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A trapezoidal cantilever density sensor based on MEMS technology*

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Abstract: A trapezoidal cantilever density sensor is developed based on micro-electro-mechanical systems (MEMS) technology. The sensor measures fluid density through the relationship between the density and the resonant frequency of the cantilever immersed in the fluid. To improve the sensitivity of the sensor, the modal and harmonic response analyses of trapezoidal and rectangular cantilevers are simulated by ANSYS software. The higher the resonant frequency of the cantilever immersed in the fluid, the higher the sensitivity of the sensor; the higher the resonant strain value, the easier the detection of the output signal of the sensor. Based on the results of simulation, the trapezoidal cantilever is selected to measure the densities of dimethyl silicone and toluene at the temperature ranges of 30 to 55 °C and 26 to 34 °C, respectively. Experimental results show that the trapezoidal cantilever density sensor has a good performance.

Key words: Micro-electro-mechanical systems (MEMS), Density sensor, Trapezoidal cantilever, Resonant frequency
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1 Introduction

Density is a basic parameter for measuring other fluid thermophysical parameters such as viscosity and flow rate (Igarashi *et al.*, 2007; Waszczuk *et al.*, 2011). To accurately assess the quality and features of the products, density should be measured in situ in many fields such as petroleum industry, food production, and automotive industry (Zribi *et al.*, 2005; Cao-Paz *et al.*, 2012).

The density sensor based on micro-electro-mechanical systems (MEMS) technology has attracted considerable interest owing to its miniaturization, integration, and batch production (Igarashi *et al.*, 2007; Sparks *et al.*, 2011). Some researchers have investigated the MEMS density sensors with the sensitive components of the micro silicon tube (Corman *et al.*, 2000; Sparks *et al.*, 2003; Najmzadeh *et al.*, 2007) or cantilever (Harrison *et al.*, 2006; Brandstetter *et al.*, 2009; Heinisch *et al.*, 2011). A common method to measure fluid density is detecting the resonant frequency of the cantilever or the micro silicon tube immersed in the fluid and then calculating the fluid density value through the relationship between the density and the resonant frequency (Wilson *et al.*, 2007; Rezazadeh *et al.*, 2010; Rust and Dual, 2011).

To improve the sensitivity of the sensor, Waugh *et al.* (2011) used the nonlinear dynamic behavior of the beam to measure the fluid density; the cantilever was designed to work with torsional vibrating mode in Li *et al.* (2011), and Liao *et al.* (2011) designed a

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cantilever with a square hole in the free end of the cantilever. The resonant frequency of the cantilever is related to its shape and dimension. In this paper, a trapezoidal cantilever density sensor is researched. The optimal size of the cantilever is obtained by harmonic response analysis and the cantilever is used to measure the densities of dimethyl silicone and toluene.

2 Modal and harmonic response analysis

Fig. 1 shows the simple models of rectangular and trapezoidal cantilevers. The left ends of the two cantilevers are fixed.

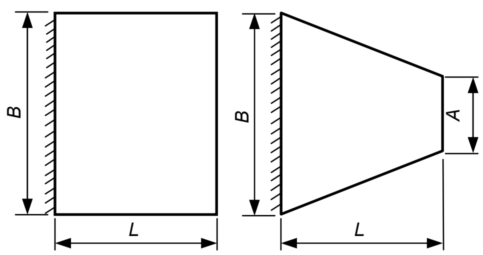

Fig. 1 Models of the rectangular and trapezoidal cantilevers
A: free edge width of the trapezoidal cantilever; *B*: bottom edge width of the cantilever; *L*: length of the cantilever. For both cantilevers, *L*=1500 μm; *B* represents five different bottom widths of 2100, 2300, 2500, 2700, and 2900 μm; *A*=1000 μm

Modal analyses of the rectangular and trapezoidal cantilevers are simulated by ANSYS software. Fig. 2 shows the curves of the first-order resonant frequency to the bottom edge width of the two cantilevers.

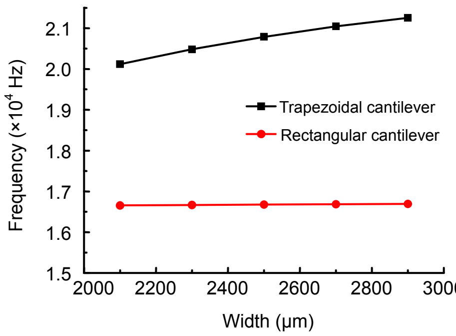

Fig. 2 Curves of first-order resonant frequency to the bottom edge width of the two cantilevers

Harmonic response analysis results show that the first-order resonant maximum strain of the trapezoidal cantilever with width *B* of 2500 μm obtains the maximum value in five different widths, including 2100, 2300, 2500, 2700, and 2900 μm. Fig. 3 shows the maximum strains of the two cantilevers changing with the increase of *B*.

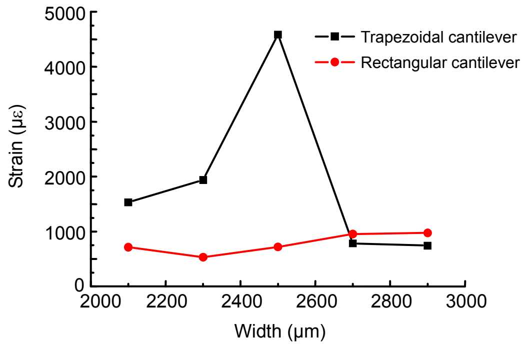

Fig. 3 First-order resonant maximum strain of the two cantilevers as a function of width

The higher the maximum strain, the easier the detection of the output signal voltage of the sensor. From the analysis, the optimal size *A*×*B*×*L* of the trapezoidal cantilever is obtained with 1000 μm×2500 μm×1500 μm.

3 Working principle of the density sensor

Fig. 4 shows the schematic diagram of the density sensor with the trapezoidal cantilever. Bonding pads 1 and 6 are used to supply AC current for the Ti-Pt-Au coil. Bonding pads 2 and 4 are used to supply DC current for the Wheatstone bridge. Bonding pads 3 and 5 are used to detect AC output signal voltage of the Wheatstone bridge due to the vibration of the trapezoidal cantilever. The trapezoidal cantilever is placed in the uniform magnetic field *T*. The direction of magnetic field *T* is parallel with the surface of the trapezoidal cantilever and vertical to the Ti-Pt-Au coil placed in the top edge of the trapezoidal cantilever (Fig. 4). When AC current is applied to the Ti-Pt-Au coil through bonding pads 1 and 6, the Ti-Pt-Au coil will generate AC Lorentz force, which actuates the trapezoidal cantilever to vibrate. The resonant frequency can be obtained through the method of frequency scanning.

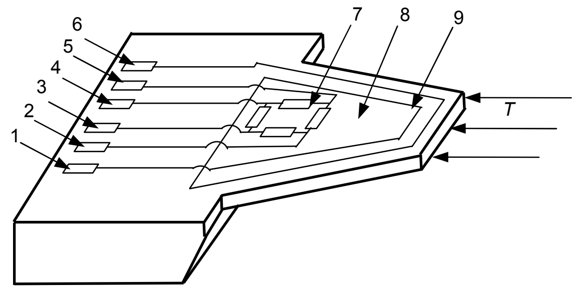

Fig. 4 Schematic diagram of the trapezoidal cantilever density sensor
1–6, bonding pads; 7, Wheatstone bridge; 8, trapezoidal cantilever; 9, Ti-Pt-Au coil

The principle of the cantilever density sensor measuring fluid density is based on the fact that the fluid generates additional inertia load on the cantilever. The additional inertia load will change the resonant frequency of the cantilever immersed in the fluid (Ghatkesar *et al*., 2008). The cantilever has different resonant frequencies in the fluids with different densities. Thus, the fluid density can be calculated through the relationship between density and the resonant frequency of the cantilever immersed in the fluid. The relationship can be expressed as follows (Goodwin *et al*., 2006):

$$\rho_f = \frac{a}{f^2} + b, \quad (1)$$

where ρ_f is the density of the fluid, f is the resonant frequency of the cantilever, and a and b are parameters related with the property of the cantilever which need to be calibrated through experiments.

4 Density measurements

The trapezoidal cantilever is packaged on the printed circuit board (PCB) with a large bonding pad (Fig. 5). Fig. 6 is the scanning electron micrograph (SEM) photo of the trapezoidal cantilever. To improve the sensitivity of the cantilever density sensor and easily detect output signal voltage of the Wheatstone bridge, a trapezoidal cantilever with the size $A \times B \times L$ being 1000 μm×2500 μm×1500 μm is selected to measure the densities of dimethyl silicone and toluene at the temperature ranges of 30 to 55 °C and 26 to 34 °C, respectively.

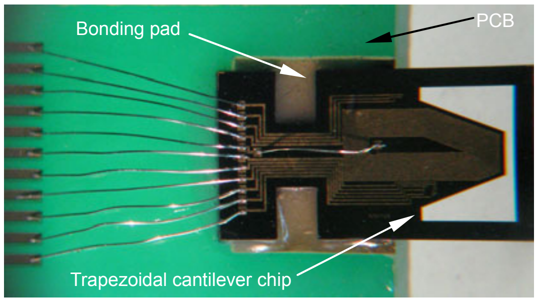

Fig. 5 Photograph of the trapezoidal cantilever sensor

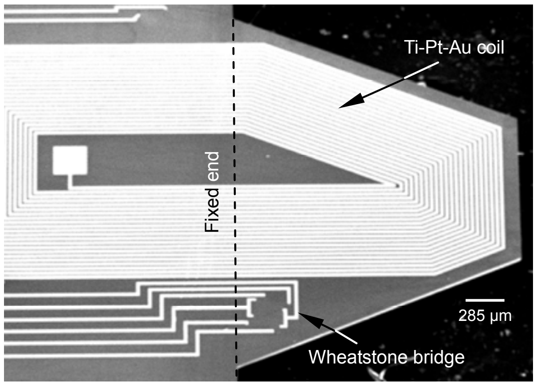

Fig. 6 Scanning electron micrograph of the trapezoidal cantilever

The resonant frequencies of the trapezoidal cantilever immersed in dimethyl silicone are detected at the temperature range of 30 to 55 °C. Reference densities are provided by Refprop9.0 software. Table 1 shows the resonant frequencies and reference densities of dimethyl silicone.

Table 1 Resonant frequencies and reference densities of dimethyl silicone

Temperature (°C)	Frequency (Hz)	Ref. density (kg/m^3)
30	8644.73962	754.10
35	8668.78029	749.29
40	8690.36093	744.44
45	8711.35328	739.55
50	8734.62692	734.51
55	8754.64902	729.62

The resonant frequency of the trapezoidal cantilever immersed in toluene is detected at the temperature range of 26 to 34 °C. Table 2 shows the resonant frequencies and reference densities of toluene.

Table 2 Resonant frequencies and reference densities of toluene

Temperature (°C)	Frequency (Hz)	Ref. density (kg/m^3)
26	8081.22428	861.31
28	8102.44811	859.44
30	8113.64067	857.57
32	8124.83397	855.70
34	8139.41582	853.83

5 Results and discussion

The resonant frequencies of the trapezoidal cantilever immersed in the dimethyl silicone and dimethyl silicone reference densities at the temperatures of 30 and 55 °C are used to calibrate constants a and b in Eq. (1):

$$a=7.332\times10^{10},\ b=-227.0153. \tag{2}$$

The resonant frequencies of the trapezoidal cantilever at the temperature range of 30 to 55 °C are substituted into Eq. (1) to calculate dimethyl silicone densities. Table 3 shows the measured densities of dimethyl silicone and deviations.

Table 3 Reference densities and measured densities of dimethyl silicone

Temperature (°C)	Ref. density (kg/m^3)	Meas. density (kg/m^3)	Deviation (%)
30	754.10	754.10	0
35	749.29	748.67	0.083
40	744.44	743.83	0.082
45	739.55	739.16	0.053
50	734.51	734.01	0.068
55	729.62	729.62	0

Then resonant frequencies of the trapezoidal cantilever immersed in the toluene and toluene reference densities at the temperatures 26 and 34 °C are used to calibrate constants a and b in Eq. (1):

$$a=3.4286\times10^{10},\ b=336.3089. \tag{3}$$

The resonant frequencies of the trapezoidal cantilever at the temperature range of 26 to 34 °C are substituted into Eq. (1) to calculate toluene densities. Table 4 shows the measured densities of toluene and deviations.

Table 4 Reference densities and measured densities of toluene

Temperature (°C)	Ref. density (kg/m^3)	Meas. density (kg/m^3)	Deviation (%)
26	861.31	861.31	0
28	859.44	858.56	0.102
30	857.57	857.12	0.052
32	855.70	855.69	0.001
34	853.83	853.83	0

The resonant frequency of the cantilever is related to its shape and dimension. The modal analyses of trapezoidal and rectangular cantilevers with different sizes are simulated. Modal analysis results show that the first-order resonant frequency of the trapezoidal cantilever is higher than that of the rectangular cantilever. The higher the resonant frequency of the cantilever, the higher the sensitivity of the sensor (Liao *et al.*, 2011). Therefore, the sensitivity of the trapezoidal cantilever sensor is higher than that of the rectangular cantilever sensor. Tables 5 and 6 show the sensitivities of the trapezoidal cantilever sensor immersed in dimethyl silicone and toluene, respectively.

Table 5 Sensitivity of the trapezoidal cantilever sensor immersed in dimethyl silicone

Temperature (°C)	Frequency (Hz)	Meas. density (kg/m^3)	Sensitivity (Hz·m^3/kg)
30	8644.73962	754.10	4.43
35	8668.78029	748.67	4.46
40	8690.36093	743.83	4.50
45	8711.35328	739.16	4.52
50	8734.62692	734.01	4.56
55	8754.64902	729.62	4.56

Table 6 Sensitivity of the trapezoidal cantilever sensor immersed in toluene

Temperature (°C)	Frequency (Hz)	Meas. density (kg/m^3)	Sensitivity (Hz·m^3/kg)
26	8081.22428	861.31	7.72
28	8102.44811	858.56	7.77
30	8113.64067	857.12	7.83
32	8124.83397	855.69	7.84
34	8139.41582	853.83	7.84

6 Conclusions

Based on simulation results, the trapezoidal cantilever with optimal size is selected to measure the densities of dimethyl silicone and toluene at the temperature ranges of 30 to 55 °C and 26 to 34 °C, respectively. The deviations of the measured densities of dimethyl silicone and toluene by the sensor from their density references are less than 0.1% and 0.2%, respectively. The sensitivities of the trapezoidal cantilever sensor immersed in dimethyl silicone and toluene are more than 4 and 7 Hz·m^3/kg, respectively. Therefore, the trapezoidal cantilever density sensor has a good performance.

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