by experience. We set β=1.2 in this study.

4.2 SCKF-STF-CN

The essence for the strong tracking is to adaptively adjust the prediction error covariance $\boldsymbol{P}_{k|k-1}$ by use of the fading factor (Zhou *et al*., 1999; Zhou and Ye, 2000). There is a substantial difference for $\boldsymbol{P}_{k|k-1}$ between SCKF and methods such as UKF and EKF; i.e., SCKF depends on $\boldsymbol{S}_{k|k-1}$ and need not compute $\boldsymbol{P}_{k|k-1}$. Thus, we should adjust $\boldsymbol{P}_{k|k-1}$ to achieve the adaptive effect using a fading factor.

Theorem 2 (SCKF-STF-CN algorithm) The running formulae for the SCKF-STF-CN algorithm are similar to those for the SCKF-CN shown by Eqs. (4)–(21). The only difference is that Eq. (9) should be rewritten as

$$\boldsymbol{S}_{k|k-1} = \sqrt{\lambda_k}\,\mathrm{Tria}([\boldsymbol{X}^*_{k|k-1},\ \boldsymbol{S}_{Q,k}]). \tag{31}$$

Proof According to the strong tracking concept and Eq. (3), we have

$$\begin{aligned} \lambda_k \boldsymbol{P}_{k|k-1} &= \lambda_k \boldsymbol{A}\boldsymbol{A}^{\mathrm{T}} = \lambda_k \boldsymbol{R}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{R} \\ &= \lambda_k \boldsymbol{R}^{\mathrm{T}}\boldsymbol{R} = \sqrt{\lambda_k}\boldsymbol{R}^{\mathrm{T}}\sqrt{\lambda_k}\boldsymbol{R} = \boldsymbol{S}_{k|k-1}\boldsymbol{S}^{\mathrm{T}}_{k|k-1}, \end{aligned} \tag{32}$$

where

$$\boldsymbol{S}_{k|k-1} = \sqrt{\lambda_k}\boldsymbol{R}^{\mathrm{T}}. \tag{33}$$

Then, combining Eq. (9) for SCKF-STF-CN, one can obtain

$$\boldsymbol{S}_{k|k-1} = \sqrt{\lambda_k}\,\mathrm{Tria}([\boldsymbol{X}^*_{k|k-1},\ \boldsymbol{S}_{Q,k}]). \tag{34}$$

The proof is finished.

A more strict and exact solution to $\boldsymbol{S}_{k|k-1}$ is possibly $\mathrm{Tria}([\sqrt{\lambda_k}\boldsymbol{X}^*_{k|k-1}, \boldsymbol{S}_{Q,k}])$. However, there is not clear difference on the estimation accuracy between the two solutions. Then, the running sequence of the SCKF-STF-CN algorithm is Eqs. (4), (5), (7), (8), (23)–(30), (31), (11)–(14), (16), (18)–(20).

5 Brief analysis

By considering one step correlation between the process and measurement noises and introducing the time-varying sub-optimal fading factor of strong tracking filtering, the current SCKF in Arasaratnam and Haykin (2009) is improved for nonlinear state estimation. As a result, SCKF-STF-CN is proposed for the system with a kind of noise correlation and state maneuver. First, we give the formulae for SCKF-CN, especially Eq. (16) which represents one step noise correlation. Second, the square-rooting factor of the prediction error covariance with a fading factor is derived based on SCKF-CN and STF. Finally, the integrated running formulae for SCKF-STF-CN are presented.

Compared with the current nonlinear estimators such as SCKF and UKF-STF-CN given in Li and Ge (2010), the proposed SCKF-STF-CN estimator has the following advantages:

1. SCKF can estimate only the conventional nonlinear state for the system with uncorrelated noises, and does not have the ability to track the target maneuver. In contrast, the proposed nonlinear filter can easily deal with the one step correlated noises, and achieve the strong tracking ability in dealing with the target maneuver or abrupt state change, and uncertain system models. Therefore, SCKF-STF-CN is superior to SCKF on estimation accuracy, and has strong robustness on the mismatching of model parameters and low sensitivity to initial statistical properties. Its computational complexity is also moderate.

2. Although UKF-STF-CN is derived for the system with one step noise correlation and possible abrupt state change, the use of UKF leads to a weak estimation performance for high-dimensional nonlinear tracking systems.

Therefore, to estimate the state of a high-dimensional nonlinear system, by applying SCKF and STF and considering the one step correlation between process and measurement noises, the proposed SCKF-STF-CN achieves more extensive overall properties, better performance, and stronger tracking ability to deal with the target state maneuver.

6 Computer simulation

In this section, the bearings-only tracking is considered to validate the proposed SCKF-CN and SCKF-STF-CN. All results are the mean of 1000 Monte-Carlo simulations.

6.1 Bearings-only tracking system

The bearings-only tracking system (Fig. 1) is composed of two sensors to track a moving target state, where each sensor can obtain only an angle measurement to the state, denoted as $\alpha_1(k)$ and $\alpha_2(k)$, respectively. The two sensors A and B are fixed, and the distance between them is d (m).

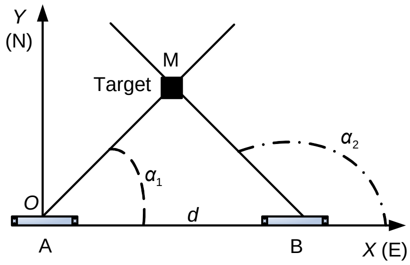

Fig. 1 Bearings-only tracking system

6.2 Tracking model

Considering the system described in Section 2, the state is denoted as $\boldsymbol{x}_k=[x_{1,k}, x_{2,k}, y_{1,k}, y_{2,k}]^{\mathrm{T}}$, where $x_{1,k}$ and $y_{1,k}$ (m) are the displacement components of east and north, respectively, and $x_{2,k}$ and $y_{2,k}$ are the respective velocity components. The target moves according to a constant velocity (CV) model, and the transfer matrix in Eq. (1) is

$$\boldsymbol{F}_{k,k-1}=\begin{bmatrix}1 & T & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & T\\ 0 & 0 & 0 & 1\end{bmatrix}, \tag{35}$$

where the tracking cycle T=1 s.

In terms of the principle of two-position cross-point, the measurement equation can be established as

$$\boldsymbol{h}(x_k)=\begin{bmatrix}\arccos\left(\dfrac{x_{1,k}}{\sqrt{x_{1,k}^2+y_{1,k}^2}}\right)\\ \arccos\left(\dfrac{x_{1,k}-d}{\sqrt{(x_{1,k}-d)^2+y_{1,k}^2}}\right)\end{bmatrix}, \tag{36}$$

and the measurement noise is a Gaussian sequence with zero mean.

6.3 Examples

Example 1 We compare the estimation precisions of SCKF-CN and SCKF for the nonlinear state system with one step correlated noises. Here, $\boldsymbol{Q}_{k,k-1}$= diag{100, 1, 100, 1}. The scheme to create the noise correlation is as follows: Let $\boldsymbol{v}_k=\boldsymbol{C}\boldsymbol{w}_{k,k-1}$, where $\boldsymbol{C}$ is a known coefficient matrix with an appropriate dimensionality. Then, we easily have

$$\begin{aligned}\boldsymbol{R}_k &= E(\boldsymbol{v}_k\boldsymbol{v}_k^{\mathrm{T}})=E(\boldsymbol{C}\boldsymbol{w}_{k,k-1}\boldsymbol{w}_{k,k-1}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}})\\ &=\boldsymbol{C}E(\boldsymbol{w}_{k,k-1}\boldsymbol{w}_{k,k-1}^{\mathrm{T}})\boldsymbol{C}^{\mathrm{T}}=\boldsymbol{C}\boldsymbol{Q}_{k,k-1}\boldsymbol{C}^{\mathrm{T}}.\end{aligned} \tag{37}$$

The initial state is $\boldsymbol{x}_0=[100, 4.62, 100, 9.62]^{\mathrm{T}}$, the covariance matrix is

$$\boldsymbol{P}_0=\begin{bmatrix}100 & 1 & 0 & 0\\ 1 & 1 & 0 & 0\\ 0 & 0 & 100 & 1\\ 0 & 0 & 1 & 1\end{bmatrix},$$

and the known coefficient matrix is

$$\boldsymbol{C}=\begin{bmatrix}0.000001 & 0.0001 & 0.0001 & 0.0001\\ 0.0001 & 0.000001 & 0.00001 & 0.0001\end{bmatrix}.$$

In addition, for the SCKF, $\boldsymbol{D}_k=\mathbf{0}$. The simulation results are shown in Fig. 2 and Table 1.

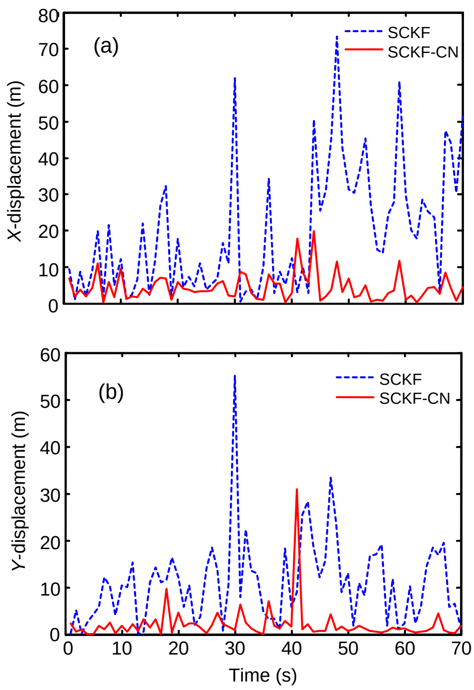

Fig. 2 Absolute estimation errors of *X*-displacement (a) and *Y*-displacement (b) in Example 1

Table 1 Absolute displacement estimation error

Displacement	Absolute error (m)		Improved percentage
	SCKF	SCKF-CN	
X	19.15	4.25	77%
Y	10.99	2.16	80%

We can easily see from the results that SCKF-CN has better estimation performance than SCKF without considering the noise correlation for the nonlinear system with one step correlated noises. Also, SCKF-CN is more robust than the conventional SCKF.

Example 2 We compare SCKF-CN with SCKF-STF-CN on the estimation performance for the nonlinear system, to show the strong tracking ability of SCKF-STF-CN towards abrupt state change. For the convenience of the comparison, we consider $\boldsymbol{D}_k=\mathbf{0}$, which means that the process and measurement Gaussian noises are independent. Thus, SCKF-CN and SCKF-STF-CN can be marked with SCKF and SCKF-STF in this example. We set two state mutations from 30 to 40 s, which make the abrupt state change from 30 to 40 s. The system parameters are

$$\boldsymbol{Q}_{k,k-1}=\begin{bmatrix}\boldsymbol{Q}_1 & \mathbf{0}\\ \mathbf{0} & \boldsymbol{Q}_1\end{bmatrix},\ \boldsymbol{Q}_1=\begin{bmatrix}T^3/3 & T^2/2\\ T^2/2 & T\end{bmatrix},$$

$$\boldsymbol{R}_k=\mathrm{diag}\{0.001,\ 0.001\}\times\pi/180,$$

$$\boldsymbol{x}_0=[100,\ 4.62,\ 100,\ 9.62]^{\mathrm{T}},$$

$$\boldsymbol{P}_0=\mathrm{diag}\{7,\ 0.01,\ 7,\ 0.01\}.$$

The simulation results are given in Fig. 3, Fig. 4, and Table 2.

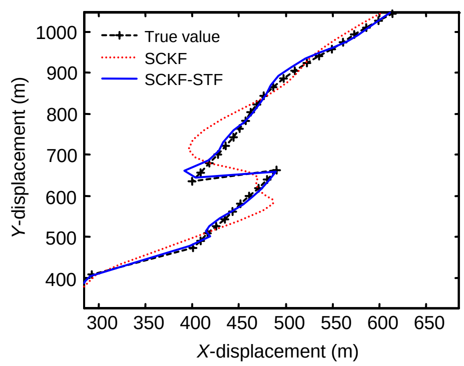

Fig. 3 Tracking results of two filters

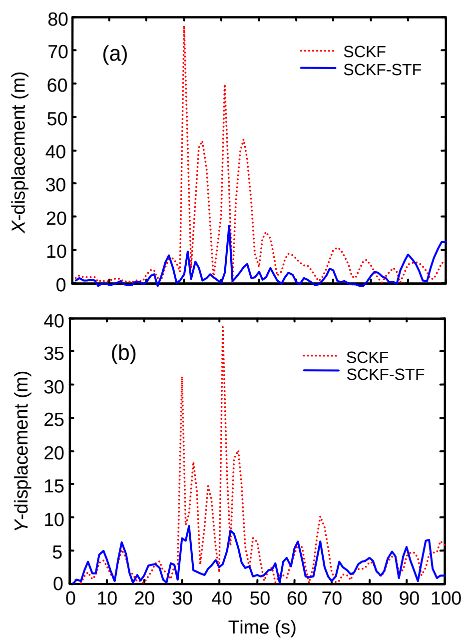

Fig. 4 Absolute estimation errors of *X*-displacement (a) and *Y*-displacement (b) in Example 2

Table 2 Absolute displacement estimation error

Displacement	Absolute error (m)		Improved percentage
	SCKF	SCKF-STF	
X	9.57	2.96	69%
Y	4.68	2.08	55%

The results show that SCKF-STF can achieve a better tracking effect besides the target state with abrupt change, and the tracking performance of SCKF is reduced because the abrupt state change will harm its estimation accuracy. This is because the fading factor in STF helps SCKF-STF to obtain the adaptive tracking ability towards the abrupt target state change. Fig. 4 also shows that SCKF-STF-CN has better robustness than SCKF-CN when there is target maneuver or abrupt state change.

Example 3 The simulation is performed to compare the estimation performance between the proposed SCKF-STF-CN and the UKF-STF-CN in Li *et al.* (2010), to show the superiority of SCKF-STF-CN on estimation performance. The setup for the abrupt state change refers to Example 2, and

$$\boldsymbol{Q}_{k,k-1}=\begin{bmatrix}\boldsymbol{Q}_1 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{Q}_1\end{bmatrix},\quad \boldsymbol{Q}_1=\begin{bmatrix}T^3/3 & T^2/2\\ T^2/2 & T\end{bmatrix},$$
$$\boldsymbol{v}_k=\boldsymbol{C}\boldsymbol{w}_{k,k-1},\quad \boldsymbol{R}_k=\boldsymbol{C}\boldsymbol{Q}_{k,k-1}\boldsymbol{C}^{\mathrm{T}},$$

initial state $\boldsymbol{x}_0=[100, 4.62, 100, 9.62]^{\mathrm{T}}$, its covariance $\boldsymbol{P}_0=\mathrm{diag}\left\{\begin{bmatrix}100 & 1\\ 1 & 1\end{bmatrix},\begin{bmatrix}100 & 1\\ 1 & 1\end{bmatrix}\right\}$, and

$$\boldsymbol{C}=\begin{bmatrix}0.000005 & 0.0005 & 0.0005 & 0.0005\\ 0.0005 & 0.000005 & 0.00005 & 0.0005\end{bmatrix}.$$

The results are shown in Fig. 5 and Table 3.

Table 3 Absolute displacement estimation error

Displacement	Absolute error (m)		Improved percentage
	UKF-STF-CN	SCKF-STF-CN	
X	17.73	12.05	32%
Y	16.54	7.75	53%

Clearly, SCKF-STF-CN has better tracking performance than UKF-STF-CN because of the advantage of SCKF, which uses the spherical cubature rule and radial rule to optimize the sigma points and weights. As a result, the capability to deal with a state with high dimensionality is enhanced. SCKF-STF-CN also has better stability because of the use of ***QR*** decomposition.

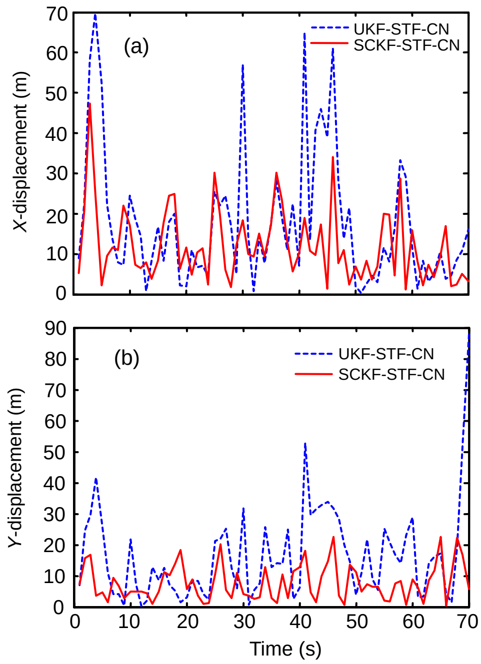

Fig. 5 Absolute estimation errors of *X*-displacement (a) and *Y*-displacement (b) in Example 3

7 Conclusions

We study nonlinear filtering for the system with one step noise correlation and abrupt state change based on the square-root cubature Kalman filter. The contribution includes two aspects. First, we present SCKF-CN that can operate with one step correlation between process and measurement noises. Second, by introducing the strong tracking concept into SCKF-CN, we establish a universal nonlinear filtering method that can not only deal with the correlated noise but also achieve outstanding tracking ability towards high-dimensional systems and abrupt change of target state. In the future, we will extend this algorithm to multisensor nonlinear systems with full noise correlation and abrupt state change for nonlinear data fusion.

References

Arasaratnam, I., Haykin, S., 2008. Square-root quadrature Kalman filtering. *IEEE Trans. Signal Process.*, **56**(6):

2589-2593. [doi:10.1109/TSP.2007.914964]

Arasaratnam, I., Haykin, S., 2009. Cubature Kalman filters. *IEEE Trans. Autom. Control*, **54**(6):1254-1269. [doi:10.1109/TAC.2009.2019800]

Arulampalam, M.S., Maskell, S., Gordon, N., Clapp, T., 2002. A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking. *IEEE Trans. Signal Process.*, **50**(2):174-188. [doi:10.1109/78.978374]

Daum, F., 2005. Nonlinear filters: beyond the Kalman filter. *IEEE Aerosp. Electron. Syst. Mag.*, **20**(8):57-69. [doi:10.1109/MAES.2005.1499276]

Deng, X.L., Xie, J.Y., Guo, W.Z., 2006. Adaptive particle filtration for state estimation. *J. South China Univ. Technol. (Nat. Sci. Ed.)*, **34**(1):57-61 (in Chinese).

He, Y., Song, Q., Dong, Y.L., Yang, J., 2006. Adaptive Tracking Algorithm Based on Modified Strong Tracking Filter. Int. Conf. on Radar, p.1-4.

Julier, S.J., Uhlmann, J.K., 1997. A new extension of the Kalman filter to nonlinear systems. *SPIE*, **3068**:182-193. [doi:10.1117/12.280797]

Julier, S.J., Uhlmann, J.K., 2004. Unscented filtering and nonlinear estimation. *Proc. IEEE*, **92**(3):401-422. [doi:10.1109/JPROC.2003.823141]

Kalman, R.E., 1960. A new approach to linear filtering and prediction problems. *Trans. ASME J. Basic Eng.*, **82**(1):35-45.

Li, W.B., Ge, Q.B., 2010. UKF-STF Tracking with Correlated Noises for the Nonlinear System. 8th World Congress on Intelligent Control and Automation, p.3466-3471.

Pan, Q., Yang, F., Liang, Y., 2005. Survey of a kind of nonlinear filters. *Control Dec. China*, **20**(5):481-494 (in Chinese).

Wan, E.A., van der Merwe, R., 2000. The Unscented Kalman Filter for Nonlinear Estimation. IEEE Adaptive Systems for Signal Processing, Communications, and Control Symp., p.153-158. [doi:10.1109/ASSPCC.2000.882463]

Wu, Y.X., Hu, D.W., 2005. Unscented Kalman filtering for additive noise case: augmented vs. non-augmented. *IEEE Signal Process. Lett.*, **12**(5):357-360. [doi:10.1109/LSP.2005.845592]

Xiong, K., Zhang, H.Y., Chan, C.W., 2006. Performance evaluation of UKF-based nonlinear filtering. *Automatica*, **42**(2):261-270. [doi:10.1016/j.automatica.2005.10.004]

Xu, W.M., Kuang, L.L., Lu, J.H., 2009. Positioning algorithm with joint space-time constraints for unmanned network-flying vehicles. *IEEE Trans. Geosci. Remote Sens.*, **47**(8):2694-2704. [doi:10.1109/TGRS.2009.2015655]

Zhou, D.H., Ye, Y.Z., 2000. Modern Fault Diagnosis and Fault-Tolerant Control. Tsinghua University Press, Beijing, China, p.60-116 (in Chinese).

Zhou, D.H., Xi, Y.G., Zhang, Z.J., 1999. A suboptimal multiple fading extended Kalman filter. *Acta Autom. Sin. China*, **17**(6):689-695.

Zhu, T.J., Zheng, H.G., 2008. Application of Unscented Kalman Filter to Vehicle State Estimation. ISECS Int. Colloquium on Computing, Communication, Control, and Management, p.135-139. [doi:10.1109/CCCM.2008.124]