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A multiple maneuvering targets tracking algorithm based on a generalized pseudo-Bayesian estimator of first order*#

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Abstract: We describe the design of a multiple maneuvering targets tracking algorithm under the framework of Gaussian mixture probability hypothesis density (PHD) filter. First, a variation of the generalized pseudo-Bayesian estimator of first order (VGPB1) is designed to adapt to the Gaussian mixture PHD filter for jump Markov system models (JMS-PHD). The probability of each kinematic model, which is used in the JMS-PHD filter, is updated with VGPB1. The weighted sum of state, associated covariance, and weights for Gaussian components are then calculated. Pruning and merging techniques are also adopted in this algorithm to increase efficiency. Performance of the proposed algorithm is compared with that of the JMS-PHD filter. Monte-Carlo simulation results demonstrate that the optimal subpattern assignment (OSPA) distances of the proposed algorithm are lower than those of the JMS-PHD filter for maneuvering targets tracking.

Key words: Gaussian mixture PHD filter, Jump Markov system, Generalized pseudo-Bayesian estimator of first order (GPB1), Multi-target tracking

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1 Introduction

Multi-target tracking is important and difficult in radar, sonar, and navigation systems. Not only does the number of targets need to be estimated from cluttered measurements, but also the states and trajectories of targets need to be provided with the tracking algorithm. Furthermore, the time-varying number of targets and the uncertainty of target maneuvers increase the complexity in practical applications. Therefore, more theoretical and experimental studies are ongoing to deal with this task.

Recently, a probability hypothesis density (PHD) filter has been presented by Mahler (2003; 2007) based on finite set statistics (FISST), and is an active method (Zhang *et al.*, 2009; Wu *et al.*, 2010; Pollard *et al.*, 2011; Vo *et al.*, 2012) for multi-target tracking. Gaussian mixture PHD filter is a closed-form solution for implementation of PHD recursion, which uses weighted Gaussian components to propagate an intensity of target using PHD recursion (Vo and Ma, 2006). The mean and covariance of each Gaussian component can be propagated analytically using the extended Kalman filter or unscented Kalman filter.

How to extend the PHD filter to the interacting multiple model (IMM) remains a challenging and interesting problem (Vo *et al.*, 2006). A Gaussian jump Markov system can be combined flexibly with

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other tracking algorithms to estimate maneuvering target states (Blom and Bar-Shalom, 1988; Bar-Shalom *et al.*, 1989); e.g., combining jump Markov system models with the Gaussian mixture PHD filter forms the JMS-PHD filter (Pasha *et al.*, 2009). In the JMS-PHD filter, jump Markov system models are adopted to manage the transition among different models. However, the Gaussian component with the largest weight among models is chosen as the output of the JMS-PHD filter, which may result in the degradation of the tracking performance. Actually, the target maneuvering mode cannot be represented appropriately with one model, which has the largest weight, at each time step. The level of measurement noise, tracking delay, parameter non-adaptivity, and missing detection could result in the wrong selection of the model in the JMS-PHD filter. In fact, each model in the model set has a certain likelihood in the multiple model estimation system. Therefore, target parameters are more suitable to be estimated under the joint action of different models.

The generalized pseudo-Bayesian estimator of first order (GPB1) has been proven an effective method for target estimation in a multiple model estimation system. Motivated by this, a modified algorithm for the JMS-PHD filter is presented in this paper. Variation of GPB1 (VGPB1) under the framework of the JMS-PHD filter is described and the mathematical proof is also given. Then it is combined with the JMS-PHD filter to update the model probability of each kinematic model. The weighted sum state, associated covariances, and weights of Gaussian components are then calculated. Pruning and merging techniques are also adopted in the presented algorithm for effectively reducing the combined explosion about the Gaussian components.

2 Background

In this section, the Gaussian mixture PHD filter for jump Markov system models is summarized. Furthermore, the generalized pseudo-Bayesian estimator is introduced.

2.1 JMS-PHD filter

A jump Markov system can be described as a stochastic process that transmits from one state to another according to a finite state Markov chain. Let $\boldsymbol{\xi}_k \in \mathbb{R}^n$ and $\boldsymbol{z}_k \in \mathbb{R}^m$ denote the kinematic state and observation at time k, respectively. The observation likelihood is denoted by $g_k(\boldsymbol{z}_k|\boldsymbol{\xi}_k, r_k^l)$ and the transition density of state dynamics is denoted by $\tilde{f}_{k|k-1}(\boldsymbol{\xi}_k|\boldsymbol{\xi}_{k-1}, r_k^l)$. r_k^l denotes model r^l in the model set at time step k.

A linear Gaussian jump Markov system is a Gaussian jump Markov system with linear Gaussian models, which is listed as follows:

$$\begin{aligned}&\tilde{f}_{k|k-1}(\boldsymbol{\xi}_k|\boldsymbol{\xi}_{k-1}, r_k^l)\\&=\mathcal{N}\left(\boldsymbol{\xi}_k;\boldsymbol{F}_{k-1}(r_k^l)\boldsymbol{\xi}_{k-1},\boldsymbol{Q}_{k-1}(r_k^l)\right),\end{aligned} \tag{1}$$

$$g_k(\boldsymbol{z}_k|\boldsymbol{\xi}_k, r_k^l)=\mathcal{N}\left(\boldsymbol{z}_k;\boldsymbol{H}_k(r_k^l)\boldsymbol{\xi},\boldsymbol{R}_k(r_k^l)\right), \tag{2}$$

where $\mathcal{N}(\cdot;\mu,\sigma)$ is a Gaussian density function with mean μ and covariance σ. $\boldsymbol{F}_{k-1}(r_k^l)$ is the state transition matrix and $\boldsymbol{H}_k(r_k^l)$ is the measurement matrix for target dynamic model r^l. $\boldsymbol{Q}_{k-1}(r_k^l)$ and $\boldsymbol{R}_k(r_k^l)$ are the process noise covariance matrix and measurement noise covariance matrix, respectively.

The PHD filter proposed by Mahler (2003; 2007) to solve a multi-target tracking problem is based on finite set statistics. The first-order intensity moment is utilized to approximate the PHD of the target in this filter. Combining Gaussian mixture properties with PHD recursion forms the Gaussian mixture PHD filter, which is a closed-form solution for implementing the PHD filter (Vo and Ma, 2006).

Unfortunately, a single model in the Gaussian mixture PHD filter would be inappropriate to accommodate maneuvering targets that switch between several kinematic modes. A JMS-PHD filter (Pasha *et al.*, 2009) was designed to overcome this limitation. The main idea of the filter is to utilize a jump Markov system to manage the Gaussian components, which are generated with the different kinematic models. Two propositions are proven during the deviated process. One is for the prediction step and the other for the update step. These two propositions are given under the framework of the linear Gaussian jump Markov system. The prediction and update steps are summarized as follows.

It is assumed that the posterior intensity v_{k-1} at time $k-1$ has the form $v_{k-1}(\boldsymbol{\xi}_{k-1}, r_k^l)$. Then the predicted intensity $v_{k|k-1}$ is given by

$$\begin{aligned}v_{k|k-1}(\boldsymbol{\xi}_{k-1}, r_k^l)=&\gamma_k(\boldsymbol{\xi}_{k-1}, r_k^l)\\&+v_{f,k|k-1}(\boldsymbol{\xi}_{k-1}, r_k^l)+v_{\beta,k|k-1}(\boldsymbol{\xi}_{k-1}, r_k^l),\end{aligned} \tag{3}$$

where $\gamma_k(\boldsymbol{\xi}_{k-1}, r_k^l)$ denotes the intensity of the birth target at time k, and $v_{\beta,k|k-1}(\boldsymbol{\xi}_{k-1}, r_k^l)$ denotes the intensity of the spawn target at time k. Similarly, the posterior intensity $v_{k|k-1}(\boldsymbol{\xi}_{k-1}, r_k^l)$ is given as

$$v_k(\boldsymbol{\xi}_{k-1}, r_k^l) = \left(1 - p_{\mathrm{D},k}(r_k^l)\right) v_{k|k-1}(\boldsymbol{\xi}_{k-1}, r_k^l) + \sum_{z \in Z_k} v_{g,k}(\boldsymbol{\xi}_{k-1}, r_k^l; z). \tag{4}$$

Details of Eqs. (3) and (4) and the JMS-PHD filter can be found in Pasha *et al.* (2009).

2.2 Generalized pseudo-Bayesian estimator

The generalized pseudo-Bayesian estimator is a fusion technique based on total probability Bayesian theory. Two estimators are included in this kind of technology: generalized pseudo-Bayesian estimator of first order (GPB1) and of second order (GPB2). A total of n hypotheses are needed in GPB1 while n^2 in GPB2, where n is the number of models in the model set. Details can be found in Bar-Shalom *et al.* (2001). The main steps of GPB1 are listed as follows.

Step 1: model-matched filtering

Initial target states $\hat{\boldsymbol{\xi}}_{k-1|k-1}$ and corresponding covariances $\boldsymbol{P}_{k-1|k-1}$ are given in this step; running of each model-matched filter is also performed in this step. Target states, associated covariances, and likelihood of each model in the model set, are calculated based on Bayesian recursion. The likelihood of model j is calculated as follows:

$$\lambda_k^j = p[\boldsymbol{z}_k | r_k^j, \boldsymbol{Z}^{k-1}]. \tag{5}$$

Step 2: model probability update

The updated model probability at time k is

$$\begin{aligned} \mu_k^j &\triangleq p[r_k^j | \boldsymbol{Z}^k] \\ &= p[r_k^j | \boldsymbol{z}_k, \boldsymbol{Z}^{k-1}] \\ &= \frac{1}{c} p[\boldsymbol{z}_k | r_k^j, \boldsymbol{Z}_{k-1}] p[r_k^j | \boldsymbol{Z}_{k-1}] \\ &= \frac{1}{c} \lambda_k^j \sum_{i=1}^n p_{ij} \mu_{k-1}^i, \end{aligned} \tag{6}$$

where $c = \sum_{j=1}^n \lambda_k^j \sum_{i=1}^n p_{ij} \mu_{k-1}^i$ is the generalized factor, μ_{k-1}^i is the probability of model i at time $k-1$, and p_{ij} is the ith row and jth column element of the model probability transition matrix.

Step 3: fusion of target states and covariances

Target states are integrated as

$$\hat{\boldsymbol{\xi}}_{k|k} = \sum_{j=1}^n \hat{\boldsymbol{\xi}}_{k|k}^j \mu_k^j. \tag{7}$$

And the associated covariances are

$$\boldsymbol{P}_{k|k} = \sum_{j=1}^n \mu_k^j [\boldsymbol{P}_{k|k}^j + (\hat{\boldsymbol{\xi}}_{k|k}^j - \hat{\boldsymbol{\xi}}_{k|k})(\hat{\boldsymbol{\xi}}_{k|k}^j - \hat{\boldsymbol{\xi}}_{k|k})^{\mathrm{T}}], \tag{8}$$

where $\boldsymbol{P}_{k|k}^j$ and $\hat{\boldsymbol{\xi}}_{k|k}^j$ are covariances and states of model j at time step k, respectively.

3 Improved JMS-PHD filter

3.1 Target and measurement model

Consider a case for M targets tracking, and assume that the dynamics of the jth target is modeled as a jump Markov system:

$$\boldsymbol{\xi}_k^j = \boldsymbol{F}^j \boldsymbol{\xi}_{k-1}^j + \boldsymbol{a}^j \boldsymbol{\omega}_{k-1}^j, \ j = 1, 2, ..., M, \tag{9}$$

where $\boldsymbol{\xi}_k^j$ is a $d \times 1$ target state vector, $\boldsymbol{F}^j$ is a $d \times d$ target state transition matrix, $\boldsymbol{a}^j$ is a $d \times d'$ matrix, and $\boldsymbol{\omega}_{k-1}^j$ is a sequence of independent identically distributed (i.i.d.) standard Gaussian variables with dimension d'.

It is assumed that the measurement corresponding to state $\boldsymbol{\xi}_k^j$ is also modeled as a jump Markov system:

$$\boldsymbol{z}_k^j = \boldsymbol{H}^j \boldsymbol{\xi}_k^j + \boldsymbol{b}^j \boldsymbol{v}_k^j, \ j = 1, 2, ..., M, \tag{10}$$

where $\boldsymbol{z}_k^j$ is an $m \times 1$ target measurement vector, $\boldsymbol{H}^j$ is an $m \times d$ target measurement matrix, $\boldsymbol{b}^j$ is an $m \times m$ matrix, and $\boldsymbol{v}_k^j$ is a sequence of i.i.d. standard Gaussian variables with dimension m.

The aim of multi-target tracking is to estimate the number and states of targets. The key factor to the accuracy of target state estimation is the adaptivity of the target dynamic model. It has been verified that the GPB estimator is an effective method for the multiple model estimation system. Therefore, it can be combined with the PHD filter.

3.2 Mathematical justification

States of the target at time k are weighted according to the probabilities, which are generated with different models, according to the theory of GPB1. The total probability theorem is listed as follows:

$$\begin{aligned} p[\hat{\boldsymbol{\xi}}_k|\boldsymbol{Z}_k] &= \sum_{l=1}^{n} p[\hat{\boldsymbol{\xi}}_k|r_k^l, \boldsymbol{Z}_k]p[r_k^l|\boldsymbol{Z}_k] \\ &= \sum_{l=1}^{n} p[\hat{\boldsymbol{\xi}}_k|r_k^l, \boldsymbol{z}_k, \boldsymbol{Z}_{k-1}]\mu_k(r_k^l) \\ &\approx \sum_{l=1}^{n} p[\hat{\boldsymbol{\xi}}_k|r_k^l, \boldsymbol{z}_k, \hat{\boldsymbol{\xi}}_{k-1}, \hat{\boldsymbol{P}}_{k-1}]\mu_k(r_k^l). \end{aligned} \tag{11}$$

Considering the case of the JMS-PHD filter, if there are J_k Gaussian components at time step k, then the variation of GPB1 is described as follows:

Theorem 1 If the states of the jth Gaussian component are updated with the ith measurement, then the probability of the jth component under the update of the ith measurement is

$$p[\hat{\boldsymbol{\xi}}_k^{i,j}|\boldsymbol{Z}_k] \approx \sum_{l=1}^{n} p[\hat{\boldsymbol{\xi}}_k^j|r_k^l, \boldsymbol{z}_k^i, \hat{\boldsymbol{\xi}}_{k-1}^j, \hat{\boldsymbol{P}}_{k-1}^j]\mu_k^i(r_k^l),$$

where $i \in \{1, 2, ..., |\boldsymbol{z}_k|\}$, $j \in \{1, 2, ..., J_k\}$. $|\boldsymbol{z}_k|$ is the cardinality of the set of measurements at time step k, $\mu_k^i(r_k^l)$ denotes the likelihood of model r^l on the condition of measurement i, and J_k is the number of Gaussian components at time step k.

Proof From Eq. (11), it can be seen that

$$\begin{aligned} p[\hat{\boldsymbol{\xi}}_k^{i,j}|\boldsymbol{Z}_k] &= \sum_{l=1}^{n} p[\hat{\boldsymbol{\xi}}_k^j|r_k^l, \boldsymbol{Z}_k]p[r_k^l|\boldsymbol{Z}_k] \\ &= \sum_{l=1}^{n} p[\hat{\boldsymbol{\xi}}_k^j|r_k^l, \boldsymbol{z}_k^i, \boldsymbol{Z}_{k-1}]\mu_k^i(r_k^l) \\ &\approx \sum_{l=1}^{n} p[\hat{\boldsymbol{\xi}}_k^j|r_k^l, \boldsymbol{z}_k^i, \hat{\boldsymbol{\xi}}_{k-1}^j, \hat{\boldsymbol{P}}_{k-1}^j]\mu_k^i(r_k^l). \end{aligned} \tag{12}$$

This theorem demonstrates that target states, covariances, and the weight of Gaussian component j updated by the ith measurement are weighted sums, using outputs of the different models according to their probabilities.

After the update step of the JMS-PHD filter, the total n^2 hypotheses for the same Gaussian component are merged into one single combined hypothesis, which is similar to using an integral target model to approximate target kinematics based on GPB1.

3.3 Algorithm

After the update step of the JMS-PHD filter, several Gaussian components are generated with different models for the same measurement, which resembles the update step in the GPB1 estimator. Thus, the variated GPB1 technology is adopted to manage Gaussian components generated by different models in this work. One cycle of the proposed Gaussian mixture PHD filter for multiple maneuvering targets tracking, which is an extension of the one proposed by Zhang *et al.* (2012), is described as follows.

Step 1: initialization

Assume the posterior intensity $v_{k-1}(\hat{\boldsymbol{\xi}}_{k-1}, r_{k-1}^l)$ at time $k-1$ is a Gaussian mixture for each r_{k-1}^l, i.e.,

$$\begin{aligned} v_{k-1}(\hat{\boldsymbol{\xi}}_{k-1}, r_{k-1}^l) = &\sum_{i=1}^{J_{k-1}(r_{k-1}^l)} \omega_{k-1}^i(r_{k-1}^l) \\ &\cdot \mathcal{N}\Big(\hat{\boldsymbol{\xi}}_{k-1}; \hat{\boldsymbol{m}}_{k-1}^i(r_{k-1}^l), \hat{\boldsymbol{P}}_{k-1}^i(r_{k-1}^l)\Big), \end{aligned} \tag{13}$$

where r_{k-1}^l denotes that model r^l is active at time step $k-1$, $\hat{\boldsymbol{\xi}}_{k-1}$ is its estimated state, and $\hat{\boldsymbol{m}}_{k-1}^i(r_{k-1}^l)$ and $\hat{\boldsymbol{P}}_{k-1}^i(r_{k-1}^l)$ are the mean and covariance of the ith Gaussian component for model r^l at time $k-1$, respectively.

Step 2: prediction

The predicted intensity $v_{k|k-1}(\hat{\boldsymbol{\xi}}_{k|k-1}, r_{k-1}^l)$ is also a Gaussian mixture for each model r^l:

$$\begin{aligned} &v_{k|k-1}(\hat{\boldsymbol{\xi}}_{k|k-1}, r_{k-1}^l) \\ &= \sum_{r=1}^{n} \sum_{i=1}^{J_{k-1}^{r_{k-1}^l}} \omega_{k|k-1}^i(r_{k-1}^l)\Big[p_{(s,k)}(r_{k-1}^l)\tau_{k|k-1}(r_k^l|r_{k-1}^{l'}) \\ &\quad \cdot \mathcal{N}\Big(\hat{\boldsymbol{\xi}}_{k|k-1}; \hat{\boldsymbol{m}}_{k|k-1}^i(r_k^l, r_{k-1}^{l'}), \hat{\boldsymbol{P}}_{k|k-1}^i(r_k^l, r_{k-1}^{l'})\Big) \\ &\quad + \sum_{j=1}^{J_{\beta,k|k-1}(r_k^l, r_{k-1}^{l'})} \omega_{\beta,k|k-1}(r_k^l, r_{k-1}^{l'})\pi_{k|k-1}(r_k^l|r_{k-1}^{l'}) \\ &\quad \cdot \mathcal{N}\Big(\hat{\boldsymbol{\xi}}; \hat{\boldsymbol{m}}_{k|k-1}^{i,j}(r_k^l, r_{k-1}^{l'}), \hat{\boldsymbol{P}}_{k-1}^i(r_{k-1}^{l'})\Big)\Big] \\ &\quad + \gamma_k(\hat{\boldsymbol{\xi}}_{k|k-1}, r_{k-1}^l), \end{aligned} \tag{14}$$

where $\tau_{k|k-1}(r_k^l|r_{k-1}^{l'})$ is the probability of a transition from model $r_{k-1}^{l'}$ at time $k-1$ to model r_k^l at time k, $\pi_{k|k-1}(r_k^l|r_{k-1}^{l'})$ is the probability distribution of model r_k^l at time k spawned from model $r_{k-1}^{l'}$ at time $k-1$, $p_{(s,k)}(r_{k-1}^{l'})$ is the probability of survival for model $r_{k-1}^{l'}$ at time k, $J_{k-1}^{r_{k-1}^{l'}}$ is the Gaussian component number for model $r_{k-1}^{l'}$, and

$\gamma_k(\hat{\boldsymbol{\xi}}_{k|k-1}, r_{k-1}^l)$ is the intensity of the birth target.

$$\hat{\boldsymbol{m}}_{k|k-1}^i(r_k^l, r_{k-1}^{l'}) = \boldsymbol{F}_k^{r_k^l} \hat{\boldsymbol{m}}_{k-1}^i(r_{k-1}^{l'}), \tag{15}$$

$$\hat{\boldsymbol{P}}_{k|k-1}^i(r_k^l, r_{k-1}^{l'}) = \boldsymbol{F}_k^{r_k^l} \hat{\boldsymbol{P}}_{k-1}^i(r_{k-1}^{l'}) \left(\boldsymbol{F}_k^{r_k^l}\right)^{\mathrm{T}} (r_{k-1}^l) + \boldsymbol{Q}_{k-1}^{r_{k-1}^l}, \tag{16}$$

$$\hat{\boldsymbol{m}}_{k|k-1}^{i,j}(r_k^l, r_{k-1}^{l'}) = \boldsymbol{F}_{\beta,k-1}^j(r_k^l, r_{k-1}^{l'}) \hat{\boldsymbol{m}}_{k-1}^i(r_{k-1}^{l'}) + d_{\beta,k-1}^j(r_k^l, r_{k-1}^{l'}), \tag{17}$$

where $\boldsymbol{F}_{k-1}^{r_{k-1}^l}$ is the state transition matrix for target dynamic model r^l at time $k-1$, and $\boldsymbol{Q}_{k-1}^{r_{k-1}^l}$ is the corresponding process noise covariance matrix. $\omega_{\beta,k|k-1}^j(r_k^l, r_{k-1}^{l'})$, $\boldsymbol{F}_{\beta,k-1}^{(j)}$, $d_{\beta,k-1}^j(r_k^l, r_{k-1}^{l'})$, $\boldsymbol{Q}_{\beta,k-1}^j$, and $J_{\beta,k|k-1}(r_{k-1}^l, r_{k-1}^{l'})$ determine the shape of the spawning intensity of a target with previous state $\hat{\boldsymbol{\xi}}_{k|k-1}$. The intensity of the birth random finite set (RFS) can be expressed as Gaussian mixtures of the form

$$\gamma_k(\hat{\boldsymbol{\xi}}_{k|k-1}, r_{k-1}^l) = \pi_k(r_{k-1}^l) \sum_{j=1}^{J_{\gamma,k}(r_{k-1}^l)} \left[\omega_{\gamma,k}^j(r_{k-1}^l) \cdot \mathcal{N}\left(\hat{\boldsymbol{\xi}}_{k|k-1}; \hat{\boldsymbol{m}}_{\gamma,k}^j(r_{k-1}^l), \hat{\boldsymbol{P}}_{\gamma,k}^j(r_{k-1}^l)\right)\right]. \tag{18}$$

Similar to the posterior intensity at time $k-1$, it is assumed that the predicted intensity $v_{k|k-1}(\hat{\boldsymbol{\xi}}_{k|k-1}, r_{k-1}^l)$ is a Gaussian mixture:

$$v_{k|k-1}(\hat{\boldsymbol{\xi}}_{k|k-1}, r_{k-1}^l) = \sum_{i=1}^{J_{k|k-1}} \left[\omega_{k|k-1}^i(r_{k-1}^l) \cdot \mathcal{N}\left(\hat{\boldsymbol{\xi}}_{k|k-1}; \hat{\boldsymbol{m}}_{k|k-1}^i(r_{k-1}^l), \hat{\boldsymbol{P}}_{k|k-1}^i(r_{k-1}^l)\right)\right]. \tag{19}$$

Step 3: update

The posterior intensity $v_k(\hat{\boldsymbol{\xi}}_k, r_k^l)$ at time k is still a Gaussian mixture for each model r^l:

$$v_k(\hat{\boldsymbol{\xi}}_k, r_k^l) = \left(1 - p_{\mathrm{D},k}(r_k^l)\right) v_{k|k-1}(\hat{\boldsymbol{\xi}}_k, r_k^l) + \sum_{\boldsymbol{z} \in \boldsymbol{Z}_k} v_{\mathrm{D},k}(\hat{\boldsymbol{\xi}}_k, \boldsymbol{z}), \tag{20}$$

where

$$v_{\mathrm{D},k}(\hat{\boldsymbol{\xi}}_k, \boldsymbol{z}) = \sum_{j=1}^{J_{k|k-1}} \omega_k^j(\boldsymbol{z}) \mathcal{N}\left(\hat{\boldsymbol{\xi}}_k; \hat{\boldsymbol{m}}_{k|k}^j(\boldsymbol{z}, r_k^l), \hat{\boldsymbol{P}}_{k|k}^j(r_k^l)\right), \tag{21}$$

with

$$\omega_k^j = \left(p_{\mathrm{D},k}(r_k^l) \omega_{k|k-1}^j q_k^j(\boldsymbol{z}, r_k^l)\right) \Big/ \left(\kappa_k(\boldsymbol{z}) + \sum_{r_k^{l'}} p_{\mathrm{D},k}(r_k^{l'}) \sum_{\ell=1}^{J_{k|k-1}(r_k^{l'})} \omega_{k|k-1}^{\ell}(r_k^{l'}) q_k^{\ell}(\boldsymbol{z}, r_k^{l'})\right), \tag{22}$$

and

$$\hat{\boldsymbol{m}}_{k|k}^j = \hat{\boldsymbol{m}}_{k|k-1}^j(r_k^l) + \boldsymbol{K}_k^j \left(\boldsymbol{z} - \boldsymbol{H}_k(r_k^l) \hat{\boldsymbol{m}}_{k|k-1}^j(r_k^l)\right), \tag{23}$$

$$\hat{\boldsymbol{P}}_{k|k}^j(r_k^l) = \left(\boldsymbol{I} - \boldsymbol{K}_k^j(r_k^l) \boldsymbol{H}_k(r)\right) \hat{\boldsymbol{P}}_{k|k-1}^j(r_k^l), \tag{24}$$

$$\boldsymbol{K}_k^j(r_k^l) = \hat{\boldsymbol{P}}_{k|k-1}^j(r_k^l) \boldsymbol{H}_k^{\mathrm{T}}(r_k^l) \cdot \left(\boldsymbol{H}_k(r_k^l) \hat{\boldsymbol{P}}_{k|k-1}^j(r_k^l) \boldsymbol{H}_k^{\mathrm{T}}(r_k^l) + \boldsymbol{R}_k(r_k^l)\right)^{-1}, \tag{25}$$

$$q_k^j(\boldsymbol{z}, r_k^l) = \mathcal{N}\left(\boldsymbol{z}; \boldsymbol{H}_k(r_k^l) \hat{\boldsymbol{m}}_{k|k-1}^j(r_k^l), \boldsymbol{H}_k(r_k^l) \hat{\boldsymbol{P}}_{k|k-1}^j(r_k^l) \boldsymbol{H}_k^{\mathrm{T}}(r_k^l) + \boldsymbol{R}_k(r_k^l)\right). \tag{26}$$

$p_{\mathrm{D},k}(r_k^l)$ is the probability of target detection at time k for model r^l, and $\boldsymbol{z}$ is multi-target observation.

Step 4: model probability update

If the existing probability of Gaussian component j at time k for model r^l is

$$\lambda_k^j(r_k^l) = p[\boldsymbol{z}_k | r_k^l, \boldsymbol{Z}_{k-1}] = p[C_k^j(r_k^l) | \boldsymbol{z}_k^{(\ell)}, \boldsymbol{Z}_{k-1}], \tag{27}$$

where $r_k^l = 1, 2, ..., n$, $j = 1, 2, ..., J_k$, $C_k^j(r_k^l)$ is the jth component for model r^l at time k, then the updated probabilities of the component for model r^l are evaluated as

$$\mu_k^{\ell_g + j}(r_k^l) = p[r_k^l | \boldsymbol{Z}_k] = \frac{1}{c} \lambda_k^j(r_k^l) \sum_{l'=1}^{n} p[r_k^l | r_k^{l'}, \boldsymbol{Z}_{k-1}] \cdot p[C_k^j(r_k^l) | \boldsymbol{z}_k^{\ell}, \boldsymbol{Z}_{k-1}], \tag{28}$$

where $\boldsymbol{z}_k^{\ell}$ is the ℓth measurement at time k, and c is the normalization constant:

$$c = \sum_{r^l=1}^{n} \lambda_k^j(r_k^l) \sum_{r^{l'}=1}^{n} p_{r_k^{l'} r_k^l} \lambda_k^j(r_k^l). \tag{29}$$

Step 5: fusion

The combined Gaussian weight, mean, and covariance of the target are estimated as

$$\omega_k^j = \sum_{r_k^l=1}^{n} \mu_k^j(r_k^l) \omega_k^j(r_k^l) / n. \tag{30}$$

If the Gaussian weight is $\omega_k^j(r_k^l)$ for model r^l after update time step k for Gaussian component j, then the Gaussian weight should be $\omega_k^j(r_k^l)/n$ before the pruning and merging step.

Proof of Eq. (30) From Eq. (14), it is evident that n Gaussian components would be generated with one Gaussian component under the management of the jump Markov system. Therefore, if there are n models, n^2 components would be generated. Under the condition of GPB1, n components are combined into a unit. Thus, the Gaussian weight is n times the original one and should be normalized by n.

$$\hat{\boldsymbol{\xi}}_k^j = \sum_{r_k^l=1}^{n} \hat{\boldsymbol{m}}_k^j(r_k^l)\mu_k^j(r_k^l), \tag{31}$$

$$\hat{\boldsymbol{P}}_k^j = \sum_{r_k^l=1}^{n} \mu_k^j(r_k^l)\Big[\hat{\boldsymbol{P}}_k^j(r_k^l) + \left(\hat{\boldsymbol{m}}_k^j(r_k^l) - \hat{\boldsymbol{\xi}}_k^j\right) \cdot \left(\hat{\boldsymbol{m}}_k^j(r_k^l) - \hat{\boldsymbol{\xi}}_k^j\right)^{\mathrm{T}}\Big]. \tag{32}$$

Step 6: pruning and merging

Pruning is a heuristic technique used in this proposed filter to restrict the increasing number of Gaussian components during the recursive process. After pruning, if the weight is larger than the preset threshold, it will be retained; otherwise, it will be deleted.

$$S = \left\{\Gamma_i | \omega_k^i \geq \Gamma_{\mathrm{Th}}, i = 1, 2, ..., n\right\}. \tag{33}$$

Merging is another heuristic technique to reduce the computation. Adjacent components will be merged into one Gaussian component if they are subject to

$$\left(\hat{\boldsymbol{\xi}}_k^i - \hat{\boldsymbol{\xi}}_k^j\right)^{\mathrm{T}} (\hat{\boldsymbol{P}}_k^i)^{-1} \left(\hat{\boldsymbol{\xi}}_k^i - \hat{\boldsymbol{\xi}}_k^j\right) \leq U_{\mathrm{merge}}, \tag{34}$$

where $\Gamma_i, \Gamma_j \in S, i \neq j$, $\Gamma_{\mathrm{Th}}, U_{\mathrm{merge}}, \Gamma_i$ are the pruning threshold, merging threshold, Gaussian component, respectively.

Step 7: state extraction

The mean of the Gaussian component, whose weight is larger than a certain threshold Γ_{Gate}, acts as the final output of the target state.

4 Simulation

Targets spawn and birth are not considered in this paper due to space limitations. Performance of the proposed algorithm is analyzed by comparison with the JMS-PHD filter.

4.1 Simulation initialization

In this simulation scenario, there are five targets at the beginning, including three maneuvering targets and two non-maneuvering targets in the field of view. Then one target disappears after time step 140. All measurements are received in range, azimuth, elevation, and velocity, which can be obtained by radar. The initial mean $\hat{\boldsymbol{\xi}}_0 = [\hat{x}_0, \hat{\dot{x}}_0, \hat{\ddot{x}}_0, \hat{y}_0, \hat{\dot{y}}_0, \hat{\ddot{y}}_0, \hat{z}_0, \hat{\dot{z}}_0, \hat{\ddot{z}}_0]$. The first target begins to maneuver at constant acceleration with $A_x = -10\ \mathrm{m/s^2}, A_y = 15\ \mathrm{m/s^2}$ from 80th to 120th time step, the second with $A_x = -10\ \mathrm{m/s^2}, A_y = 15\ \mathrm{m/s^2}, A_z = 3\ \mathrm{m/s^2}$ from 90th to 120th time step. The third target maneuvers with different accelerations during different stages, with $A_x = -5\ \mathrm{m/s^2}, A_y = 5\ \mathrm{m/s^2}$ from the beginning to the 40th time step, with $A_x = -10\ \mathrm{m/s^2}, A_y = -20\ \mathrm{m/s^2}$ from the 41th time step to the 80th time step, while with $A_x = 6\ \mathrm{m/s^2}, A_y = -6\ \mathrm{m/s^2}$ from the 81th time step to the 140th time step, which is the end of this target. The other two targets move uniformly at all time steps. The initial number of targets in the simulation program is unknown and presumed to be 50 in this scenario; the radar is located at the origin of the coordinates.

The methods of adding measurement noise and the clutter are as follows.

First, the target states are converted from Cartesian to polar, and noise is added in range, azimuth, elevation, and velocity. The level of the noise standard is $\delta_{\mathrm{r}} = 150$ m in range, $\delta_{\mathrm{az}} = \delta_{\mathrm{el}} = 0.005$ rad in azimuth, elevation, and $\delta_{\mathrm{v}} = 3$ m/s in velocity.

Then, clutters are added uniformly in range, azimuth, elevation, and velocity. The clutters are subject to Poisson RFS with the intensity function

$$\kappa_k(z) = \lambda_{\mathrm{c}} V u(z). \tag{35}$$

Clutter number $\lambda_{\mathrm{c}} = 1.8 \times 10^{-8}$, which gives about 60 measurements for every time step.

The scope for the range is [16 000, 53 000] m, azimuth [0.87, 1.48] rad, elevation [0.08, 0.32] rad, and velocity [−400, 600] m/s. Three-dimensional trajectories of targets are shown in Fig. 1, and the two-dimensional ones are shown in Fig. 2. Fig. 3 displays the measurements of the received observation. Pruning parameter threshold $\Gamma_{\mathrm{Th}} = 10^{-5}$, Gaussian component merging threshold $U_{\mathrm{merge}} = 4$, weight threshold $\Gamma_{\mathrm{Gate}} = 0.5$, detection probability $p_{\mathrm{D},k} = 0.98$, survival probability $p_{\mathrm{s}} = 0.99$, and sampling time

$T_s = 0.5$ s are adopted in this simulation.

Three models are adopted in both algorithms, including one constant velocity (CV) model and two constant acceleration (CA) models with different process noise coefficients. Detailed model set information can be found in Li and Jilkov (2003); the Markov transition probability is

$$\boldsymbol{\tau}_{k|k-1} = \begin{bmatrix} 0.98 & 0.01 & 0.01 \\ 0.01 & 0.98 & 0.01 \\ 0.01 & 0.01 & 0.98 \end{bmatrix}.$$

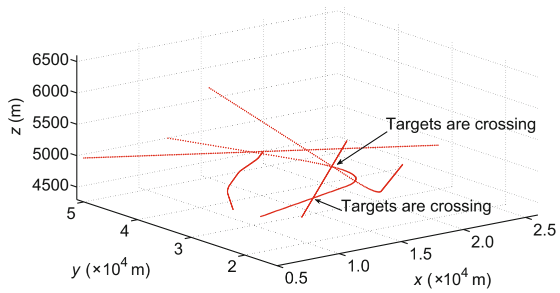

Fig. 1 Three-dimensional true target trajectories

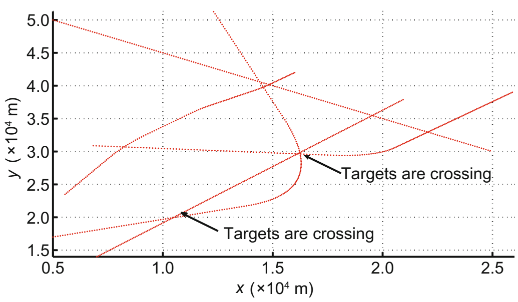

Fig. 2 Two-dimensional true target trajectories

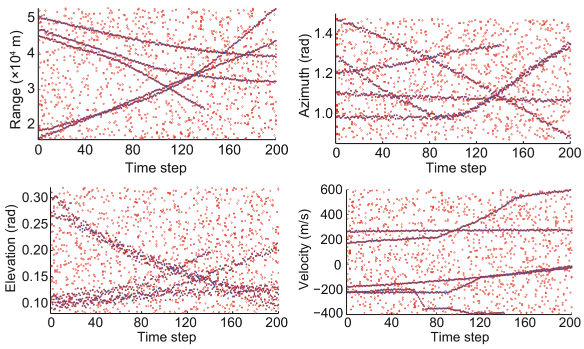

Fig. 3 Measurements obtained for tracking algorithms

4.2 Simulation results and analysis

To evaluate the performance of the proposed algorithm, 200 runs of Monte-Carlo simulations are executed, and the optimal subpattern assignment (OSPA) metric (Schuhmacher *et al.*, 2008) is adopted in this simulation. The OSPA metric has been proven a reasonable and intuitive interpretation of the localization and cardinality errors in the multi-target tracking scenario.

The parameters of order $p = 2$ and cut-off $e = 400$ are set. Details on the definition of the OSPA metric can be found in Schuhmacher *et al.* (2008).

The average OSPA distances of the 200 runs of Monte Carlo simulations are shown in Fig. 4. In the first several steps in Fig. 5, it can be seen that the target number estimation errors of the two algorithms are large, which results in the OSPA distances being close to the constant value of cut-off e. During the benign flight or flight stage with weak maneuverability, which begins at the 80th time step, OSPA distances in the proposed algorithm are slightly lower than those of the JMS-PHD filter (Fig. 6). After the benign flight, the OSPA distances of the proposed algorithm are lower than those of the JMS-PHD filter.

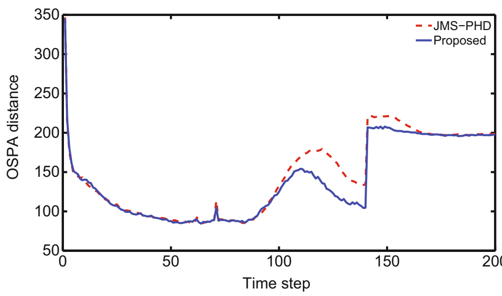

Fig. 4 Optimal subpattern assignment (OSPA) distances for two algorithms

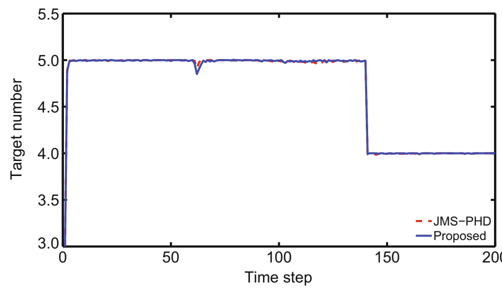

Fig. 5 Target number estimation for two algorithms

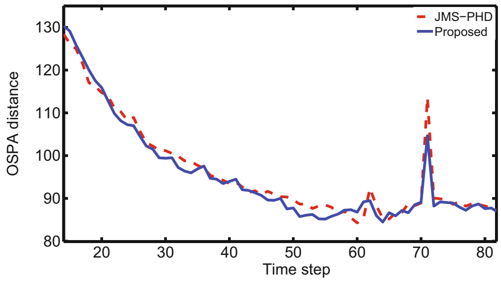

Fig. 6 Optimal subpattern assignment (OSPA) distances during little maneuver flight for two algorithms

Among the components generated with different models, the Gaussian component with the largest weight is selected as the output of the filter in the original JMS-PHD filter. In fact, the dynamic model with the largest Gaussian component weight could not denote the actual target mode at all time steps because the intensity of measurement noise, tracking delay, non-adaptivity of model parameters in the model set, and missing detection would impact the Gaussian component weight in the JMS-PHD filter. In the presented algorithm, VGPB1 is used to manage different Gaussian components. Therefore, components updated with each model make contributions to target parameters, such as states, covariances, and Gaussian weights. This technique may avoid the risk of wrong model selection, which results in the improvement of tracking performance for the proposed algorithm.

5 Conclusions

In this paper, the development of a multiple maneuvering targets tracking algorithm under the framework of the Gaussian mixture PHD filter is described. First, the variation of GPB1 was derived under the framework of the JMS-PHD filter. Then, based on GPB1, a modified Gaussian mixture PHD filter was provided. The proposed algorithm was evaluated and compared with the JMS-PHD filter. Monte-Carlo simulation results showed that the OSPA distances of the proposed algorithm are lower than those of the JMS-PHD filter for maneuvering target tracking.

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