Journal of Zhejiang University-SCIENCE C (Computers & Electronics)
ISSN 1869-1951 (Print); ISSN 1869-196X (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Supply chain network design under uncertainty with new insights from contracts*

Mohammad Mohajer TABRIZI, Behrooz KARIMI†‡

(*Department of Industrial Engineering, AmirKabir University of Technology, Tehran 1591634311, Iran*)

†E-mail: b.karimi@aut.ac.ir

Received Oct. 6, 2013; Revision accepted June 23, 2014; Crosschecked Nov. 13, 2014

Abstract In this paper, the classical problem of supply chain network design is reconsidered to emphasize the role of contracts in uncertain environments. The supply chain addressed consists of four layers: suppliers, manufacturers, warehouses, and customers acting within a single period. The single owner of the manufacturing plants signs a contract with each of the suppliers to satisfy demand from downstream. Available contracts consist of long-term and option contracts, and unmet demand is satisfied by purchasing from the spot market. In this supply chain, customer demand, supplier capacity, plants and warehouses, transportation costs, and spot prices are uncertain. Two models are proposed here: a risk-neutral two-stage stochastic model and a risk-averse model that considers risk measures. A solution strategy based on sample average approximation is then proposed to handle large scale problems. Extensive computational studies prove the important role of contracts in the design process, especially a portfolio of contracts. For instance, we show that long-term contract alone has similar impacts to having no contracts, and that option contract alone gives inferior results to a combination of option and long-term contracts. We also show that the proposed solution methodology is able to obtain good quality solutions for large scale problems.

Key words: Supply chain network design, Contracts, Uncertainty, Conditional value at risk
doi:10.1631/jzus.C1300279 **Document code:** A **CLC number:** TP393; F22

1 Introduction

A supply chain is a system of suppliers, manufacturers, distributors, retailers, and customers. In this system, materials move from suppliers to customers, and information flows in both directions (Simchi-Levi *et al.*, 2007). Since uncertainty plays a strong role in today's world, the importance of effectively connecting entities in a supply chain has been highlighted in recent years, and is known in the literature as the supply chain network design (SCND) problem. A crucial component of SCND is the way in which manufacturers are tied to their suppliers to ensure the stable supply of raw materials. These ties are conventionally constructed using contracts among manufacturers and suppliers. The uncertainty that dominates today's decision making process affects many SCND parameters, such as plant capacities, demand rates, transportation fees, spot prices, and exchange rates. The existence of uncertainty has a two-fold effect on the importance of contracts, since the stationary provision of products to customers is under pressure from highly uncertain parameters. One day a supplier fails to fulfill an order, another day transportation prices go up because of an increase in fuel prices. Despite these effects, it seems that the literature on SCND has not yet paid enough attention to the incorporation of contracts, a factor that can affect the success of the design process. Many contracts have been introduced for different situations and for various industries. Examples include long-term, option,

‡ Corresponding author
* Project supported by the Faculty of Industrial Engineering and Management Systems, AmirKabir University of Technology, Iran
ORCID: Mohammad Mohajer TABRIZI, http://orcid.org/0000-0002-9117-0632; Behrooz KARIMI, http://orcid.org/0000-0003-3556-8643

and capacity reservation contracts. In a long-term contract, the supplier guarantees to send the manufacturer a pre-specified amount of products at an agreed-upon price by a pre-determined time in the future. An obvious disadvantage of this contract is rooted in the inventory holding costs such as depreciation and warehousing. To illustrate, in 2001, Cisco, a popular supplier in the communication industry, encountered a drop in its demand rates. However, due to having long-term contracts with its suppliers, Cisco was forced to write-off $2.2 billions of its inventory and fire 8500 of its personnel. In contrast, in an option contract, the buyer has the right to receive the product in future up to a pre-specified amount. The buyer pays a fraction of the price at the time the contract is agreed (premium fee) and the remainder at the time an order is placed with the supplier (execution fee). Although an option contract has the advantage of hedging the buyer against risks such as demand decline, the total price in this contract is higher than that in a long-term contract. Note that for some commodity parts there may always be the opportunity to supply these materials from the spot market, which operates under completely uncertain conditions because of variable spot prices and even availability of products. However, to hedge against many types of such uncertainties, some companies, like HP, have used a portfolio of these contracts successfully to avoid the disadvantages of an uncertain environment while avoiding contract-specific problems. In 2000, the demand for flash memory grew at a high rate, but uncertainty in supply along with uncertain spot prices led HP to sign long-term contracts for 50%, and option contracts for 30% of its demand. The remaining supply was obtained by purchasing from the spot market (Simchi-Levi *et al.*, 2007). In this paper, we consider a multi-site manufacturer who supplies a single product to a set of distributors (warehouses) and from there to end users (customers). This product needs a raw material (e.g., a core component) that should be contracted to a set of suppliers, mainly with long-term and option contracts, and unsatisfied demand will be supplied from the spot market. The capacity of suppliers, plants and warehouses, transportation fees, and spot purchasing prices are uncertain and stochastic with known distributions. The location and number of suppliers and customers are known, however, these items for plants and warehouses should be determined by the model. We have modeled this problem in two cases: a risk-neutral model and a risk-averse model. In both cases the modeling approach used is a two-stage stochastic programming. However, in the risk-neutral model the objective is to minimize average SCND costs, while in the risk-averse model, the objective is to minimize a measure of cost risk in addition to average cost as a way of forcing risk-averse decision making, which seems to be logical in the current uncertain environment. A solution procedure based on sample average approximation (SAA) is used to handle these stochastic models. Sensitivity analysis of model parameters was carried out to show the usefulness of the proposed model, and especially to highlight the important role of contracts in SCND decisions. Another contribution of this paper is the incorporation of risk in this problem. Although the application of a risk measure is not new in general, in the context of SCND, the impact of risk measures has not been studied thoroughly. More specifically, we are unaware of any such application in the context of SCND problems while considering contracts in the model.

2 Literature review

The SCND problem has attracted the attention of many researchers for years. The main aspect of this problem is related to the classical location-allocation problem, in which a set of manufacturing plants and warehouses should be installed and then allocated to customers with minimum cost, combined with a transportation problem. Many researchers have considered this problem in a deterministic environment so that all parameters related to the problem are known in advance. Thanh *et al.* (2010) addressed the problem of designing and planning a multi-period, multi-echelon, multi-commodity supply chain whereby each product has a bill of material and all parameters are deterministic. Each plant and warehouse has limited capacity and lower and upper bounds on the utilization level. The comprehensive model they developed captures many modeling issues including: opening, closing, or expanding facilities, selecting suppliers, and planning the distribution flow. As for the solution strategy, a heuristic based on successive linear relaxation of original mixed integer program (MIP) and rounding strategies was proposed. Nagurney (2010) considered the problem of designing and redesigning a supply chain from a different modeling approach, i.e., variational inequalities. The research investigated optimal levels of capacity, production quantities, storage volumes, and shipments. This approach has the merit of handling congestion in the network as well as nonlinear and nonseparable cost structures.

Badri *et al.* (2013) developed a new mathematical model for multi-echelon, multi-commodity dynamic SCND considering expansions of the supply chain according to cumulative net profits and funds supplied by external sources, in contrast to the common approach in which expansion is restricted to a predetermined fund or to a fixed number of facilities. Other aspects include: bounds on utilization rates for facilities, public warehouses, and potential private warehouses. Correia *et al.* (2013) presented two new models for a two-echelon dynamic system. The main features are: location of new facilities, installation of warehouses, and distribution of products. Decisions are bound to a given budget and different product families are considered. While the first model is a cost minimization model, the second is a profit maximization model. To investigate the implications of the choice of performance measure on the network, an extensive computational study was also presented.

In other stream of SCND papers, researchers have tried to make the decision making process more realistic through the incorporation of uncertainty in their models. Santoso *et al.* (2005) addressed a single-period, single-commodity network design problem in which a two-stage model determines which facilities should be opened, in addition to the technology of each facility. They also proposed a hybrid of Benders decomposition and SAA as the solution approach (Santoso, 2003). Tiwari *et al.* (2010) proposed a novel model for SCND with a cost minimization objective satisfying uncertain demands at a specified service level. Other features include: the availability of various transportation options for manufacturers, warehouses, and distribution centers, nonlinearity of transportation and inventory holding costs, and consideration of economies of scale for these costs. Then a solution procedure based on the Taguchi method hybridized with an artificial immune system was proposed. Pan and Nagi (2010) considered an SCND with the new feature of considering emerging new markets while demand is uncertain in an agile manufacturing setting. The main decisions are: selection of facilities, alliances among different facilities, production, and distribution. A robust scenario approach was used to handle the uncertainty of demand with the objective consisting of the mean total costs, cost dispersion, and the mean penalty for unmet demand. A heuristic based on the *k*-shortest path was also suggested. Georgiadis *et al.* (2011) proposed a new mathematical model for SCND with multiple products flowing in the network in multiple time periods and under uncertain demand. The model considers time-dependent uncertainties in demand through scenario planning. A case study regarding the establishment of a Europe-wide supply chain was presented to show the applicability of the proposed model. Das (2011) introduced a new model for the integration of capacity, product mix, distribution, and input supply flexibility in strategic supply chain design. Other novelties of the paper include: modeling capacity, product mix, customer service level, and input flexibility based on quantifiable performance measures. It was the first paper to include product mix flexibility. Rajgopal *et al.* (2011) proposed a new two-stage stochastic model for the design and operation of a remnant industry, motivated by the metal industry. Uncertainties arise in several aspects: demand rate, raw material costs, cutting costs, salvage costs, capacity of facilities, and transportation fee. The first stage decisions concern fixed cost of operating facilities and the second stage costs are composed of metal cutting and distribution. They proposed a solution based on a modified version of the *L*-shaped method.

Goetschalckx *et al.* (2013) considered an SCND problem with uncertainties in demand, capacities of suppliers, manufacturers, warehouses, and fixed location costs. Uncertainty is modeled as a set of scenarios. The risk of the system is modeled as the two-sided standard deviation of the profits of the various scenarios. They showed that the common risk mitigation strategy to increase the overall capacity may not always have the desired effect. Tabrizi and Razmi (2013) considered an SCND problem in a multi-commodity, multi-stage, multi-capacity, and multi-source mode with sources of uncertainty in supply, demand, and processing sides, and modeled it using fuzzy logic. The non-linear MIP model was then solved using Benders decomposition.

Despite the fact that contracts may have very important effects on designing supply chains in an uncertain environment, most authors have ignored these effects. Melo *et al.* (2009), in their review paper, concluded that there is a gap in the literature related to consideration of decisions in a supply chain such as transportation modes, routing, and purchasing (contracts). Wever *et al.* (2012) noted that most previous studies examined only supply/demand risks and contract issues in the context of bilateral relations. They also showed that, when supply chain actors follow the instructions from these traditional models, their risks may increase rather than decrease. Further information on supply chain contracts may be found by Feng *et al.* (2013). In addition, most studies suppose that a pre-specified amount of product will reach a facility from its predecessor facility, which implies long-term contracts. Paksoy *et al.* (2013) proposed a novel model, consisting of multiple suppliers, manufacturers, distribution centers, and retailers, and dealt with the effects of product quality on supply chain design. Raw materials from suppliers are divided into high, low, or bad quality. Low quality material is cheaper but requires additional rework time and cost and could cause the failer of reaching target quality for manufacturers, and thus may affect the ability to satisfy demand. To hedge against these uncertainties the manufacturers use option contracts. The model answers the questions of with which suppliers these contracts should be signed and for which quality level. Although the authors declared that they used option contracts, actually they meant long-term contracts since in option contracts there is an option to buy or not to buy the amount agreed in the contract. However, in this paper no such concept was observed. Xu and Nozick (2009) considered a multi-period single-commodity supply chain with demand as the only uncertain parameter. Manufacturers belong to a single owner, and the problem considered is basically the supplier selection in conjunction with customer allocation and tries to find the right amount of raw material to purchase from each supplier using long-term and option contracts while unmet demand is purchased from the spot market with fixed prices. The modeling approach used is a two-stage stochastic programming. They used a hybrid solution strategy based on the standard *L*-shaped algorithm and Lagrangean relaxation. The main advantages of our paper over this work are: First, our model considers the main design decisions in a standard SCND problem, i.e., location of suppliers, manufacturers, and warehouses, in addition to allocation. However, the model of Xu and Nozick (2009) is mainly a supplier selection and does not consider warehouses and distribution decisions. Second, we have relaxed the assumption of deterministic values for many parameters that are exposed to uncertainty in reality. In our model, the capacity of all facilities including suppliers, manufacturers, and warehouses, in addition to demand and spot prices, is uncertain. Also, transportation fees from all facilities to all others are considered uncertain in our model, and nowadays are subject to the oscillation of fuel prices. The solution procedure of Xu and Nozick (2009) is suitable only for discrete distributions where the number of scenarios is not too large. However, this is not the case for our solution strategy. Feng *et al.* (2013) proposed a two-stage stochastic programming model for coordinated contract design in a supply chain. They considered a multi-site manufacturer with uncertain capacity, which supplies various raw materials from multiple suppliers, with stochastic capacity and price, and, in turn, supplies multiple products to customers with random demand and market price. The manufacturer signs contracts with both suppliers and customers, which is a new feature in this study. Spot purchasing and selling is also allowed. Four types of contracts from the manufacturer's side are used: (1) price only; (2) periodical minimum commitment; (3) periodical commitment with order band; (4) periodical stationary commitment. The only contract with suppliers is 'total minimum quantity commitments'. Another uncertain parameter, which is a novelty in their work, is that the manufacturer presents the set of contracts to buyers and the behavior of these buyers can be approximated through the distribution of the customer's choice. Contract selections construct the first stage variables, i.e., whether to offer a type of contract to customers or sign a special contract with suppliers. Production and distribution constitute the second stage variables. The three main weaknesses of this paper are: First, there are no network design decisions regarding the location of manufacturers and distribution centers in the model. Second, as pointed out by the authors, they have used only long-term contracts with suppliers, although option contracts have more power to hedge against uncertainty. Third, the capacities of suppliers and transportation fees have been regarded as certain. An important drawback of previous two papers (Xu and Nozick, 2009; Feng *et al.*, 2013) is that they have made their decisions without considering the associated risks, which makes their solutions unstable against possible bad scenarios. However, we have refined this by incorporation of a risk measure in the objective of our model. Furthermore, the risk measure we used is a coherent risk measure. Table 1 summarizes the main features of our paper in contract to the abovementioned literature with respect to design and modeling features.

3 Problem description

A multi-site manufacturer that supplies a single product to a set of warehouses and from there to its customers needs to locate its manufacturing plants

Table 1 Comparison of our research with previous studies, based on design and modeling aspects

	<=2010					2011			2013						This paper
	Xu and Nozick	Tiwari *et al.*	Thanh *et al.*	Nagurney	Pan and Nagi	Rajgopal *et al.*	Das	Georgiadis *et al.*	Feng *et al.*	Correia *et al.*	Badri *et al.*	Tabrizi and Razmi	Paksoy *et al.*	Goetschalckx *et al.*	
Decisions															
Supplier selection	✓	✓	✓	✓	✓	-	✓	-	✓	-	✓	✓	✓	✓	✓
Location															
-Plants	-	✓	✓	✓	✓	✓	✓	-	-	-	-	✓	-	✓	✓
-Warehouses	-	✓	✓	✓	✓	-	✓	✓	-	✓	-	✓	-	✓	✓
Operational															
-Production		✓	✓	✓	✓	✓	✓	✓	✓	-	✓	✓	✓	✓	✓
-Distribution		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Uncertainty															
Demand	-	✓	✓	-	✓	✓	✓	-	✓	-	-	✓	-	✓	✓
Capacity															
-Suppliers	-	✓	-	-	-	-	-	-	✓	-	-	✓	-	✓	✓
-Plants	-	✓	-	-	-	✓	-	-	✓	-	-	✓	-	✓	✓
-Warehouses	-	✓	-	-	-	-	-	-	✓	-	-	✓	-	✓	✓
Transportation fee	-	-	-	-	-	✓	-	-	-	-	-	-	-	✓	✓
Spot price	-	-	-	-	-	-	-	-	-	-	-	-	-	✓	✓
Other sources	-	-	-	-	-	✓	-	-	✓	-	-	-	-	✓	-
Architecture															
Static	-	✓	-	✓	-	✓	-	-	-	-	-	✓	✓	✓	✓
Dynamic	✓	-	✓	-	✓	-	✓	✓	✓	✓	-	-	-	-	-
Products															
-Single	✓	✓	-	✓	✓	-	-	-	-	-	-	-	✓	-	✓
-Multiple	-	-	✓	-	-	✓	✓	✓	✓	✓	✓	✓	-	✓	-
Risk aversion															
Coherent	-	-	-	-	-	-	-	-	-	-	-	-	-	-	✓
Non-coherent	-	-	-	-	✓	-	-	-	-	-	-	-	-	✓	-
Contracts															
Long term	✓	-	-	-	-	-	-	-	✓	-	-	-	✓	-	✓
Option	✓	-	-	-	-	-	-	-	✓	-	-	-	-	-	✓
Spot purchase	✓	-	-	-	-	-	-	-	✓	-	-	-	-	-	✓
Other types	-	-	-	-	-	-	-	-	✓	-	-	-	-	-	-

and warehouses and allocate customers to warehouses. This product needs a strategic raw material or raw material, the supply of which should be contracted to a set of suppliers, mainly with long-term and option contracts, and unsatisfied demand would be outsourced to the spot market. The capacity of suppliers, plants, and warehouses, transportation fees, and spot purchasing prices are uncertain and stochastic with known distributions. Each supplier proposes a contract list which specifies the price for long-term contracts and another list showing different prices for the volume purchased under option contracts. In addition to location and allocation decisions, i.e., the design phase, the manufacturer needs to determine which supplier should supply each plant and with which contract, option, long-term, or both? Note that as the capacity of suppliers, plants, warehouses, and the demand are uncertain and considering the fact that contracts signed with suppliers constrain available products, it is assumed that unmet demand will be outsourced from the spot market but with stochastic prices. The volume of materials flowing from suppliers to plants, from plants to warehouses, and from warehouses to customers should be determined as well, keeping in mind that transportation prices are stochastic. Modeling approach used here is a two-stage stochastic programming. Our first model is risk-neutral, i.e., the objective function considers only the average SCND cost, while the second model incorporates risk of decisions with a risk measure. In the two-stage stochastic programming, there is a timeline before which the uncertain parameters are unknown and after which they will be known. The decisions before this timeline are called the first stage and those after the timeline are called second stage. In this approach, the objective function consists of the costs due to the first stage decisions plus expected value of the costs due to the second stage decisions, assuming that the first stage decisions are fixed. Suppose that vector $\boldsymbol{y}$ is the aggregate of variables in the first stage with the cost vector $\boldsymbol{d}$, and $\boldsymbol{x}$ is the aggregate of variables in the second stage with cost vector $\boldsymbol{c}$. Also, assume that $\boldsymbol{\xi}$ represents the aggregate of uncertain parameters with realizations ξ_n. We have

$$\min_{\boldsymbol{y}} \left\{ \boldsymbol{d}^{\mathrm{T}} \boldsymbol{y} + E\left(Q(\boldsymbol{y}, \xi_n)\right) \right\} \text{ s.t. } \boldsymbol{A}\boldsymbol{y} \le \boldsymbol{b}, \quad (1)$$

in which $Q(\boldsymbol{y}, \xi_n)$ is defined as follows:

$$Q(\boldsymbol{y}, \xi_n) = \min_{\boldsymbol{x}} \{\boldsymbol{c}_n^{\mathrm{T}} \boldsymbol{x}\} \text{ s.t. } \boldsymbol{W}_n \boldsymbol{x}_n = \boldsymbol{h}_n - \boldsymbol{T}_n \boldsymbol{y}. \quad (2)$$

We may call problem 1 the first stage problem and problem 2 the second stage problem in our implementation. The notations used in our mathematical model and the corresponding descriptions are illustrated in Table 2.

3.1 Model assumptions

The main assumptions of this research are listed as follows:

1. There is a single product and each unit of the product is made out of one unit of raw materials.

2. The locations and numbers of suppliers and customers are known, respectively.

3. Each supplier has proposed a price for long-term contracts and an option list for price-volume pairs (the volume-up-to levels that plants are allowed to order).

4. Unmet demand will be outsourced from the spot market.

5. Capacities of all facilities, transportation fees, demand, and spot prices are stochastic.

6. A scenario based approach is used to model uncertainty (although this assumption does not restrict us due to an adapted solution strategy, i.e., SAA, as will be explained later).

7. The sets of locations for plants and warehouses are known a priori.

3.2 Case I: risk-neutral two-stage stochastic model

In this case, and in case II, the set of first stage variables consists of binary variables corresponding to the act of engaging with each supplier, buying an option level from option list, installing a plant, or establishing a warehouse in potential locations, and continuous variables for long-term contracts signed with suppliers. Second stage variables consist of distribution decisions, i.e., the amount of products that flow from suppliers to plants under different contracts and scenarios or the amount of products produced at each plant and sent to warehouses and from there to customers under different scenarios. Also, products that should be outsourced under each scenario will be

Table 2 Illustration for notations used in our mathematical model

Notation	Description
Set	
S	Set of suppliers indexed by s
P	Set of possible plants indexed by p
W	Set of possible warehouses indexed by w
C	Set of customers indexed by c
I	Set of option levels indexed by i
N	Set of scenarios indexed by n
Parameter	
β	Number of available suppliers
Fo_{is}	Premium cost of buying option i from supplier s
Fl_s	Unit cost of purchasing an item from supplier s with long-term contract
Fs_s	Fixed cost to engage with supplier s
Fp_p	Fixed cost for establishing a plant in site p
Fw_w	Fixed cost for establishing a warehouse in site w
Tsp_{spn}	Unit transportation cost from supplier s to plant p in scenario n
Tpw_{pwn}	Unit transportation cost from plant p to warehouse w in scenario n
Twc_{wcn}	Unit transportation cost from warehouse w to customer c in scenario n
Cs_{sn}	Total capacity of supplier s in scenario n
Cp_{pn}	Total capacity of plant p in scenario n
Cw_{wn}	Total capacity of warehouse w in scenario n
eo_s	Unit exercise cost for using option contract from supplier s
go_i	Available items under option level i from each supplier
P_{cn}	Spot price of final product to supply customer c in scenario n
d_{cn}	Demand from customer c in scenario n
Variable	
ys_s	1 if we decide to engage with supplier s, and 0 otherwise
yp_p	1 if we decide to establish a plant in site p, and 0 otherwise
yw_w	1 if we decide to establish a warehouse in site w, and 0 otherwise
yo_{is}	1 if we decide to buy option level i from sup plier s, and 0 otherwise
yl_s	Amount we decide to buy from supplier s with long-term contract
xo_{spn}	Amount sent from supplier s to plant p in scenario n with option contract
xl_{spn}	Amount sent from supplier s to plant p in scenario n with long-term contract
xpw_{pwn}	Amount sent from plant p to warehouse w in scenario n
xwc_{wcn}	Amount sent from warehouse w to customer c in scenario n
z_{cn}	Unmet demand of customer c in scenario n satisfied by purchasing from the spot market

determined. The two-stage model for case I is as follows:

The first stage problem:

$$\min_{y} \left\{ \sum_s \mathrm{Fs}_s \mathrm{ys}_s + \sum_{i,s} \mathrm{Fo}_{is} \mathrm{yo}_{is} + \sum_s \mathrm{Fl}_s \mathrm{yl}_s + \sum_p \mathrm{Fp}_p \mathrm{yp}_p + \sum_w \mathrm{Fw}_w \mathrm{yw}_w + E(Q(\boldsymbol{y},\xi)) \right\} \quad (3\text{-}0)$$

$$\text{s.t.} \sum_i \mathrm{yo}_{is} \le \mathrm{ys}_s,\ \forall s \quad (3\text{-}1)$$

$$\text{and} \begin{cases} \sum_s \mathrm{ys}_s \le \beta, \\ \mathrm{ys}_s,\ \mathrm{yo}_{is},\ \mathrm{yp}_p,\ \mathrm{yw}_w \in \{0,1\},\ \mathrm{yl}_s \ge 0. \end{cases} \quad (3\text{-}2)$$

The second stage problem:

$$\min_{x,z} \left\{ \sum_{s,p,n} \mathrm{Tsp}_{spn} \mathrm{xl}_{spn} + \sum_{s,p,n} (\mathrm{eo}_s + \mathrm{Tsp}_{spn}) \mathrm{xo}_{spn} + \sum_{p,w,n} \mathrm{Tpw}_{pwn} \mathrm{xpw}_{pwn} + \sum_{w,c,n} \mathrm{Twc}_{wcn} \mathrm{xwc}_{wcn} + \sum_{c,n} P_{cn} z_{cn} \right\} \quad (3\text{-}3)$$

$$\text{s.t.} \sum_p (\mathrm{xl}_{spn} + \mathrm{xo}_{spn}) \le \mathrm{Cs}_{sn} \mathrm{ys}_s,\ \forall s,n, \quad (3\text{-}4)$$

$$\sum_w \mathrm{xpw}_{pwn} \le \mathrm{Cp}_{pn} \mathrm{yp}_p,\ \forall p,n, \quad (3\text{-}5)$$

$$\sum_c \mathrm{xwc}_{wcn} \le \mathrm{Cw}_{wn} \mathrm{yw}_w,\ \forall w,n, \quad (3\text{-}6)$$

$$\sum_s (\mathrm{xl}_{spn} + \mathrm{xo}_{spn}) = \sum_w \mathrm{xpw}_{pwn},\ \forall p,n, \quad (3\text{-}7)$$

$$\sum_p \mathrm{xpw}_{pwn} = \sum_c \mathrm{xwc}_{wcn},\ \forall w,n, \quad (3\text{-}8)$$

$$\sum_p \mathrm{xo}_{spn} \le \sum_i \mathrm{go}_i \mathrm{yo}_{is},\ \forall s,n, \quad (3\text{-}9)$$

$$\sum_p \mathrm{xl}_{spn} \le \mathrm{yl}_s,\ \forall s,n, \quad (3\text{-}10)$$

$$\sum_w \mathrm{xwc}_{wcn} + z_{cn} \ge d_{cn},\ \forall c,n, \quad (3\text{-}11)$$

$$\mathrm{xo}_{spn},\ \mathrm{xl}_{spn},\ \mathrm{xpw}_{pwn},\ \mathrm{xwc}_{wcn},\ z_{cn} \ge 0, \quad (3\text{-}12)$$

where (3-4)–(3-6) are capacity constraints for suppliers, plants, and warehouses, respectively, (3-7) and (3-8) are flow conservation constraints, (3-9) and (3-10) are contractual constraints, (3-11) is the demand constraint, and (3-12) is the non-negativity constraint.

With respect to the first stage problem, the objective function consists of fixed costs for engaging suppliers, a premium option cost for the option level acquired, a purchasing cost for long-term contracts, and a fixed cost for establishing plants and warehouses. Constraint (3-1) ensures that only one of the capacity levels in the option list should be reserved from each supplier, if we have any relation at all with that supplier. Constraints (3-2) enforce the number of suppliers to be less than a certain number. In the second stage, the objective is comprised of four terms: (1) the cost of carrying goods from suppliers to plants under different contracts, i.e., goods with long-term or option contracts, (2) the cost of exercising options purchased, (3) the cost of carrying goods from plants to warehouses, and (4) the cost of carrying goods from warehouses to customers. Constraint (3-4) states that the total items bought from a supplier under long-term and option contracts should not exceed its capacity under different scenarios, provided that we have engaged with that supplier. Constraint (3-5) ensures that the amount of goods produced at plants is within the capacity of plants under different scenarios, and constraint (3-6) limits the amount of products that flow from warehouses to less than their capacity levels in various scenarios. Constraints (3-7) and (3-8) guarantee that the total amount of goods entering a plant or warehouse is equal to the total amount leaving. Constraints (3-9) and (3-10) obligate the circumstances of the contracts signed. Specifically, constraint (3-9) mandates the option exercised in all scenarios to be less than the option level purchased and constraint (3-10) ensures that the amount sent from a supplier to all plants, under long-term contract is less than the amount agreed upon. Constraint (3-11) requires the demand for each customer in each scenario to be satisfied, either with products from warehouses or from the spot market.

3.3 Case II: risk-averse model

Traditional two-stage models consider expectation as the main preference measure to compare random variables. This strategy results in risk-neutral models. In decision theory, an important issue is the comparison of random variables. For these variables, one should take the dispersion effect into consideration, in addition to the location effect. In other words, both the mean and variability of random variables should be considered. This had led to the important concept of risk measures. Of course, merely emphasizing the location effect may result in solutions with high variability and emphasizing only the dispersion effect may result in too conservative solutions. Thus, it is conventional to use mean-risk models as follows:

$$\min_{y}\left\{E(\mathcal{F}(\boldsymbol{y},\xi))+\lambda\rho(\mathcal{F}(\boldsymbol{y},\xi))\right\}, \tag{4}$$

where ρ is the measure of variability, i.e., the risk measure, λ is the risk preference coefficient stated by the decision maker. Traditionally, variance was used as the measure of variability; however, this risk measure has some major deficiencies. For instance, it considers variability in both tails of the distribution of a random variable, but when we are talking about costs we do not care about the left tail. Another weakness arises from the nonlinearity of this measure. These and other deficiencies led us to use a more suitable and fairly recent measure, CVaR. This measure quantifies the mean of values in the distribution function of a random variable $\mathcal{Z}$ that exceed a certain point, as illustrated below (Rockafellar and Uryasev, 2000):

$$\mathrm{CVaR}_{\alpha}(\mathcal{Z})=E(\mathcal{Z}|\mathcal{Z}\geq\mathrm{VaR}_{\alpha}(\mathcal{Z})),$$

in which $\mathrm{VaR}_{\alpha}(\mathcal{Z})$ denotes a point at which α percent of the distribution of $\mathcal{Z}$ is to the left of it. Rockafellar and Uryasev (2000) gave a more tractable version of this definition:

$$\mathrm{CVaR}_{\alpha}(\mathcal{Z})=\inf_{\eta}\left\{\eta+\frac{1}{1-\alpha}E([\mathcal{Z}-\eta]_{+})\right\}, \tag{5}$$

where $[a]_{+}=\max(a, 0)$. Therefore, in this paper, we are looking to solve the following mean-risk model:

$$\min_{y}\left\{E(\mathcal{F}(\boldsymbol{y},\xi))+\lambda\mathrm{CVaR}_{\alpha}(\mathcal{F}(\boldsymbol{y},\xi))\right\}. \tag{6}$$

In our two-stage model, however,

$$\mathcal{F}(\boldsymbol{y},\xi)=\boldsymbol{d}^{\mathrm{T}}\boldsymbol{y}+Q(\boldsymbol{y},\xi)\ \text{s.t.}\ \boldsymbol{A}_1\boldsymbol{y}\leq\boldsymbol{b}_1.$$

Noyan (2012) showed that the general two-stage model for discrete (scenario-based) random variables could be written in the following linear form:

$$\min_{y}\left\{(1+\lambda)\boldsymbol{d}^{\mathrm{T}}\boldsymbol{y}+\sum_{n}p_{n}\boldsymbol{c}_{n}^{\mathrm{T}}\boldsymbol{x}_{n}+\lambda\left(\eta+\frac{1}{1-\alpha}\sum_{n}p_{n}v_{n}\right)\right\} \quad (7\text{-}0)$$

$$\text{s.t.}\ \boldsymbol{W}_{n}\boldsymbol{x}_{n}=\boldsymbol{h}_{n}-\boldsymbol{T}_{n}\boldsymbol{y},\ \forall n, \quad (7\text{-}1)$$

$$v_{n}\geq\boldsymbol{c}_{n}^{\mathrm{T}}\boldsymbol{x}_{n}-\eta,\ \forall n, \quad (7\text{-}2)$$

$$\boldsymbol{A}_{1}\boldsymbol{y}\leq\boldsymbol{b}_{1}, \quad (7\text{-}3)$$

$$\boldsymbol{x}_{n},v_{n}\geq 0,\ \forall n, \quad (7\text{-}4)$$

$$\eta\in\mathbb{R}. \quad (7\text{-}5)$$

In this formulation, p_n denotes the probability of occurrence of the nth scenario. Risk aversion here is modeled in the form of a mean-risk model. For convenience, note that in the next sections, as stated by Noyan (2012), since the variable η does not depend on scenarios while variable v_n does, we can consider η as the first stage and v_n as the second stage auxiliary variables. Define

$$\tilde{\mathcal{F}}(\boldsymbol{y},\eta,\boldsymbol{\xi})=(1+\lambda)\boldsymbol{d}^{\mathrm{T}}\boldsymbol{y}+\lambda\eta+\tilde{\mathcal{Q}}(\boldsymbol{y},\eta,\xi^{n}) \quad (8)$$

$$\text{s.t. } \boldsymbol{A}_{1}\boldsymbol{y}\leq\boldsymbol{b}_{1} \text{ and } \eta\in\mathbb{R},$$

where

$$\tilde{\mathcal{Q}}(\boldsymbol{y},\eta,\xi^{n})=\min_{\boldsymbol{x},v}\left\{\boldsymbol{c}_{n}^{\mathrm{T}}\boldsymbol{x}_{n}+\frac{\lambda}{1-\alpha}v_{n}\right\}$$

$$\text{s.t. } \boldsymbol{W}_{n}\boldsymbol{x}_{n}=\boldsymbol{h}_{n}-\boldsymbol{T}_{n}\boldsymbol{y} \text{ and } v_{n}-\boldsymbol{c}_{n}^{\mathrm{T}}\boldsymbol{x}_{n}\geq-\eta.$$

Therefore, the mean-risk model (6) could be redefined in the same way as traditional risk-neutral models, as follows:

$$\min_{y}\ \{(1+\lambda)\boldsymbol{d}^{\mathrm{T}}\boldsymbol{y}+\lambda\eta+E(\tilde{\mathcal{Q}}(\boldsymbol{y},\eta,\xi^{n}))\} \quad (9)$$

$$\text{s.t. } \boldsymbol{A}_{1}\boldsymbol{y}\leq\boldsymbol{b}_{1} \text{ and } \eta\in\mathbb{R}.$$

In the following, we will show that considering risk in the model has a great impact on the cost of supply chain. Also, the impact of parameter λ will be discussed.

4 Solution procedure

In this section we illustrate our strategy to solve the problem. In stochastic models like ours, it is necessary to determine whether the distribution of the stochastic parameters is continuous or discrete. The problem with continuous functions is that we may not be able to take expectations and they may not be good approximations for real distributions. The problem with discrete distributions, if we consider them as real distributions, is the large number of scenarios in real cases. To illustrate the importance of this, we discuss the example by Santoso *et al.* (2005). Consider a supply chain with 50 facilities, which is normal for realistic cases. Each facility has just one uncertain parameter. Suppose that this uncertain parameter has only three scenarios. In this situation, we have to deal with $3^{50}\cong 7\times 10^{23}$ scenarios, which is far more than current technology can handle. In this paper, we have adopted SAA to handle a very large, theoretically infinite number of scenarios (Santoso, 2003). This approach also enables us to use continuous distributions, since it draws samples from the original distribution, whether it is discrete or continuous. This procedure has its roots in Monte Carlo simulation and statistics, and we will have to deal with a smaller sample of the real problem in each stage of the procedure. In the next subsection we will illustrate the procedure of SAA in detail and its adaption to our problem.

4.1 Sample average approximation

In a stochastic program like Eq. (1) or (9), the main difficulty arises from the expected value terms, since we do not have the function $Q(\boldsymbol{y}, \xi_n)$ or $\tilde{\mathcal{Q}}(\boldsymbol{y},\eta,\xi^{n})$ explicitly and in a closed analytical form. Even if we did, computing such an expectation consists of multiple integrals or solving too many linear programs. To avoid these difficulties, a good idea is to use the sample average (mean) statistic, instead of the original expectation. To do this, Shapiro and Homem-de-Mello (1998) proposed performing M independent experiments. In each experiment, one must take N independent samples from the main problem. In fact, each sample consists of solving the following linear program, called the sample average problem:

$$\hat{l}_m = \min_y \frac{1}{N}\sum_{n=1}^{N}\tilde{\mathcal{F}}(\boldsymbol{y},\eta,\xi_n^m). \tag{10}$$

We call the optimal solution to this problem $\hat{\boldsymbol{y}}_\text{m}$ or $\hat{\eta}_m$, and it is proven that taking an average over $\hat{l}_m s$ gives a lower bound on the optimal value (Wang, 2007).

$$\overline{l} = \frac{1}{M}\sum_{m=1}^{M}\hat{l}_m.$$

The variance to this estimator is calculated as follows:

$$s_\text{l}^2 = \frac{1}{M(M-1)}\sum_{m=1}^{M}(\hat{l}_m - \overline{l})^2. \tag{11}$$

To obtain a good quality solution, an upper bound is also derived through a feasible first stage solution, such as one of $\hat{\boldsymbol{y}}_m$ and $\hat{\eta}_m s$ obtained in the last step, and we call them $\overline{\boldsymbol{y}}$ and $\overline{\eta}$, respectively. Then a new and large sample will be derived with size N' and we compute the following equation to obtain the sample upper bound:

$$\overline{u} = \frac{1}{N'}\sum_{n=1}^{N'}\tilde{\mathcal{F}}(\overline{\boldsymbol{y}},\overline{\eta},\xi_n). \tag{12}$$

Note that in the above, we are solving N' single-scenario linear programmings (LPs) $\left(\tilde{\mathcal{F}}(\overline{\boldsymbol{y}},\overline{\eta},\xi_n)\right)$ which are easy to handle. Thus, since the accuracy of the sample upper bound increases with the increase in N', we may use a much larger sample size than the one used as N. The suggestion is N'=1000 compared with about 30 or 60 for N. Eventually, the variance of the above estimator is as

$$s_\text{u}^2 = \frac{1}{N'(N'-1)}\sum_{n=1}^{N'}(\tilde{\mathcal{F}}(\overline{\boldsymbol{y}},\overline{\eta},\xi_n) - \overline{u})^2. \tag{13}$$

This procedure continues until the gap between the upper and lower bounds, reaches an a priori determined level. This gap and its corresponding variance for our problem appear as follows:

$$\mu_\text{gap} = \overline{u} - \overline{l}, \tag{14}$$

$$\sigma_\text{gap}^2 = s_\text{u}^2 + s_\text{l}^2. \tag{15}$$

To assess the quality of the solutions obtained so far, Santoso *et al.* (2005) proposed to construct a confidence interval based on the normal distribution, which is depicted as

$$[0, \mu_\text{gap} + z_{\alpha/2}\sigma_\text{gap}]. \tag{16}$$

For our problem, Eqs. (10) and (12) could be expressed, respectively, as

$$\hat{l}_m = \min_y \left\{\frac{1}{N}\sum_{n=1}^{N}(1+\lambda)\boldsymbol{d}^T\boldsymbol{y} + \lambda\eta + \boldsymbol{c}_n^\text{T}\boldsymbol{x}_n + \frac{1}{1-\alpha}v_n\right\}$$
$$\text{s.t. (7-1)–(7-5)},$$
$$\overline{u} = \frac{1}{N'}\sum_{n=1}^{N'}\left((1+\lambda)\boldsymbol{d}^\text{T}\overline{\boldsymbol{y}} + \lambda\overline{\eta} + \tilde{\mathcal{Q}}(\overline{\boldsymbol{y}},\overline{\eta},\xi_n)\right).$$

Note that the above arguments are true for traditional two-stage problems with elimination of terms related to η, v_n, and λ.

5 Computational results

5.1 Test problem generation

To test the above approach on our problem, we generated four classes of random problems and from each class some random instances were derived (Table 3). It is obvious that class D is larger than class C, and class C larger than B, and so forth. Scenarios for all stochastic parameters were generated from a lognormal distribution while deterministic parameters were generated based on a uniform distribution, as illustrated in Table 4. Santoso *et al.* (2005) pointed out some reasons to use a lognormal distribution: First, it generates only positive values for parameters, which is suitable for parameters such as capacities, prices, and demands. Second, Kamath and Pakkala (2002) argued that this distribution is a good approximation for parameters such as demand. It should be clarified that in Table 4, the parameter μ_d represents the average demand for a customer, and CsMean$_s$ represents the average capacity of supplier s. Also, recall that in the process of SAA we need to solve M problems each with size N to obtain a lower bound. Meanwhile, we need to solve a problem with much bigger sample size, i.e., N' to achieve an upper bound on the true problem. In this paper, a sample of size N' is shown to be enough for achieving a good quality solution.

Standard deviations of stochastic parameters were considered as a percentage of their means and shown as θ in Table 4. Experiments were run on a Dell 6400 laptop, using C# in conjunction with CPLEX Concert Technology.

5.2 Quality of stochastic solutions: case I

This section is devoted to a comparison between stochastic and deterministic solutions from the aspect of solution quality. Here, by quality, we mean centrality and diversity of cost distribution. A distribution with higher mean has lower quality and one with a lower standard deviation (SD) has higher quality. To elaborate, we compare the distribution of costs obtained in two modes: First, we solve the stochastic model in case I using SAA, and then we replace the stochastic parameters with their mean and solve the resulting deterministic problem, called mean value problem (MVP). Then we have two sets of first stage decisions at hand (y's), one from solving the stochastic model and the other from MVP. By fixing each set of first stage solutions, i.e., design decisions, we can determine the cost of operating the supply chain under different scenarios by solving the second stage problem. This leads to two sets of SCND cost distributions. By comparing the centrality and diversity of these solutions, we can decide whether the stochastic model or the MVP design leads to a supply chain with less cost (on average). Also, we need to compare these distributions with respect to their stability, i.e., which distribution has a lower SD and therefore a more stable solution? Another aspect of quality is the

Table 3 Characteristics of test problems

Class	$\lvert S\rvert$	$\lvert I\rvert$	$\lvert P\rvert$	$\lvert W\rvert$	$\lvert C\rvert$	Number of samples
A	4	3	2	4	10	2
B	5	4	2	6	20	2
C	10	5	3	8	50	2
D	20	5	3	10	100	2

$\lvert\cdot\rvert$ represents the number of elements in the set

Table 4 Parameter generation in test problems

Parameter	Specification
Fo_{is}	$0.2\times eo_s\times go_i$
Fl_s	~Uniform [10, 15]
Fs_s	~Uniform [10^4, 3×10^4]
Fp_p	~Uniform [10^6, 10^7]
Fw_w	~Uniform [5×10^4, 10^5]
Tc_{spn}	~Lognormal [μ, σ^2] with μ~uniform [0.5, 1] and $\sigma=\theta\mu$
Tc_{pwn}	~Lognormal [μ, σ^2] with μ~uniform [0.5, 1] and $\sigma=\theta\mu$
Tc_{wcn}	~Lognormal [μ, σ^2] with μ~uniform [0.5, 1] and $\sigma=\theta\mu$
$CsMean_s$	$\sim \text{Uniform}\left[0.8\times\left(\frac{\sum_c \mu_{d_c}}{\lvert S\rvert}\right), 1.2\times\left(\frac{\sum_c \mu_{d_c}}{\lvert S\rvert}\right)\right]$
Cs_{sn}	~Lognormal [μ, σ^2] with μ=$CsMean_s$ and $\sigma=\theta\mu$
Cp_{pn}	$\sim \text{Lognormal}\,[\mu,\sigma^2]$ with $\mu \sim \text{Uniform}\left[0.8\times\left(\frac{\sum_c \mu_{d_c}}{\lvert P\rvert}\right), 1.2\times\left(\frac{\sum_c \mu_{d_c}}{\lvert P\rvert}\right)\right]$ and $\sigma=\theta\mu$
Cw_{wn}	$\sim \text{Lognormal}\,[\mu,\sigma^2]$ with $\mu \sim \text{Uniform}\left[0.8\times\left(\frac{\sum_c \mu_{d_c}}{\lvert W\rvert}\right), 1.2\times\left(\frac{\sum_c \mu_{d_c}}{\lvert W\rvert}\right)\right]$ and $\sigma=\theta\mu$
eo_s	~Uniform [18, 20]
go_i	$(i\times0.8\times CsMean_s)/\lvert I\rvert$
P_{cn}	~Lognormal [μ, σ^2] with μ~uniform [800, 900] and $\sigma=\theta\mu$
D_{cn}	~Lognormal [μ, σ^2] with μ~uniform [7×10^5, 10^6] and $\sigma=\theta\mu$
Location	~Uniform [0, 100] and distances are based on Euclidean measure

precision of the SAA solutions obtained, i.e., how much the solution of SAA deviates from the true optimal solution? Since the gap computed above is a random variable itself, we may be able to compare the mean and variance of the gap obtained for different values of M, N, and N', and determine the best values for these parameters to obtain the required precision. We use M=20 to solve the model in case I using SAA, which was shown to be good enough for convergence of the algorithm to a 1% gap. The SD for demand is considered to be 20% of its mean and for other costs, 10% of their means. For the stochastic model, we compute the three best solutions (with respect to the mean of the gaps reached) and calculate descriptive statistics for these solutions. Then these statistics are compared with those of MVP. Table 5 shows results for N=30. The mean costs of the stochastic solutions, shown as SAA1–SAA3 for the three best solutions, are smaller than the cost associated with MVP. This means that, on average, the stochastic model results in SCNDs with less cost. Moreover, the SDs of the stochastic solutions indicate slightly better performance, which can be interpreted as more stable solutions. The range of costs and the maximum (worst case) observed in the cost distribution show a better quality for the stochastic designs. Finally, 'Gap mean' and 'Gap SD', which represent the mean and SD, respectively, of the gap between solutions obtained by solving SAA or MVP and the true optimal solution, show higher quality of solutions from the stochastic model. To validate our observations on the quality of solutions obtained, we constructed a 95% confidence interval on the mean of the costs and the mean of the gaps from the true solutions and, as Table 6 shows clearly, the solutions of the model in case I are statistically superior to that of MVP. In addition, the confidence interval for the mean of the gap becomes tighter as sample size increases, which means that solutions obtained are valid for different sample sizes. The results in Table 5 also show that statistical measures for the three candidate solutions are very similar, suggesting that the solution approach adopted is robust. We also examined the impact of variability of parameters on the quality of solutions. Up to now, results were based on the same amount of variability in uncertain parameters. However, in this section, these values are changed according to the categories in Table 7. Fig. 1 reveals that changing variability enhances the value of stochastic solutions over their deterministic counterparts significantly and, on average, the quality of solutions remains more stable in this case.

Table 5 Cost statistics for stochastic and deterministic solutions (M=20, N=30, N'=1000)

	Cost (billion $)			
	SAA1	SAA2	SAA3	MVP
Mean	1.822	1.823	1.823	2.061
SD	0.480	0.480	0.480	0.494
Range	2.612	2.611	2.612	2.739
Worst case	3.721	3.719	3.720	3.841
Gap mean	0.00007	0.00093	0.00099	0.23875
Gap SD	0.02196	0.02195	0.02195	0.02226

5.3 Quality of stochastic solutions: case II

In this section, we consider a risk-averse model and solve it using SAA. The purpose is to show that,

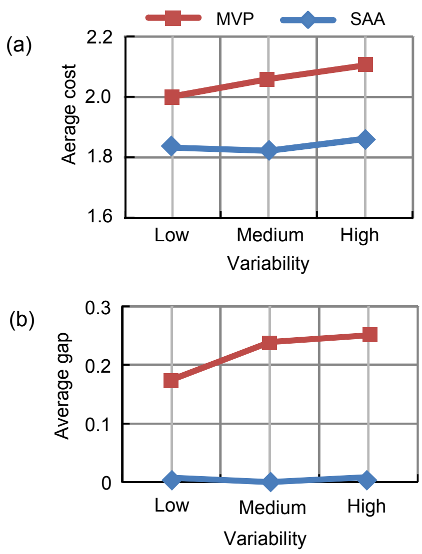

Fig. 1 Impact of changing the variability of parameters on the quality of stochastic solutions vs. MVP
(a) Average total cost; (b) Optimality gap mean

Table 6 The 95% confidence intervals for the mean of the total cost and the mean of the gap (in billion $)

	N=10	N=15	N=20	N=25	N=30	MVP
Cost mean	[1.8, 1.86]	[1.83, 1.89]	[1.79, 1.85]	[1.8, 1.86]	[1.79, 1.85]	[2.03, 2.09]
Gap mean	[0, 0.06]	[0, 0.06]	[0, 0.05]	[0, 0.05]	[0, 0.04]	[0, 0.28]

although solutions of case I were superior to MVP, we have not obtained designs from these solutions that are safeguarded against possible bad scenarios, i.e., risky situations. Again, 20 problems modeled in the form of case II, with different sample sizes and different levels of variability, from low to high, were solved. Risk coefficients α and λ were set to 0.9 and 1, respectively. Tables 8 and 9 report the results for this case. The quality of stochastic solutions in case II is superior to that of MVP in all variability categories and all cases. To compare the models of case I and case II, note that, although the mean of the costs in case I is slightly better than in case II, the CVaR shows better risk performance for case II solutions. Thus, there is an indication that SCND solutions achieved from case II are more robust to uncertainty, i.e., costs will be lower in case II compared to case I in risky situations.

In case II, we have also analyzed the impact of changing the risk coefficient α on the quality of stochastic solutions obtained (Table 10 and Fig. 2). Increasing the size of α increases CVaR and decreases the worst case cost, but decreasing α decreases CVaR and increases the worst case cost (Fig. 2). Also, larger values for α lead to a lower range but a larger mean. However, an increase in the mean is not that significant compared to other criteria. Fig. 2 also shows that the CVaR line is always below the worst case line; however, they are closest when α=0.95. This implies that higher values for α give more robust solutions, i.e., our model results in solutions with the lowest worst case costs. However, solutions in robust models are very conservative and may rarely happen. In any instance, our results show that our models lead to much better solutions than MVP, on average by (2.06−1.83)/2.06=11%. To decide which model to use, i.e., case I or case II and with which α, one should make a tradeoff among these criteria. For example, if all criteria have the same importance for the decision maker, all these values may be summed and, as the row ‘Sum’ in Table 10 illustrates, α=0.95 is the most suitable.

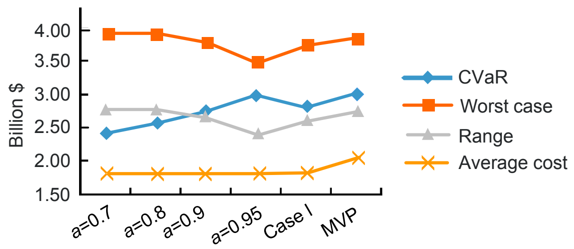

Fig. 2 Impact of increasing α on solution quality statistics (λ=1 and variability=medium)

Table 7 Different variance categories for uncertain parameters as a percentage of their means

Category	Low	Medium	High
Demand STD	10%	20%	40%
Other STD	5%	10%	20%

Table 8 Comparison of mean total cost among MVP, case I, and case II models (α=0.9, λ=1)

Mean	MVP	Case I	Case II
Low	2.00	1.83	1.81
Medium	2.06	1.82	1.83
High	2.10	1.86	1.87

Table 9 Comparison of risk (CVaR) of total cost among MVP, case I, and case II models (α=0.9, λ=1)

CVaR	MVP	Case I	Case II
Low	2.52	2.42	2.38
Medium	3.02	2.82	2.76
High	3.69	3.40	3.38

Table 10 Comparative statistics for MVP, case I, and II with changing α (λ=1 and variability=medium)

Medium	α=0.7	α=0.8	α=0.9	α=0.95	Case I	MVP
Mean	1.81	1.82	1.83	1.83	1.82	2.06
Range	2.78	2.76	2.67	2.38	2.61	2.74
Worst case	3.92	3.90	3.80	3.49	3.72	3.84
CVaR	2.41	2.57	2.76	2.99	2.82	3.02
Sum	10.91	11.06	11.06	10.69	10.97	11.66

5.4 Impact of considering contracts: case I

In this section, the impact of considering contracts in SCND models is studied through a series of experiments. To do this, we first consider case I, with a risk-neutral model. In the beginning, we consider a model which allows using all types of contracts, i.e., long-term and option contracts. Then a model with no contracts is considered, which is similar to traditional SCND models, and we continue experimentations with models that allow for long-term or option contracts alone. Table 11 shows numerical results for these experiments with random data. As a managerial insight, parts (a) and (b) of this table show that cost indexes for SCND are all unrealistic when we do not consider the impact of contracts in the model. In fact, the absence of contracts misleads us to solutions with dramatically higher mean costs and much more risk (since the range and worst case costs of SCND increase drastically as well). Thus, if we use traditional models, they may force us to omit solutions and designs which may be of interest to decision makers from aspects other than cost, or they may report a lower return on investment and lead to rejection of plans for new designs. Also, the quality of stochastic solutions in models that consider contracts is better than those of traditional models since they have smaller SDs, and thus are more stable solutions. Parts (c) and (d) of Table 11 show that long-term and option contracts have a great impact on the mean total costs and if we omit these types of contracts from our contract list, we may lose a great deal. Also, if we omit these contracts, the SD and other deviation measures, such as range, increase, which supports our assertion about the impact of contracts. Moreover, the means in parts (c) and (d) of Table 11 show that although omitting long-term contracts results in an increase in costs, the impact of option contracts is significantly greater, implying a more important role for option contracts. From the corresponding SD results we may conclude that option contracts have more of a risk hedging impact, which is an empirical prove for a famous result.

Table 11 Impact of considering different types of contracts on SCND cost structure (billion \$, case I)

(a) All contracts

	Low	Medium	High
Mean	1.83	1.82	1.86
Standard deviation	0.32	0.48	0.69
Range	1.82	2.61	4.95
Worst case	3.06	3.72	5.89

(b) No contracts

	Low	Medium	High
Mean	11.00	10.64	9.89
Standard deviation	0.42	0.86	1.65
Range	2.62	5.35	10.71
Worst case	12.26	13.80	16.36

(c) No long-term contracts

	Low	Medium	High
Mean	3.99	3.64	3.16
Standard deviation	0.29	0.53	0.92
Range	1.64	3.22	7.78
Worst case	4.80	5.46	8.75

(d) No option contracts

	Low	Medium	High
Mean	11.02	10.62	9.84
Standard deviation	0.43	0.86	1.49
Range	3.00	6.21	9.35
Worst case	12.61	14.32	15.20

5.5 Impact of considering contracts: case II

We also studied the impact of contracts in case II and observed the same pattern as in case I. The average (mean) cost of SCND is lowest when we consider all types of contracts and is highest when we do not consider any type of contracts (Fig. 3). Note that the behavior of the model when we omit option contracts and use only long-term contracts is very close to that of traditional models, i.e., models with no contracts. However, the best performance comes from the case of all contracts. Considering only option contracts and omitting long-term contracts does not result in the best case. Thus, from our experiments we can conclude that the cost of SCND in uncertain environments is lowest only if we use a portfolio of contracts, i.e., a combination of long-term and option contracts and outsourcing, which is again an empirical proof of the well-known performance of HP, as detailed in Section 1. This figure also implies that SCND solutions that consider all types of contracts have lower SDs and that the gap intensifies with increasing uncertainty. Also, option contracts show better protection properties than long-term contracts. However, as discussed above, the best performance comes from a portfolio of contracts. Risk-averse decision makers may also prefer a portfolio solution, since CVaR is lower in this case, followed by the option contract. Finally, the worst case cost is again the lowest in portfolio and option solutions, and thus these solutions are also more robust than the traditional non-contract approach.

6 Managerial insights

To clarify the impact of contracts on SCND decisions, we conducted some simulation studies. In

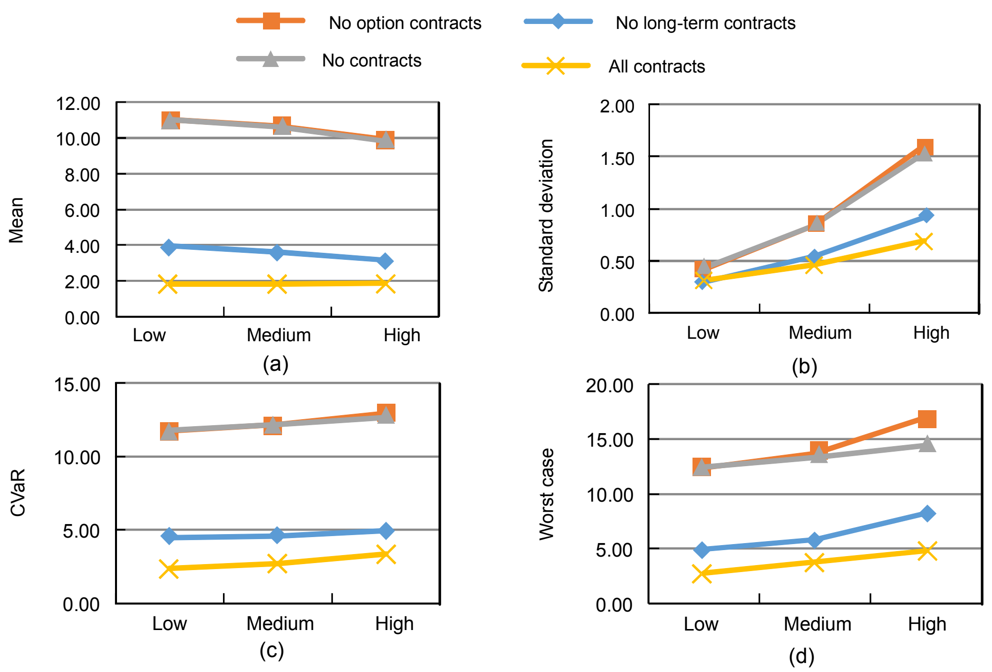

Fig. 3 Impact of considering different types of contracts on SCND cost structure (billion \$, case II: λ=1, α=0.9)
(a) Mean; (b) Standard deviation; (c) CVaR; (d) Worst case

these studies, we first ran the traditional model to obtain the corresponding decisions on the optimal structure of the supply chain (remember that these models do not support any type of contracts). Results of the simulations are depicted in Fig. 4, which has two parts illustrating the total inventory and the total shortage costs in the supply chain, respectively. Goldratt and Fox (1996) showed that the impact of inventory on an organization goes well beyond holding costs and has a direct impact on the competitive edge of an organization. Thus, as shown by the total inventory part of Fig. 4, traditional models provide us with false information on the amount of inventory in the supply chain pipeline and therefore mislead us to design our supply chains with a weak competitive edge. This figure shows also that the total inventory is highest for long-term-only contracts, followed by portfolio and option-only contracts. This result may seem natural since in long-term contracts all items contracted belong to the buyer, whereas in option contracts this is not the case. In portfolio contracts, the total inventory is in between since we have fewer long-term items in inventory than in long-term-only contracts, but more than in option-only contracts. Since manufacturers and their suppliers at the beginning of a relationship may not trust each other, they usually opt for long-term contracts only. However, as time passes and trust constructs, they may agree on option or portfolio contracts which have been shown to need fewer inventory levels. Thus, it might be wise to rent warehouses instead of buying them till the stability of the supply chain is established. The

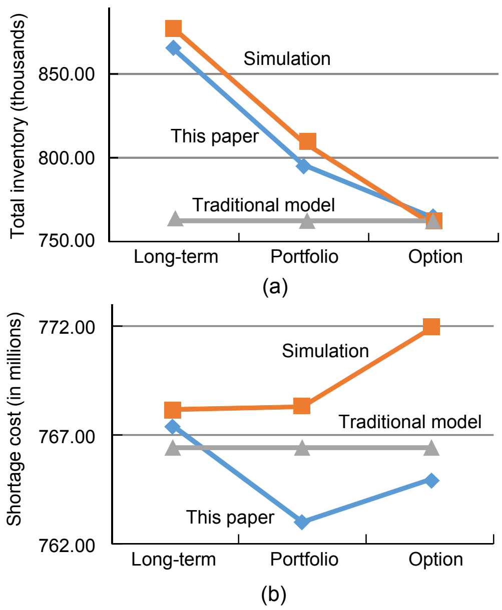

Fig. 4 Results of traditional and this paper's models
(a) Total inventory; (b) Shortage cost

shortage part of Fig. 4 shows that the total shortage cost obtained from traditional models might be totally misleading. This phenomenon is important since it implies the inability of traditional models to design responsive supply chains and that the incorporation of contracts into SCND models results in networks with a higher service level to customers. Also, it shows the superiority of portfolio contracts in fulfilling customer orders, i.e., firms that apply this type of contract will deliver a higher service level to their users.

7 Conclusions and future research directions

In this paper, we have, for the first time, considered contracts as a part of SCND models and showed that they can have a great impact on the costs of SCND. Also, our model considers many realistic assumptions such as the uncertainty of parameters. Uncertain parameters considered include the demand of customers, capacity of suppliers, manufacturers and warehouses, and transportation and spot purchasing prices. The modeling approach applied is two-stage stochastic pricing with recourse, and we have used sample average approximation to solve the model. In addition, we have developed this risk-neutral model to consider risk, with CVaR as an important risk measure. Studies of SCND considering risk measures are still rare. To show the effectiveness of the stochastic models we compared them with their deterministic counterparts and showed, through randomly generated instances, that in many ways stochastic solutions have advantages over the deterministic problem called MVP. First, the cost of SCND is lower, on average, with stochastic solutions. Second, the quality of solutions is higher in the stochastic model, that is, they are more robust since their SDs are lower. Moreover, they show a much smaller gap to the true solution. When risk was incorporated in our model, we observed that the higher the risk coefficient α the more conservative was the model. Furthermore, increasing α results in smaller values for worst case costs and the range of the solutions obtained. Finally, we analyzed the impact of incorporating contracts in the SCND model, and showed that if we do not consider contracts, especially the option contract, unrealistic costs appear in the SCND process which may mislead us to degraded solutions. The most important result here is that portfolio solutions, which consider all types of contracts, have the best performance from all cost aspects, followed by designs with option contracts. Also, considering only long-term contracts gives results similar to those of traditional models, which consider no types of contracts. This research can be extended in many ways in the future. Consideration of a multi-period model as well as inventory, and devising an appropriate solution approach for this model is one possible extension. Also, incorporating contracts between manufacturers and customers in the model will make it more realistic.

References

Badri, H., Bashiri, M., Hejazi, T.H., 2013. Integrated strategic and tactical planning in a supply chain network design with a heuristic solution method. *Comput. Oper. Res.*, **40**(4):1143-1154. [doi:10.1016/j.cor.2012.11.005]

Correia, I., Melo, T., Saldanha-da-Gama, F., 2013. Comparing classical performance measures for a multi-period, two-echelon supply chain network design problem with sizing decisions. *Comput. Ind. Eng.*, **64**(1):366-380. [doi:10.1016/j.cie.2012.11.001]

Das, K., 2011. Integrating effective flexibility measures into a strategic supply chain planning model. *Eur. J. Oper. Res.*, **211**(1):170-183. [doi:10.1016/j.ejor.2010.12.006]

Feng, Y., Martel, A., D'Amours, S., *et al.*, 2013. Coordinated contract decisions in a make-to-order manufacturing supply chain: a stochastic programming approach. *Prod. Oper. Manag.*, **22**(3):642-660. [doi:10.1111/j.1937-5956.2012.01385.x]

Georgiadis, M.C., Tsiakis, P., Longinidis, P., *et al.*, 2011. Optimal design of supply chain networks under uncertain transient demand variations. *Omega*, **39**(3):254-272. [doi:10.1016/j.omega.2010.07.002]

Goetschalckx, M., Huang, E., Mital, P., 2013. Trading off supply chain risk and efficiency through supply chain design. *Procedia Comput. Sci.*, **16**:658-667. [doi:10.1016/j.procs.2013.01.069]

Goldratt, E.M., Fox, R.E., 1996. The Race. North River Press.

Kamath, K.R., Pakkala, T.P.M., 2002. A Bayesian approach to a dynamic inventory model under an unknown demand distribution. *Comput. Oper. Res.*, **29**(4):403-422. [doi:10.1016/S0305-0548(00)00075-7]

Melo, M.T., Nickel, S., Saldanha-da-Gama, F., 2009. Facility location and supply chain management—a review. *Eur. J. Oper. Res.*, **196**(2):401-412. [doi:10.1016/j.ejor.2008.05.007]

Nagurney, A., 2010. Optimal supply chain network design and redesign at minimal total cost and with demand satisfaction. *Int. J. Prod. Econ.*, **128**(1):200-208. [doi:10.1016/j.ijpe.2010.07.020]

Noyan, N., 2012. Risk-averse two-stage stochastic programming with an application to disaster management. *Comput. Oper. Res.*, **39**(3):541-559. [doi:10.1016/j.cor.2011.03.017]

Paksoy, T., Özceylan, E., Weber, G.W., 2013. Profit oriented supply chain network optimization. *Cent. Eur. J. Oper. Res.*, **21**(2):455-478. [doi:10.1007/s10100-012-0240-0]

Pan, F., Nagi, R., 2010. Robust supply chain design under uncertain demand in agile manufacturing. *Comput. Oper. Res.*, **37**(4):668-683. [doi:10.1016/j.cor.2009.06.017]

Rajgopal, J., Wang, Z., Schaefer, A.J., *et al.*, 2011. Integrated design and operation of remnant inventory supply chains under uncertainty. *Eur. J. Oper. Res.*, **214**(2):358-364. [doi:10.1016/j.ejor.2011.04.039]

Rockafellar, R.T., Uryasev, S., 2000. Optimization of conditional value-at-risk. *J. Risk*, **2**(3):21-42.

Santoso, T., 2003. A Comprehensive Model and Efficient Solution Algorithm for the Design of Global Supply Chains Under Uncertainty. PhD Thesis, Georgia Institute of Technology, USA.

Santoso, T., Ahmed, S., Goetschalckx, M., *et al.*, 2005. A stochastic programming approach for supply chain network design under uncertainty. *Eur. J. Oper. Res.*, **167**(1):96-115. [doi:10.1016/j.ejor.2004.01.046]

Shapiro, A., Homem-de-Mello, T., 1998. A simulation-based approach to two-stage stochastic programming with recourse. *Math. Program.*, **81**(3):301-325. [doi:10.1007/BF01580086]

Simchi-Levi, D., Kaminsky, P., Simchi-Levi, E., 2007. Designing and Managing the Supply Chain: Concepts, Strategies, and Case Studies (3rd Ed.). McGraw-Hill International Edition, USA.

Tabrizi, B.H., Razmi, J., 2013. Introducing a mixed-integer non-linear fuzzy model for risk management in designing supply chain networks. *J. Manuf. Syst.*, **32**(2):295-307. [doi:10.1016/j.jmsy.2012.12.001]

Thanh, P.N., Péton, O., Bostel, N., 2010. A linear relaxation-based heuristic approach for logistics network design. *Comput. Ind. Eng.*, **59**(4):964-975. [doi:10.1016/j.cie.2010.09.007]

Tiwari, M.K., Raghavendra, N., Agrawal, S., *et al.*, 2010. A hybrid taguchi-immune approach to optimize an integrated supply chain design problem with multiple shipping. *Eur. J. Oper. Res.*, **203**(1):95-106. [doi:10.1016/j.ejor.2009.07.004]

Wang, W., 2007. Sample Average Approximation of Risk-Averse Stochastic Programs. PhD Thesis, Georgia Institute of Technology, USA.

Wever, M., Wognum, P.M., Trienekens, J.H., *et al.*, 2012. Supply chain-wide consequences of transaction risks and their contractual solutions: towards an extended transaction cost economics framework. *J. Supply Chain Manag.*, **48**(1):73-91. [doi:10.1111/j.1745-493X.2011.03253.x]

Xu, N., Nozick, L., 2009. Modeling supplier selection and the use of option contracts for global supply chain design. *Comput. Oper. Res.*, **36**(10):2786-2800. [doi:10.1016/j.cor.2008.12.013]